Skills Review Handbook

Problem Solving

One of your primary goals in mathematics should be to become a good problem solver. It will help to approach every problem with an organized plan.

STEP 1 **Understand the problem.**
Read the problem carefully. Decide what information you are given and what you need to find. Check whether some of the information given is unnecessary, or whether you need additional information to solve the problem. Supply missing facts, if possible.

STEP 2 **Make a plan to solve the problem.**
Choose a strategy. (You can get ideas from the list on page 784.) Choose the correct operations. Decide if you will use a tool such as a calculator, a graph, or a spreadsheet.

STEP 3 **Carry out the plan to solve the problem.**
Use the strategy and any tools you have chosen. Estimate before you calculate, if possible. Do any necessary calculations. Answer the question that the problem asks.

STEP 4 **Check to see if your answer is reasonable.**
Reread the problem. See if your answer agrees with the given information and with your estimate if you calculated one.

EXAMPLE In how many ways can two students be chosen to receive an award from a list of ten nominees?

Solution

1. You are given the number of nominated students and the number of students to be chosen. You need to determine how many ways there are to do this.
2. Some strategies to consider are the following: make an organized list, look for a pattern, and solve a simpler problem.
3. Consider the problem when fewer students are nominated. Look for a pattern.

Number of students	2: A, B	3: A, B, C	4: A, B, C, D	5: A, B, C, D, E
Number of ways to choose 2 students	**1**	**3**: AB; AC; BC	**6**: AB; AC; AD; BC; BD; CD	**10**: AB; AC; AD; AE; BC; BD; BE; CD; CE; DE
Pattern	**1**	**1 + 2**	**1 + 2 + 3**	**1 + 2 + 3 + 4**

Continue the pattern to find the number of ways to choose 2 out of 10 students.

▶ There are 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45 ways to choose two students from a list of ten to receive an award.

4. You can check your solution by using an organized list.

Step 2 of the problem-solving plan on the previous page asks you to select a strategy. When you solve a problem, you may want to consider these strategies.

PROBLEM SOLVING STRATEGIES

• **Guess, check, and revise.**	Use when you do not seem to have enough information.
• **Draw a diagram or a graph.**	Use when words describe a visual representation.
• **Make a table or an organized list.**	Use when you have data or choices to organize.
• **Use an equation or a formula.**	Use when you know a relationship between quantities.
• **Use a proportion.**	Use when you know that two ratios are equal.
• **Look for a pattern.**	Use when you can examine several cases.
• **Break the problem into simpler parts.**	Use when you have a multi-step problem.
• **Solve a simpler problem.**	Use when smaller numbers make the problem easier to understand.
• **Work backwards.**	Use when you are looking for a fact leading to a known result.
• **Act out the situation.**	Use when visualizing the problem is helpful.

PRACTICE

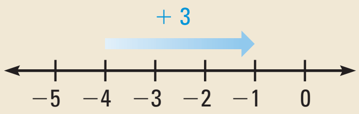
Solve, if possible.

1. During the month of May, Rosa made deposits of $128.50 and $165.19 into her checking account. She wrote checks for $55.12, $25, and $83.98. If her account balance at the end of May was $327.05, what was her balance at the beginning of May? **$197.46**
2. You make 20 silk flower arrangements and plan to sell them at a craft show. Each flower arrangement costs $12 in materials, and your booth at the craft show costs $30. If you sell the arrangements for $24 each, how many must you sell to make at least $100 profit? **at least 16 flower arrangements**
3. A store sells sweatshirts in small, medium, large, and extra large. A customer can choose a long sleeve sweatshirt or sweatshirt with a hood There are four choices of colors: white, blue, gray, and black. How many different kinds of sweatshirts are available at the store? **32 kinds**
4. If 4.26 lb of chicken costs $6.77, what would 3.75 lb of chicken cost? **$5.96**
5. Roger bought some 33¢ stamps and some 20¢ stamps, and spent $4.50. How many of each type of stamp did he buy? **10 33¢ stamps and 6 20¢ stamps**
6. Anita, Betty, Carla, and Dominique are competing in a race. In how many different orders can the four athletes cross the finish line? **24 different orders**
7. Stan and Margaret Wu are planning to paint their living room walls. The living room is 18 ft long and 12 ft wide, and the walls are 10 ft high. If a can of paint costs $8.75, what will it cost to paint the living room walls? **not enough information (You need to know how much area a can of paint will cover.)**

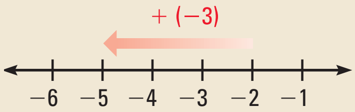
Positive and Negative Numbers

You can use a number line to find the sum of two numbers. Add a positive number by moving to the right. Add a negative number by moving to the left.

EXAMPLES



a. To find the sum $-4 + 3$, start at -4. Go 3 units to the right. End at -1. So, $-4 + 3 = -1$.



b. To find the sum $-2 + (-3)$, start at -2. Go 3 units to the left. End at -5. So, $-2 + (-3) = -5$.

To subtract a number, add its opposite.

EXAMPLES

a. $7 - 9 = 7 + (-9) = -2$

b. $2.1 - (-5.5) = 2.1 + 5.5 = 7.6$

When you multiply or divide numbers, use these rules.

- The product or quotient of two numbers with the same sign is positive.
- The product or quotient of two numbers with opposite signs is negative.

EXAMPLES

a. $-2(4) = -8$

b. $-\frac{1}{2}\left(-\frac{2}{3}\right) = \frac{1 \cdot 2}{2 \cdot 3} = \frac{1}{3}$

c. $18 \div (-9) = -2$

d. $\frac{-3.5}{-7} = 0.5$

Practice

Add, subtract, multiply, or divide.

1. $0 + (-3.3)$ **-3.3**

2. $-7 + (-2)$ **-9**

3. $5.2 + (-2.5)$ **2.7**

4. $\frac{1}{2} + \left(-\frac{1}{2}\right)$ **0**

5. $-\frac{1}{3} + \left(-\frac{3}{4}\right)$ **$-1\frac{1}{12}$**

6. $-75 + 48$ **-27**

7. $-1 - (-1)$ **0**

8. $\frac{1}{2} - \left(-\frac{1}{2}\right)$ **1**

9. $8 - 9$ **-1**

10. $8 - (-9)$ **17**

11. $-4.8 - 3.2$ **-8**

12. $24 - 67$ **-43**

13. $(-3)(-3)$ **9**

14. $-8 \cdot 9$ **-72**

15. $7 \cdot 0 \cdot (-12)$ **0**

16. $(-4)(-4)(-4)$ **-64**

17. $6\left(-\frac{1}{6}\right)$ **-1**

18. $-\frac{1}{4} \cdot 20$ **-5**

19. $0.65(-0.24)$ **-0.156**

20. $-120(-46)$ **5520**

21. $-33 \div 11$ **-3**

22. $0 \div (-1)$ **0**

23. $45 \div (-5)$ **-9**

24. $\frac{-108}{8}$ **-13.5**

25. $7.2 \div (-2.5)$ **-2.88**

26. $\frac{-128}{-16}$ **8**

27. $-\frac{1}{3} \div (-10)$ **$\frac{1}{30}$**

28. $-\frac{7}{12} \div \frac{2}{3}$ **$-\frac{7}{8}$**

EVALUATING EXPRESSIONS

To evaluate a **numerical expression** involving more than one operation, follow the *order of operations*.

- First do operations that occur within grouping symbols.
- Then evaluate *powers* (expressions with exponents, such as $3^2 = 3 \cdot 3$).
- Then do multiplications and divisions in order from left to right.
- Finally, do additions and subtractions in order from left to right.

