

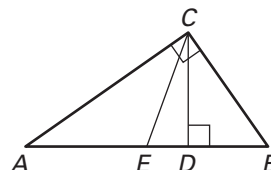
Challenge: Skills and Applications

For use with pages 527–534

In Exercises 1 and 2, use the following information.

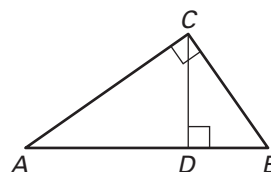
A triangle inscribed in a circle is a right triangle if and only if the longest side of the triangle is a diameter of the circle.

1. Given point D on line segment \overline{AB} , explain how to use a compass and straightedge to construct a line segment whose length is the geometric mean of AD and BD .
2. Refer to the diagram. In $\triangle ABC$, \overline{CD} is an altitude and \overline{CE} is a median.
 - a. Explain why CE is the arithmetic mean of AD and BD .
 - b. Use the diagram to show that the arithmetic mean of AD and BD is greater than the geometric mean of AD and BD .
 - c. Use your argument from part (b) to show that the arithmetic mean of any two distinct positive numbers is greater than the geometric mean.



In Exercises 3 and 4, refer to the diagram.

3. Prove that $\frac{(AC)^2}{(BC)^2} = \frac{AD}{BD}$.
4. If $AD = x^2$ and $BD = y^2$, use the Geometric Mean Theorems to find AC , BC , and CD in terms of x and y . (Assume that x and y are positive.)



In Exercises 5–10, find the possible values of x .

