LESSON 9.3

NAME _____ DATE _____

Challenge: Skills and Applications

For use with pages 543–549

1. Here is a formula for generating Pythagorean triples. If m and n are positive integers, with $m < n$, let $a = n^2 - m^2$, $b = 2mn$, and $c = n^2 + m^2$.

a. Show that a, b, and c form a Pythagorean triple.

b. List the Pythagorean triples that are generated using $n \leq 5$.

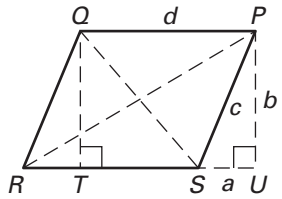
c. It can be shown that *every* Pythagorean triple can be generated in this manner. Find expressions for m and n in terms of a, b, and c.

d. If you are given the three numbers of a Pythagorean triple and asked to find the corresponding values of m and n, how can you decide which number is a, which is b, and which is c?

e. Find the values of m and n for the Pythagorean triple 56, 90, 106.

f. Find the values of m and n for the Pythagorean triple 48, 55, 73.

2. Let $PQRS$ be a parallelogram with side lengths $QR = PS = c$ and $QP = RS = d$, and diagonal lengths $PR = e$ and $QS = f$.

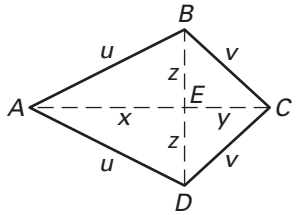


a. Justify drawing auxiliary line segments $\overline{QT}$, $\overline{SU}$, and $\overline{UP}$, as shown.

b. Use the Pythagorean Theorem and the properties of algebra to evaluate $e^2 + f^2$ in terms of c and d.

c. Based on your work, write a general statement about the relationship between the lengths of the sides and the diagonals of a parallelogram.

d. Using the diagram, show that the relationship you found in part (c) does *not* hold true for a kite.



In Exercises 3–8, find the possible values of x.

3. $\triangle ABC$ is a right triangle; $AB = x$, $BC = x + 1$, $AC = x + 9$.

4. $\triangle DEF$ is a right triangle; $DE = 12$, $EF = x - 1$, $DF = x + 1$.

5. $\triangle GHI$ is a right triangle; $GH = 5$, $HI = x + 4$, $GI = 2x - 3$.

6. $\triangle JKL$ is a right triangle; $JK = 3x - 6$, $KL = 2x + 11$, $JL = 20$.

7. $\triangle MNO$ is an acute triangle; $MN = x - 1$, $NO = x + 1$, $MO = 8$.

8. $\triangle PQR$ is an obtuse triangle; $PQ = x$, $QR = x + 1$, $PR = 5$.