EXAMPLES Evaluate the expression.

a. $2 - (4 - 7)^2 \div (-6)$ **b.** $\frac{(2 + 4)^2}{2 + 4^2}$

SOLUTION **a.** $2 - (4 - 7)^2 \div (-6) = 2 - (-3)^2 \div (-6) = 2 - 9 \div (-6) = 2 - (-1.5) = 3.5$

b. A fraction bar acts as a grouping symbol. Simplify the numerator and the denominator. Then divide.

$$\frac{(2 + 4)^2}{2 + 4^2} = \frac{6^2}{2 + 16} = \frac{36}{18} = 2$$

To evaluate a **variable expression,** substitute values for the variables, and simplify the resulting numerical expression using the order of operations.

EXAMPLES Evaluate the expression when $x = 3$.

a. $x(2x - 8)$ **b.** $\frac{12}{x} + \frac{1}{2}x$

SOLUTION **a.** $x(2x - 8) = \mathbf{3}(2 \cdot \mathbf{3} - 8) = 3(6 - 8) = 3(-2) = -6$

b. $\frac{12}{x} + \frac{1}{2}x = \frac{12}{\mathbf{3}} + \frac{1}{2} \cdot \mathbf{3} = 4 + 1.5 = 5.5$

PRACTICE

Evaluate the expression.

1. $8^2 + (-6)^2$ **100**
2. $-4 \cdot 5 - 8 \div 2$ **−24**
3. $-7 + 2^3 - 9$ **−8**
4. $18 \div [(4 - 7) + 5]$ **9**
5. $9(7 - 2)^2$ **225**
6. $\frac{3}{4} \cdot 24 + 4^2 - 1$ **33**
7. $156 - 3^2 \cdot 5 - 8^2$ **47**
8. $4.2 \div (0.7 \div 0.1)$ **0.6**
9. $17^2 - 15^2$ **64**
10. $\frac{5 + 7 \cdot 3}{6 + 7}$ **2**
11. $\frac{2 - 9}{8 - 8^2}$ $\mathbf{\frac{1}{8}}$
12. $[(1 - 7)^2 + 4] \div 8$ **5**

Evaluate the expression when $x = -4$ and $y = 3$.

13. $-3x^2$ **−48**
14. $(-3x)^2$ **144**
15. $\frac{x + 2}{x - 2}$ $\mathbf{\frac{1}{3}}$
16. $\frac{1}{2}x^3$ **−32**
17. $x(x + 7)$ **−12**
18. $20 - \frac{16}{x}$ **24**
19. $x^2 - x + 5$ **25**
20. $(x + 3)(x - 3)$ **7**
21. $xy \div (x + y)$ **12**
22. $5(2y - x)$ **50**
23. $-2x^2 + y^2$ **−23**
24. $-2(x + 4y)^2$ **−128**

THE DISTRIBUTIVE PROPERTY

Here are four forms of the **distributive property**:

$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$

$a(b - c) = ab - ac$ and $(b - c)a = ba - ca$

EXAMPLES

a. $x(x + 4) = x \cdot x + 4x = x^2 + 4x$

b. $-(2a - 3b) = -1(2a - 3b) = -1 \cdot 2a - (-1)(3b) = -2a + 3b$

c. $5 - 3(n - 2) = 5 - (3 \cdot n - 3 \cdot 2) = 5 - 3n - (-6) = 5 - 3n + 6 = 11 - 3n$

When an expression is written as a sum, the parts that are added are the **terms** of the expression. **Like terms** are constant terms or terms that have the same variable raised to the same power. The distributive property allows you to combine like terms that have variables by adding coefficients. An expression is *simplified* if it has no grouping symbols and if all the like terms have been combined.

EXAMPLES

a. $-4x + 7x = (-4 + 7)x = 3x$

b. $2(x + y) - x(4 - y) = 2x + 2y - 4x + xy = (2 - 4)x + 2y + xy = -2x + 2y + xy$

c. $4x^2 + 5x - 7 + 2 - 3x^2 = (4 - 3)x^2 + 5x + (-7 + 2) = x^2 + 5x - 5$

PRACTICE

Use the distributive property to rewrite the expression without parentheses.

1. $2(a + 4)$ **2a + 8**

2. $(2k + 1)7$ **14k + 7**

3. $-(-3x + 2)$ **3x − 2**

4. $(7 - 2z)z$ **$7z - 2z^2$**

5. $y(y - 9)$ **$y^2 - 9y$**

6. $(j - 1)(-3)$ **−3j + 3**

7. $\frac{1}{2}(8n - 14)$ **4n − 7**

8. $(3k + 5)(-k)$ **$-3k^2 - 5k$**

9. $4b(b + 2)$ **$4b^2 + 8b$**

10. $(10 + c)d$ **10d + cd**

11. $8x(2x - 9y)$ **$16x^2 - 72xy$**

12. $(4t - q)(-3t)$ **$-12t^2 + 3tq$**

13. $2r(s + t)$ **2rs + 2rt**

14. $(b + c - 1)6$ **6b + 6c − 6**

15. $7(-x^2 + 3x - 2)$ **$-7x^2 + 21x - 14$**

16. $-\frac{2}{3}x(6x + 9y - 12)$ **$-4x^2 - 6xy + 8x$**

Simplify the expression.

17. $-m + 4 + 7m$ **6m + 4**

18. $6x - 9x + x$ **−2x**

19. $3z + 6 - 3z - 7$ **−1**

20. $\frac{1}{5}d + \frac{2}{7}d$ **$\frac{17}{35}d$**

21. $18g^3 + 9g^2 + g^3$ **$19g^3 + 9g^2$**

22. $3.1 + 7.5y - 8y$ **3.1 − 0.5y**

23. $6xy + 2x - 3y - 5xy$ **xy + 2x − 3y**

24. $2.5(4z - 18) + 12$ **10z − 33**

25. $6h - 3h(h + 1)$ **$3h - 3h^2$**

26. $3 - (2x - 7)$ **10 − 2x**

27. $8 + 3(y - 4)$ **3y − 4**

28. $9k - 2(3k - 5) - 10$ **3k**

29. $5(r + 1) - (r - 3)$ **4r + 8**

30. $x(2x - 6) + x^2$ **$3x^2 - 6x$**

31. $2(n + 8) + 3n(n - 5)$ **$3n^2 - 13n + 16$**

RECIPROCALS

The product of a nonzero number and its **reciprocal** is 1.

The reciprocal of a is $\frac{1}{a}$, and the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$. Zero has no reciprocal.

EXAMPLES $-\frac{3}{7}$ and $-\frac{7}{3}$ are reciprocals because $-\frac{3}{7}\left(-\frac{7}{3}\right) = 1$.

6 and $\frac{1}{6}$ are reciprocals because $6 \cdot \frac{1}{6} = 1$.

PRACTICE

Find the reciprocal of the number.

1. 12 $\frac{1}{12}$ **2.** -99 $-\frac{1}{99}$ **3.** $\frac{1}{4}$ 4 **4.** $-\frac{5}{2}$ $-\frac{2}{5}$ **5.** $-\frac{1}{10}$ -10

6. 1 $\frac{1}{1}$ **7.** $\frac{6}{13}$ $\frac{13}{6}$ **8.** -1 -1 **9.** 0.2 5 **10.** -0.75 $-\frac{4}{3}$

RATIOS

If a and b are two quantities measured in the *same* units, then the **ratio of a to b** is $\frac{a}{b}$, usually written in simplest form. The ratio of a to b can also be written as $a : b$. Because a ratio is a quotient, its denominator cannot be zero.

EXAMPLE $\frac{9 \text{ inches}}{2 \text{ feet}} = \frac{9 \text{ inches}}{2 \cdot 12 \text{ inches}} = \frac{9}{24} = \frac{3}{8}$

Notice that to simplify a ratio with different units, you rewrite the ratio so that the numerator and denominator have the same units. Then simplify if possible.

EXAMPLE Suppose there are 18 boys in a class of 30 students. To find the ratio of girls to boys, compute the number of girls, $30 - 18 = 12$. Then $\frac{\text{girls}}{\text{boys}} = \frac{12}{18} = \frac{2}{3}$.

PRACTICE

Simplify the ratio.

1. $\frac{48 \text{ miles}}{120 \text{ miles}}$ $\frac{2}{5}$ **2.** $\frac{72 \text{ cm}}{1.5 \text{ m}}$ $\frac{12}{25}$ **3.** $\frac{9 \text{ yards}}{15 \text{ feet}}$ $\frac{9}{5}$ **4.** $\frac{12 \text{ ounces}}{2 \text{ pounds}}$ $\frac{3}{8}$

5. $\frac{3 \text{ ft}}{36 \text{ in.}}$ $\frac{1}{1}$ **6.** $\frac{980 \text{ g}}{2 \text{ kg}}$ $\frac{49}{100}$ **7.** $\frac{40 \text{ km}}{500 \text{ m}}$ $\frac{80}{1}$ **8.** $\frac{5 \text{ mi}}{6200 \text{ ft}}$ $\frac{132}{31}$

Find the ratio of girls to boys in a class, given the number of boys and the total number of students.

9. 15 boys, 28 students $\frac{13}{15}$ **10.** 12 boys, 27 students $\frac{5}{4}$ **11.** 12 boys, 20 students $\frac{2}{3}$

SOLVING LINEAR EQUATIONS (SINGLE-STEP)

The equations $\frac{1}{2}x = 4$ and $3t - 1 = 4t$ are examples of **linear** equations.

When the variable in a single-variable equation is replaced by a number and the resulting statement is true, the number is a **solution** of the equation. You can *solve an equation* by writing an *equivalent* equation that has the variable alone on one side. One way to do this is to add or subtract the same number *from each side* of the equation.

EXAMPLES Solve the equation.

a. $x + 6 = -2$

b. $y - 7 = 3$

SOLUTION

a.

$x + 6 = -2$

$x + 6 - \mathbf{6} = -2 - \mathbf{6}$ Subtract 6 from each side.

$x = -8$ Simplify.

b.

$y - 7 = 3$

$y - 7 + \mathbf{7} = 3 + \mathbf{7}$ Add 7 to each side.

$y = 10$ Simplify.

Another way to solve a linear equation is to multiply or divide each side by the same nonzero number. Notice the use of reciprocals in the example below.

EXAMPLE Solve the equation $8 = \frac{4}{3}a$.

SOLUTION

$8 = \frac{4}{3}a$

$\frac{3}{4} \cdot 8 = \frac{3}{4} \cdot \frac{4}{3}a$ Multiply each side by the reciprocal.

$6 = a$ Simplify.

Check your solution by substituting it in the original equation.

PRACTICE

Solve the equation.

1. $x + 12 = 25$ 13

2. $k - 6 = 0$ 6

3. $-36 = -9s$ 4

4. $\frac{1}{5}n = 5$ 25

5. $-32h = 4$ $-\frac{1}{8}$

6. $4.6 + z = 3.6$ -1

7. $-\frac{3}{4}d = 24$ -32

8. $0.02v = 8$ 400

9. $w - 5 = -13$ -8

10. $4z = 132$ 33

11. $-6 = c + 4$ -10

12. $-\frac{4}{7}p = -8$ 14

13. $\frac{2}{3}y = 7$ $\frac{21}{2}$

14. $37 = r - (-9)$ 28

15. $\frac{1}{2}x = -40$ -80

16. $-m = 5$ -5

17. $\frac{n}{3} = 6$ 18

18. $330 = -15f$ -22

19. $y + 7 = -16$ -23

20. $-4.2z = 42$ -10

21. $t - \frac{1}{8} = \frac{5}{8}$ $\frac{3}{4}$

22. $\frac{9}{2}x = -1$ $-\frac{2}{9}$

23. $120 = -120b$ -1

24. $0 = 6.4k$ 0

SOLVING LINEAR EQUATIONS (MULTI-STEP)

Solving a linear equation may require several steps. You may need to simplify one or both sides of the equation, use the distributive property, or collect variable terms on one side of the equation.

EXAMPLES Solve the equation.

a. $\frac{1}{5}x + 7 = 3$

b. $5 - 2(r + 6) = 1$

SOLUTION

a. $\frac{1}{5}x + 7 = 3$

$\frac{1}{5}x + 7 - 7 = 3 - 7$ Subtract 7 from each side.

$\frac{1}{5}x = -4$ Simplify.

$5 \cdot \frac{1}{5}x = 5(-4)$ Multiply by the reciprocal.

$x = -20$ Simplify.

CHECK

$\frac{1}{5}(-20) + 7 = 3$ ✓

b. $5 - 2(r + 6) = 1$

$5 - 2r - 12 = 1$ Distributive property

$-2r - 7 = 1$ Simplify.

$-2r - 7 + 7 = 1 + 7$ Add 7 to each side.

$-2r = 8$ Simplify.

$\frac{-2r}{-2} = \frac{8}{-2}$ Divide each side by −2.

$r = -4$ Simplify.

PRACTICE

Solve the equation.

1. $3y - 4 = 20$ **8**
2. $\frac{c}{7} + 2 = 1$ **−7**
3. $6 - \frac{3a}{2} = -6$ **8**
4. $3r - (2r + 1) = 21$ **22**
5. $5(z + 3) = 12$ **−0.6, or $-\frac{3}{5}$**
6. $44 = 5g - 8 - g$ **13**
7. $75 + 7x = 2x$ **−15**
8. $14r + 81 = -r$ **−5.4, or $-\frac{27}{5}$**
9. $3n - 1 = 5n - 9$ **4**
10. $12r - 5 = 7r$ **1**
11. $4 - 6p = 2p - 3$ **$\frac{7}{8}$**
12. $7(b - 3) = 8b + 2$ **−23**
13. $60c - 54(c - 2) = 0$ **−18**
14. $22d - (6 + 2d) = 4$ **$\frac{1}{2}$**
15. $s - (-4s + 2) = 13$ **3**
16. $-\frac{1}{2}(16 - 2h) = 11$ **19**
17. $1 + j = 2(2j + 1)$ **$-\frac{1}{3}$**
18. $4x + 2(x - 3) = 0$ **1**
19. $\frac{1}{4}y + 27 = 41$ **56**
20. $\frac{3 + m}{2} = 5$ **7**
21. $\frac{x + (-2)}{2} = -6$ **−10**
22. $\frac{8 + x}{2} = 10$ **12**
23. $2 \cdot 3.14 \cdot r = 157$ **25**
24. $\frac{1}{2} \cdot 9 \cdot h = 94.5$ **21**
25. $12 \cdot b \cdot 13 = 338$ **$\frac{13}{6}$**
26. $4(t - 7) + 6 = 30$ **13**
27. $7y - 84 = 2y + 61$ **29**
28. $85 = \frac{1}{2}(226 - x)$ **56**
29. $104 = \frac{1}{2}[(360 - x) - x]$ **76**
30. $18(x + 18) = 21^2$ **6.5**
31. $18^2 = 15(x + 15)$ **6.6**
32. $12 - 23c = 7(9 - c)$ **$-\frac{51}{16}$**
33. $2.7(z - 7) + 6 = 2.1(3z + 1)$ **$-\frac{25}{6}$**
34. $7(4h + 1) - 2(2h - 3) = -23$ **−1.5**
35. $4(5n + 7) - 3n = 3(4n - 9)$ **−11**
36. $4.7(2f - 0.5) = -6(1.6f - 8.3f)$ **$-\frac{47}{616}$**

SOLVING INEQUALITIES

You can solve a linear inequality in one variable in much the same way you solve a linear equation in one variable.

EXAMPLES Solve the inequality.

a. $x + 18 > 24$

$x + 18 - 18 > 24 - 18$

$x > 6$

b. $x + (2x - 5) > x + 3$

$3x - 5 > x + 3$ **Simplify.**

$3x - 5 + 5 > x + 3 + 5$ **Add 5 to each side.**

$3x > x + 8$ **Simplify.**

$-x + 3x > -x + x + 8$ **Add $-x$ to each side.**

$2x > 8$ **Combine like terms.**

$\frac{2x}{2} > \frac{8}{2}$ **Divide each side by 2.**

$x > 4$ **Simplify.**

A **solution of an inequality** is a number that produces a true statement when it is substituted for the variable in the inequality.

EXAMPLE Decide whether 3 is a solution of the inequality $3x - 8 < 10$.

SOLUTION $3(3) - 8 < 10$ **Substitute.**

$1 < 10$ **Simplify.**

So, 3 is a solution of the inequality.

PRACTICE

Solve the inequality.

1. $24 + 32 > x$ **$x < 56$**

2. $16 + x > 21$ **$x > 5$**

3. $x + 7.8 > 15.1$ **$x > 7.3$**

4. $x < 125 + 175$ **$x < 300$**

5. $x + \frac{7}{2} < \frac{11}{2}$ **$x < 2$**

6. $55 < 5 + x$ **$x > 50$**

7. $x + 3x > 2x + 6$ **$x > 3$**

8. $(x + 4) + (x + 6) > 3x - 1$ **$x < 11$**

9. $(2x - 1) + (x + 3) > 18 - x$ **$x > 4$**

Check whether the given number is a solution of the inequality.

10. $m + 12 > 30$; 16 **no**

11. $n - 3 < 6$; 2 **yes**

12. $5 + 2p > 10$; 3 **yes**

13. $3r - 4 < 0$; 0.5 **yes**

14. $10s - 2 > 40$; 4 **no**

15. $6t - 2 < 4t$; 3 **no**

16. $7u + 7 < 38$; 5 **no**

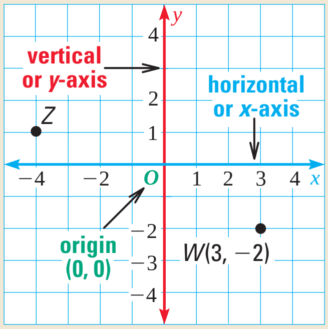
17. $8(w - 3) > 95$; 15 **yes**

18. $6.2x - 3.7 < -14$; -2.2 **yes**

19. Name three solutions of the inequality $(2x - 3) + (x + 5) > x + 8$. Is 3 a solution? Explain. **Examples: 4, 4.5, and 10. No; when $x = 3$, $(2x - 3) + (x + 5) = x + 8$.**

PLOTTING POINTS

A **coordinate plane** is formed by a horizontal ***x*-axis** and a vertical ***y*-axis** that intersect at the **origin,** forming right angles. Each point in a coordinate plane corresponds to an **ordered pair** of real numbers. Point $W(3, -2)$, shown on the graph, has an ***x*-coordinate** of 3 and a ***y*-coordinate** of -2.



EXAMPLE Use the graph to name the coordinates of point Z.

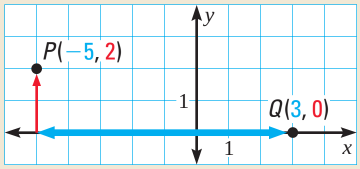
SOLUTION

The x-coordinate of Z is -4 and the y-coordinate is 1.

So, the ordered pair corresponding to Z is $(-4, 1)$.

EXAMPLES Plot each point in a coordinate plane.

a. $P(-5, 2)$ **b.** $Q(3, 0)$



SOLUTION

a. To plot the point $P(-5, 2)$, start at the origin. Move 5 units to the left and 2 units up.

b. To plot the point $Q(3, 0)$, start at the origin. Move 3 units to the right and 0 units up.

PRACTICE

Give the coordinates of each of the following points.

1. A **(−1, 0)** **2.** B **(2, 3)**

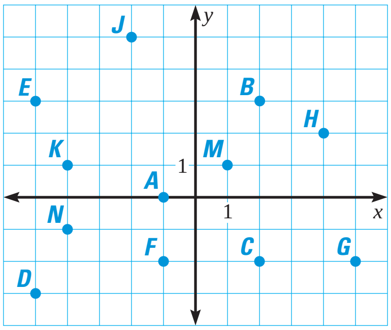
3. C **(2, −2)** **4.** D **(−5, −3)**

5. E **(−5, 3)** **6.** F **(−1, −2)**

7. G **(5, −2)** **8.** H **(4, 2)**

9. J **(−2, 5)** **10.** K **(−4, 1)**

11. M **(1, 1)** **12.** N **(−4, −1)**



Plot each point in a coordinate plane. **13–28. See margin.**

13. $A(4, 6)$ **14.** $B(-3, 2)$ **15.** $C(2, -3)$ **16.** $D(0, -1)$

17. $E(-6, -7)$ **18.** $F(5, 5)$ **19.** $G(1, 0)$ **20.** $H\left(\frac{5}{2}, \frac{5}{2}\right)$

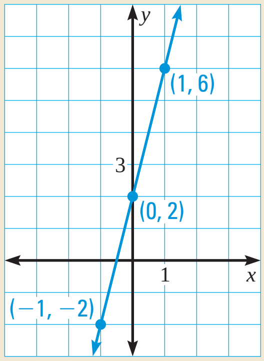
21. $J(0, 2.5)$ **22.** $K\left(\frac{5}{2}, -\frac{9}{2}\right)$ **23.** $L(-3, -2)$ **24.** $M(-2, 3)$

25. $N(-4, 6)$ **26.** $P\left(4, -\frac{3}{2}\right)$ **27.** $Q(-5, 0)$ **28.** $R\left(-\frac{9}{2}, -3\right)$

Linear Equations and Their Graphs

A **solution** of an equation in two variables x and y is an ordered pair (x, y) that makes the equation true. Equations like $2x + 3y = -6$, $y = 5x - 1$, and $y = 3$ are **linear equations**. Their graphs are lines.

EXAMPLE Graph the equation $y - 4x = 2$.

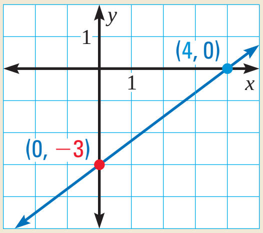


Solution You can use a table of values to graph the equation $y - 4x = 2$. Rewrite the equation in *function form* by solving for y: $y - 4x = 2$, so $y = 4x + 2$. Choose a few values of x. Substitute to find the corresponding y-value.

x	$y = 4x + 2$	(x, y)
-1	$y = 4(-1) + 2 = -2$	$(-1, -2)$
0	$y = 4 \cdot 0 + 2 = 2$	$(0, 2)$
1	$y = 4 \cdot 1 + 2 = 6$	$(1, 6)$

Plot the points in the table. Draw a line through the points.

EXAMPLE Graph the equation $3x - 4y = 12$.



Solution You can quickly draw a graph of an equation such as $3x - 4y = 12$ by using the *intercepts*. The **x-intercept** is the x-coordinate of a point where the graph crosses the x-axis. The **y-intercept** is the y-coordinate of a point where the graph crosses the y-axis.

Substitute 0 for x: $3 \cdot 0 - 4y = 12$; $y = -3$, and so the y-intercept is -3.

Substitute 0 for y: $3x - 4 \cdot 0 = 12$; $x = 4$, and so the x-intercept is 4.

Now you can graph the equation $3x - 4y = 12$ by plotting the points $(4, 0)$ and $(0, -3)$ and then drawing a line through the points.

Practice

Use a table of values to graph the equation. 1–8. See margin.

1. $y = -2x + 3$ **2.** $y = 3x - 5$ **3.** $y = \frac{1}{3}x - 2$ **4.** $y = 2.5 + 1.5x$

5. $y = -\frac{3}{4}x$ **6.** $4x + y = -8$ **7.** $x - 3y = 6$ **8.** $y = \frac{3}{2}(x + 1)$

Use the x-intercept and the y-intercept to graph the equation. Label the points where the line crosses the coordinate axes. 9–16. See margin.

9. $x + 5y = -10$ **10.** $-5x + 6y = 30$ **11.** $3x - 8y = -48$ **12.** $y = -(2x - 1)$

13. $y = 4x + 1$ **14.** $y = -x - 2$ **15.** $y = 5 - \frac{1}{2}x$ **16.** $y = 0.75x + 1.25$

SLOPE-INTERCEPT FORM

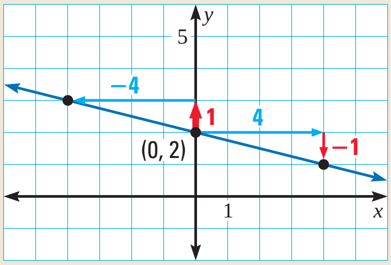
Another way to draw the graph of a linear equation is to use the slope and the y-intercept. Recall that the slope of a nonvertical line is $m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$.

The linear equation $y = mx + b$ is written in **slope-intercept form**. The slope of the line is m and the y-intercept is b.

EXAMPLE Graph the equation $\frac{1}{2}x + 2y = 4$.

SOLUTION To graph the equation $\frac{1}{2}x + 2y = 4$, write the equation in slope-intercept form:

$$2y = -\frac{1}{2}x + 4;\ y = -\frac{1}{4}x + 2.$$



The slope m is $-\frac{1}{4}$, and the y-intercept b is 2.

Plot the point (0, 2).

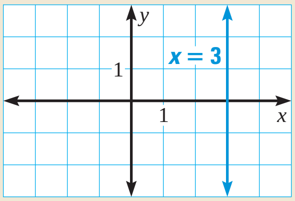
Draw a *slope triangle* to locate a second point on the line:

$$m = -\frac{1}{4} = \frac{\text{rise}}{\text{run}}.$$

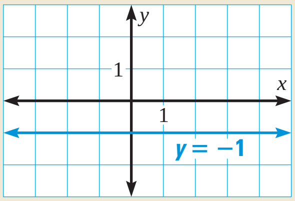
Draw a line through the two points.

EXAMPLES Graph $x = 3$ and $y = -1$.

SOLUTION



The graph of $x = 3$ is a vertical line.



The graph of $y = -1$ is a horizontal line.

PRACTICE

Use the slope and the *y*-intercept to graph the equation. **1–13. See margin.**

1. $y = \frac{1}{2}x + 4$ **2.** $y = \frac{1}{2}x - 4$ **3.** $y = -\frac{1}{2}x - 4$ **4.** $y = -\frac{1}{2}x + 4$

5. $x - y - 3 = 0$ **6.** $2x + 3y = -9$ **7.** $3x - y = 0$ **8.** $x + 2y = 5$

9. $-4x = 8$ **10.** $0.25y = 3$ **11.** $-y - 3x = 4$ **12.** $2x = 6 - 3y$

13. Graph the equation $x = -2$. Explain why the graph has no slope and no y-intercept.

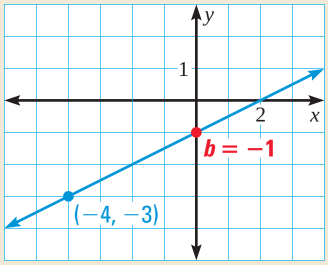
14. Graph the equation $y = 3$. Find the slope of the graph. Name three different ordered pairs that are solutions of the graph. **See margin for graph.**
The slope is 0. *Sample answer:* (−2, 3), (0, 3), and (1, 3)

WRITING LINEAR EQUATIONS

The slope of a nonvertical line is $m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$.

Given the slope and the y-intercept of a line, the slope and a point on a line, or two points on a line, you can use the slope-intercept form to write an equation of the line.

EXAMPLE Write an equation of a line that has a slope of $\frac{1}{2}$ and passes through the point $(-4, -3)$.



SOLUTION

$y = mx + b$ **Write slope formula.**

$-3 = \frac{1}{2}(-4) + b$ **Substitute.**

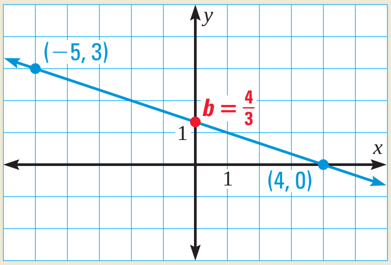
$-1 = b$ **Simplify.**

▶ So, $m = \frac{1}{2}$ and $b = -1$, and an equation of the line is $y = \frac{1}{2}x - 1$.

EXAMPLE Write an equation of a line that passes through the points $(4, 0)$ and $(-5, 3)$.

SOLUTION First find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{-5 - 4} = -\frac{1}{3}$$



Then substitute the slope and the coordinates of either point into the slope-intercept formula to find the y-intercept. Let $m = -\frac{1}{3}$, $x = 4$, and $y = 0$.

$0 = -\frac{1}{3} \cdot 4 + b$ **Substitute.**

$b = \frac{4}{3}$ **Simplify.**

▶ An equation of the line is $y = -\frac{1}{3}x + \frac{4}{3}$.

PRACTICE

Write an equation in slope-intercept form of the line that passes through the given point and has the given slope. **1–12. See margin.**

1. $(0, -4)$, $m = 1$ **2.** $(0, 8)$, $m = -3$ **3.** $(0, -0.75)$, $m = 2.5$ **4.** $(0, 1.6)$, $m = 0$

5. $(0, 0)$, $m = 0.7$ **6.** $(0, -24)$, $m = 50$ **7.** $(1, -5)$, $m = 2$ **8.** $(3, 0)$, $m = -4$

9. $(-6, -6)$, $m = 12$ **10.** $(-9, 7)$, $m = -1$ **11.** $(3, -11)$, $m = 0$ **12.** $(0.5, -1.5)$, $m = 2$

Write an equation in slope-intercept form of the line that passes through the given points. **13–24. See margin.**

13. $(1, 3)$, $(7, 4)$ **14.** $(0, -3)$, $(-5, 0)$ **15.** $(-6, -7)$, $(-5, 1)$ **16.** $(4, 2)$, $(7, -4)$

17. $(2, 0)$, $(-6, -5)$ **18.** $(11, -1)$, $(-1, -7)$ **19.** $(-5, 4)$, $(2, -3)$ **20.** $(4, -9)$, $(8, -9)$

21. $(1.4, 2.7)$, $(3.9, 1.1)$ **22.** $(0, 11)$, $(16, 87)$ **23.** $(0.5, 2)$, $(-1.25, 0.5)$ **24.** $(58, 20)$, $(80, 108)$

Solving Systems of Equations

EXAMPLE Use substitution to solve the linear system: $3x + 2y = 16$ **Equation 1**

$x + 3y = 10$ **Equation 2**

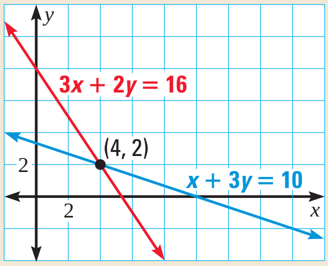
Solution

Solve for x in Equation 2 since it is easy to isolate x: $x = 10 - 3y$.

Substitute $10 - 3y$ for x in Equation 1: $3(10 - 3y) + 2y = 16$.

Solve for y to get $y = 2$. Then $x = 10 - 3y = 10 - 3 \cdot 2 = 4$.

The solution is (4, 2). One way to check this solution is to substitute 4 for x and 2 for y in each of the original equations. Another way is to graph the original equations in the same coordinate plane to see if the graphs intersect at the point (4, 2).



EXAMPLE Use linear combinations to solve the linear system.

$$4x - 3y = -5$$
$$7x + 2y = -16$$

Solution The goal is to obtain coefficients that are opposites for one of the variables.

$4x - 3y = -5$ **Multiply by 2.** → $8x - 6y = -10$

$7x + 2y = -16$ **Multiply by 3.** → $\underline{21x + 6y = -48}$

$29x = -58$ **Add the equations.**

$x = -2$ **Solve for x.**

Substitute -2 for x: $4(-2) - 3y = -5$. Solve to get $y = -1$.

▶ The solution is $(-2, -1)$. Check this in the original equations.

Practice

Use substitution to solve the system of linear equations.

1. $2x - 3y = -16$, $y = 5x + 1$ **(1, 6)**
2. $3x + y = -6$, $x = 0.5y - 3$ **(−2.4, 1.2)**
3. $7x - y = 0$, $x - y = 12$ **(−2, −14)**
4. $x + y = 8$, $2x + 5y = 3$ $\left(12\frac{1}{3}, -4\frac{1}{3}\right)$
5. $9x + 4y = 3$, $x + 8y = 6$ $\left(0, \frac{3}{4}\right)$
6. $3x + 5y = -8$, $4x - y = -3$ **(−1, −1)**
7. $x - 0.5y = 6$, $0.5x + 0.2y = 8$ $\left(\frac{104}{9}, \frac{100}{9}\right)$
8. $3x + y = 6$, $5(x + y) = 22$ **(0.8, 3.6)**

Use linear combinations to solve the system of linear equations.

9. $4x - 5y = 18$, $3x + 10y = -3$ **(3, −1.2)**
10. $7x + y = 8.5$, $-4x - 3y = 0$ $\left(1\frac{1}{2}, -2\right)$
11. $3x - 2y = -6$, $7x - 6y = 12$ **(−15, −19.5)**
12. $8x + 7y = 56$, $7x + 3y = 45$ **(5.88, 1.28)**
13. $5x + 9y = -6$, $2x - 6y = 6$ $\left(\frac{3}{8}, -\frac{7}{8}\right)$
14. $8x + y = -8$, $-2x - 3y = 35$ $\left(\frac{1}{2}, -12\right)$
15. $8x - 4y = 15$, $7x + 9y = 25$ **(2.35, 0.95)**
16. $13x - 5y = 10$, $3x - 2y = 14$ $\left(-\frac{50}{11}, -\frac{152}{11}\right)$

PROPERTIES OF EXPONENTS

An expression like 5^3 is called a **power**. The **exponent** 3 represents the number of times the **base** 5 is used as a factor: $5^3 = 5 \cdot 5 \cdot 5$ (3 factors of 5). To simplify expressions involving exponents, you often use properties of exponents. Let a and b be numbers and let m and n be integers.

- **Product of powers property:** $a^m \cdot a^n = a^{m+n}$ — **Example:** $4^2 \cdot 4^5 = 4^{2+5} = 4^7$
- **Power of a power property:** $(a^m)^n = a^{m \cdot n}$ — **Example:** $(x^4)^3 = x^{4 \cdot 3} = x^{12}$
- **Power of a product property:** $(a \cdot b)^m = a^m \cdot b^m$ — **Example:** $(-3k)^4 = (-3)^4 \cdot k^4 = 81\,k^4$
- If $a \neq 0$, then $a^0 = 1$. — **Example:** $5^0 = 1$
- If $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$. — **Example:** $x^{-5} = \frac{1}{x^5}$
- **Quotient of powers property:** If $a \neq 0$, then $\frac{a^m}{a^n} = a^{m-n}$. — **Example:** $\frac{7^6}{7} = 7^{6-1} = 7^5 = 16{,}807$
- **Power of a quotient property:** If $b \neq 0$, then $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$. — **Example:** $\left(\frac{z}{3}\right)^4 = \frac{z^4}{3^4} = \frac{z^4}{81}$

EXAMPLES Simplify the expression.

a. $(-3xy^2)^3 \cdot y$ **b.** $\frac{1}{r^7} \cdot r^4$ **c.** $\frac{1}{x^6} \cdot \left(\frac{x}{2}\right)^6$

SOLUTION

a. $(-3xy^2)^3 \cdot y = (-3)^3 \cdot x^3 \cdot (y^2)^3 \cdot y^1 = -27x^3 \cdot y^{6+1} = -27x^3y^7$

b. $\frac{1}{r^7} \cdot r^4 = \frac{r^4}{r^7} = r^{4-7} = r^{-3} = \frac{1}{r^3}$

c. $\frac{1}{x^6} \cdot \left(\frac{x}{2}\right)^6 = \frac{1}{x^6} \cdot \frac{x^6}{2^6} = \frac{x^6}{64x^6} = \frac{1}{64}$

PRACTICE

Simplify the expression. The simplified expression should have no negative exponents.

1. $\left(-\frac{2}{3}\right)^3$ $-\frac{8}{27}$

2. $x^3 \cdot x \cdot x^3$ x^7

3. $\left(\frac{1}{2}ab\right)^5$ $\frac{1}{32}a^5b^5$

4. $2n^4 \cdot (3n)^2$ $18n^6$

5. $(rst)^0$ 1

6. $\left(8^{-1}\right)^{-3}$ 512

7. $4x^{-3} \cdot y^{-6}$ $\frac{4}{x^3y^6}$

8. $c \cdot c^{-9}$ $\frac{1}{c^8}$

9. $4^3 \cdot 4^6$ $262{,}144$

10. $(3 \cdot a^2 \cdot 6)^2$ $324a^4$

11. $\left(\frac{5}{m}\right)^3$ $\frac{125}{m^3}$

12. $\frac{(-3)^5}{-3^5}$ 1

13. $(2b)^3 \cdot b$ $8b^4$

14. $(5x \cdot x^3)^4$ $625x^{16}$

15. $\left(\frac{x^4}{x^3}\right)^2$ x^2

16. $c^6 \cdot \frac{1}{c^9}$ $\frac{1}{c^3}$

17. $\frac{1}{a^{-4}}$ a^4

18. $\frac{2x^0}{8y^{-7}}$ $\frac{y^7}{4}$

19. $\left(2a^{-2}bc^3\right)^{-1}$ $\frac{a^2}{2bc^3}$

20. $\left(\frac{r}{3s}\right)^{-3}$ $\frac{27s^3}{r^3}$

21. $\left(5ab^3\right)^2 \cdot (-7b^2c)$ $-175a^2b^8c$

22. $w^5 \cdot \left(\frac{7}{w^4}\right)^2$ $\frac{49}{w^3}$

23. $4y^3z \cdot \left(\frac{y}{2z}\right)^{-3}$ $32z^4$

24. $\left(3c^{-4}d^5\right)^{-2} \cdot 12cd^{-4}$ $\frac{4c^9}{3d^{14}}$

MULTIPLYING BINOMIALS

To multiply binomials, use the distributive property. Each term in the first binomial is multiplied by each term in the second binomial. The **FOIL** method can help you remember the pattern of the distributive property. **FOIL** stands for **F**irst, **O**uter, **I**nner, and **L**ast, which is the order in which you multiply terms.

EXAMPLE $(x + 1)(2x - 4) = x(2x) + x(-4) + 1(2x) + 1(-4)$ **Multiply using FOIL.**

$= 2x^2 - 4x + 2x - 4$ **Simplify.**

$= 2x^2 - 2x - 4$ **Add like terms.**

PRACTICE

Simplify.

1. $(x + 1)(x + 1)$ $x^2 + 2x + 1$
2. $(4b + 1)(2 + b)$ $4b^2 + 9b + 2$
3. $(3c + 3)(c - 1)$ $3c^2 - 3$
4. $(t + 3)(2t - 3)$ $2t^2 + 3t - 9$
5. $(a + 5)(4a - 7)$ $4a^2 + 13a - 35$
6. $(5d + 3)(d - 2)$ $5d^2 - 7d - 6$
7. $(2f - 4)(2f + 4)$ $4f^2 - 16$
8. $(1 - 2g)(g + 3)$ $-2g^2 - 5g + 3$
9. $(6h + 3)(h + 1)$ $6h^2 + 9h + 3$

SQUARING BINOMIALS

One way to square a binomial is to use a pattern for the square of a binomial.

Patterns for the Square of a Binomial: $(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

EXAMPLES $(k + 9)^2 = k^2 + 2(k)(9) + 9^2$

$= k^2 + 18k + 81$

$(x - 4)^2 = x^2 - 2(x)(4) + 4^2$

$= x^2 - 8x + 16$

If you have trouble remembering the patterns, you can always use the distributive property to find the square of a binomial.

EXAMPLE $(r + 3)^2 = (r + 3)(r + 3)$

$= r(r + 3) + 3(r + 3)$ **Distributive property**

$= r^2 + 3r + 3r + 9$ **Distributive property**

$= r^2 + 6r + 9$ **Combine like terms.**

PRACTICE

Find the product by squaring the binomial.

1. $(x + 2)^2$ $x^2 + 4x + 4$
2. $(x - 1)^2$ $x^2 - 2x + 1$
3. $(x + 8)^2$ $x^2 + 16x + 64$
4. $(10 + x)^2$ $100 + 20x + x^2$, or $x^2 + 20x + 100$
5. $(n - 5)^2$ $n^2 - 10n + 25$
6. $(x - 0.5)^2$ $x^2 - x + 0.25$
7. $(15 - x)^2$ $225 - 30x + x^2$, or $x^2 - 30x + 225$
8. $(y + 12)^2$ $y^2 + 24y + 144$

RADICAL EXPRESSIONS

If $a^2 = b$, then b is a **square root** of a. Every positive number has two square roots: a positive square root and a negative square root. For example,

$4^2 = 16$ and $(-4)^2 = 16$, so the square roots of 16 are 4 and -4. We write:

$$\sqrt{16} = 4 \text{ and } -\sqrt{16} = -4.$$

Zero has just one square root: $\sqrt{0} = 0$.

EXAMPLES $11^2 = 121$, so $-\sqrt{121} = -11$ $\left(\frac{1}{5}\right)^2 = \frac{1}{25}$, so $\sqrt{\frac{1}{25}} = \frac{1}{5}$

$\sqrt{-4}$ is undefined, because the square of every real number is either positive or zero.

$\sqrt{5 + 4} = \sqrt{9} = 3$. Begin by simplifying an expression under the square root symbol.

When you simplify a radical expression, you will often use the following properties of radicals.

If a and b are positive numbers, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

EXAMPLES Simplify the expression.

a. $\sqrt{56}$ **b.** $\sqrt{6} \cdot \sqrt{15}$ **c.** $\frac{\sqrt{150}}{\sqrt{2}}$ **d.** $\frac{5}{\sqrt{8}}$

SOLUTION

a. $\sqrt{56} = \sqrt{4} \cdot \sqrt{14} = 2\sqrt{14}$

b. $\sqrt{6} \cdot \sqrt{15} = \sqrt{6 \cdot 15} = \sqrt{90} = \sqrt{9 \cdot 10} = \sqrt{9} \cdot \sqrt{10} = 3\sqrt{10}$

c. $\frac{\sqrt{150}}{\sqrt{2}} = \sqrt{\frac{150}{2}} = \sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$

d. $\frac{5}{\sqrt{8}} = \frac{5}{2\sqrt{2}} = \frac{5}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{4}$ **Do not leave a square root in a denominator.**

PRACTICE

Find all square roots of the number or write *no square roots*. Check the results by squaring each root.

1. 64 **8 and −8** **2.** −36 **no square roots** **3.** $\frac{49}{81}$ $\frac{7}{9}$ **and** $-\frac{7}{9}$ **4.** $\frac{7}{100}$ $\frac{\sqrt{7}}{10}$ **and** $-\frac{\sqrt{7}}{10}$ **5.** 0.09 **0.3 and −0.3**

Simplify the expression. Give the exact value in simplified form.

6. $\sqrt{36 + 64}$ **10** **7.** $\sqrt{4 + 9}$ $\sqrt{13} \approx 3.61$ **8.** $\sqrt{16 + 16}$ $4\sqrt{2} \approx 5.66$ **9.** $\sqrt{(-1)^2 + 7^2}$ $5\sqrt{2} \approx 7.07$

Simplify the expression. Give the exact value in simplified form.

10. $-\sqrt{0}$ **0** **11.** $-\sqrt{196}$ **−14** **12.** $\sqrt{54}$ $3\sqrt{6} \approx 7.35$ **13.** $\sqrt{60}$ $2\sqrt{15} \approx 7.75$

14. $\sqrt{7} \cdot \sqrt{3}$ $\sqrt{21} \approx 4.58$ **15.** $\sqrt{12} \cdot \sqrt{6}$ $6\sqrt{2} \approx 8.49$ **16.** $\sqrt{10} \cdot \sqrt{15}$ $5\sqrt{6} \approx 12.25$ **17.** $\sqrt{120 \cdot 105}$ $30\sqrt{14} \approx 112.25$

18. $\frac{\sqrt{147}}{\sqrt{3}}$ **7** **19.** $\frac{\sqrt{20}}{\sqrt{500}}$ $\frac{1}{5} = 0.2$ **20.** $\frac{\sqrt{48}}{\sqrt{6}}$ $2\sqrt{2} \approx 2.83$ **21.** $\frac{\sqrt{6}}{\sqrt{96}}$ $\frac{1}{4} = 0.25$

22. $\frac{4}{\sqrt{3}}$ $\frac{4\sqrt{3}}{3} \approx 2.31$ **23.** $\frac{6}{\sqrt{2}}$ $3\sqrt{2} \approx 4.24$ **24.** $\frac{5}{\sqrt{20}}$ $\frac{\sqrt{5}}{2} \approx 1.12$ **25.** $\frac{4}{\sqrt{27}}$ $\frac{4\sqrt{3}}{9} \approx 0.77$

SOLVING $AX^2 + C = 0$

A **quadratic equation** is an equation that can be written in the **standard form** $ax^2 + bx + c = 0$ where $a \neq 0$.

When $b = 0$, the quadratic equation has the form $ax^2 + c = 0$. In this case, you can solve for x. Solving $ax^2 + c = 0$ for x^2 you get $x^2 = \frac{-c}{a}$ and the following rules apply.

- If $\frac{-c}{a} > 0$, then $x^2 = \frac{-c}{a}$ has two solutions, $x = \sqrt{\frac{-c}{a}}$ and $x = -\sqrt{\frac{-c}{a}}$.
- If $\frac{-c}{a} = 0$, then $x^2 = \frac{-c}{a}$ has one solution, $x = 0$.
- If $\frac{-c}{a} < 0$, $x^2 = \frac{-c}{a}$ has no real solution.

EXAMPLES Solve the equation.

a. $3x^2 - 1 = 23$ **b.** $12 - x^2 = 13$ **c.** $4 + 2n^2 = 4$

SOLUTION

a.
$$3x^2 - 1 = 23$$
$$3x^2 = 24$$
$$x^2 = 8$$
$$x = \pm\sqrt{8}$$
$$x = \pm 2\sqrt{2}$$

b.
$$12 - x^2 = 13$$
$$-x^2 = 1$$
$$x^2 = -1$$
no real solution

c.
$$4 + 2n^2 = 4$$
$$2n^2 = 0$$
$$n^2 = 0$$
$$n = 0$$

EXAMPLE Solve $(x + 2)^2 = x^2 + 9$.

SOLUTION

$$(x + 2)^2 = x^2 + 9$$

$$x^2 + 4x + 4 = x^2 + 9$$ See page 798 for help with squaring a binomial.

$$4x = 5$$ The given equation simplifies to a linear equation.

$$x = 1.25$$ Simplify.

PRACTICE

Solve the equation or write *no solution*. Round solutions to the nearest hundredth.

1. $x^2 = 625$ **25, −25**
2. $x^2 = -9$ **no solution**
3. $x^2 + 6 = 11$ **$\sqrt{5} \approx 2.24$, $-\sqrt{5} \approx -2.24$**
4. $4x^2 = 0$ **0**
5. $-8 + 3r^2 = 4$ **2, −2**
6. $\frac{1}{2}k^2 + 3 = 245$ **22, −22**
7. $7a^2 + 25 = -6$ **no solution**
8. $4x^2 - 2 = 1$ **$\frac{\sqrt{3}}{2} \approx 0.87$, $-\frac{\sqrt{3}}{2} \approx -0.87$**
9. $(x + 5)^2 = x^2 + 49$ **2.4**
10. $x^2 + 81 = (x + 6)^2$ **3.75**
11. $(x + 1)^2 = 27 + x^2$ **13**
12. $(x + 4)^2 = (x - 4)^2 + 96$ **6**

SOLVING $AX^2 + BX + C = 0$

You can solve any quadratic equation by using the **quadratic formula**. This formula, which you used in Algebra, states that the solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ when } a \neq 0 \text{ and } b^2 - 4ac \geq 0.$$

EXAMPLE Solve $x^2 - 4x - 12 = 0$ by using the quadratic formula.

SOLUTION Substitute $a = 1$, $b = -4$, and $c = -12$ in the quadradic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2 \cdot 1} = \frac{4 \pm \sqrt{64}}{2}$$

▶ The solutions are $\frac{4 + 8}{2} = 6$ and $\frac{4 - 8}{2} = -2$.

Check your solutions by substituting each solution into the original equation.

$6^2 - 4(6) - 12 = 0$ $\qquad$ $(-2)^2 - 4(-2) - 12 = 0$

$36 - 24 - 12 = 0$ ✓ $\qquad$ $4 + 8 - 12 = 0$ ✓

EXAMPLE Solve $2x^2 + 6x = 1$ by using the quadratic formula.

SOLUTION

Begin by writing the equation in *standard form*: $2x^2 + 6x - 1 = 0$.

Substitute $a = 2$, $b = 6$, and $c = -1$ in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 2(-1)}}{2 \cdot 2} = \frac{-6 \pm \sqrt{44}}{4} = \frac{-6 \pm 2\sqrt{11}}{4} = \frac{-3 \pm \sqrt{11}}{2}$$

▶ The solutions are $\frac{-3 + \sqrt{11}}{2} \approx 0.16$ and $\frac{-3 - \sqrt{11}}{2} \approx -3.16$.

PRACTICE

Use the quadratic formula to solve each equation. Round solutions to the nearest hundredth.

1. $x^2 + 5x + 4 = 0$ **−4, −1**

2. $x^2 - x - 6 = 0$ **3, −2**

3. $x^2 + 6x = 0$ **0, −6**

4. $a^2 + 8 = 6a$ **4, 2**

5. $z^2 = 9z - 1$ **See margin.**

6. $-25 = x^2 + 10x$ **−5**

7. $2x^2 + 4x + 1 = 0$ **See margin.**

8. $4c^2 = 4c - 1$ **0.5**

9. $-8m + 3m^2 = -1$ **See margin.**

10. $3x^2 + 6x + 2 = 0$ **See margin.**

11. $5y^2 = 1 + 5y$ **See margin.**

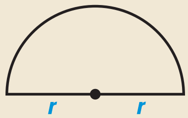
12. $4x^2 - 3x = 7$ **1.75, −1**

13. Solve the quadratic equation $x^2 - 3x + 2 = 0$. Then graph the function $y = x^2 - 3x + 2$ in a coordinate plane. Describe the relationship between the solutions of the quadratic equation and the x-intercepts of the graph. **1, 2; See margin for graph. The solutions of the quadratic equation are the same as the *x*-intercepts of the graph.**

SOLVING FORMULAS

A **formula** is an algebraic equation that relates two or more real-life quantities. You can solve a formula for one of the variables by rewriting the formula so that the required variable is isolated on one side of the equation.

EXAMPLE The formula for the perimeter of the figure shown is $P = 2r + \pi r$. Solve the formula for r.



SOLUTION

$P = 2r + \pi r$

$P = (2 + \pi)r$ — Distributive property

$\frac{P}{2 + \pi} = r$ — Divide each side by $(2 + \pi)$.

EXAMPLE Rewrite the equation $2x + 3y = -6$ so that y is a function of x.

SOLUTION

$2x + 3y = -6$

$3y = -2x - 6$

$y = \frac{-2x - 6}{3}$ or $y = -\frac{2}{3}x - 2$

PRACTICE

Solve the formula for the indicated variable.

1. Area of a parallelogram: $A = bh$. Solve for b. $b = \frac{A}{h}$
2. Volume of a pyramid: $V = \frac{1}{3}Bh$. Solve for h. $h = \frac{3V}{B}$
3. Perimeter of a triangle: $P = a + b + c$. Solve for b. $b = P - a - c$
4. Circumference of a circle: $C = 2\pi r$. Solve for r. $r = \frac{C}{2\pi}$
5. Perimeter of a parallelogram: $P = 2(a + b)$. Solve for a. $a = \frac{P - 2b}{2}$
6. Sum of the measures of the interior angles of a convex polygon with n sides: $S = (n - 2)180$. Solve for n. $n = \frac{S + 360}{180}$
7. Surface area of a rectangular solid: $S = 2\ell w + 2\ell h + 2wh$. Solve for ℓ. $\ell = \frac{S - 2wh}{2w + 2h}$
8. Surface area of a right cylinder: $S = 2\pi r^2 + 2\pi rh$. Solve for h. $h = \frac{S - 2\pi r^2}{2\pi r}$
9. Surface area of a right cone: $S = \pi r^2 + \pi r\ell$. Solve for ℓ. $\ell = \frac{S - \pi r^2}{\pi r}$
10. Area of a trapezoid: $A = \frac{1}{2}hb_1 + \frac{1}{2}hb_2$. Solve for h. $h = \frac{2A}{b_1 + b_2}$

Rewrite the equation so that y is a function of x.

11. $3x + y = 9$ $y = 9 - 3x$
12. $5x - y = 0$ $y = 5x$
13. $2y + 6 = 3 - x$ $y = -\frac{1}{2}x - \frac{3}{2}$
14. $\frac{1}{2}x + 4y = -8$ $y = -\frac{1}{8}x - 2$
15. $6x - 7y = 42$ $y = \frac{6}{7}x - 6$
16. $1.5x + 0.2y = 3$ $y = -\frac{15}{2}x + 15$
17. $ax + by = c$ $y = \frac{c - ax}{b}$
18. $ax^2 - by = c$ $y = \frac{ax^2 - c}{b}$