CHAPTER 9

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Solve problems involving similar right triangles formed by the altitude drawn to the hypotenuse of a right triangle. **(9.1)**	Find a height in a real-life structure, such as the height of a triangular roof. **(p. 528)**
Use the Pythagorean Theorem. **(9.2)**	Solve real-life problems, such as finding the length of a skywalk support beam. **(p. 537)**
Use the Converse of the Pythagorean Theorem. **(9.3)**	Use in construction methods, such as verifying whether a foundation is rectangular. **(p. 545)**
Use side lengths to classify triangles by their angle measures. **(9.3)**	Write proofs about triangles. **(p. 547)**
Find side lengths of special right triangles. **(9.4)**	Solve real-life problems, such as finding the height of a loading platform. **(p. 553)**
Find trigonometric ratios of an acute angle. **(9.5)**	Measure distances indirectly, such as the depth of a crater on the moon. **(p. 564)**
Solve a right triangle. **(9.6)**	Solve real-life problems, such as finding the glide angle and altitude of the space shuttle. **(p. 569)**
Find the magnitude and the direction of a vector. **(9.7)**	Describe physical quantities, such as the speed and direction of a ship. **(p. 574)**
Find the sum of two vectors. **(9.7)**	Model real-life motion, such as the path of a skydiver. **(p. 578)**

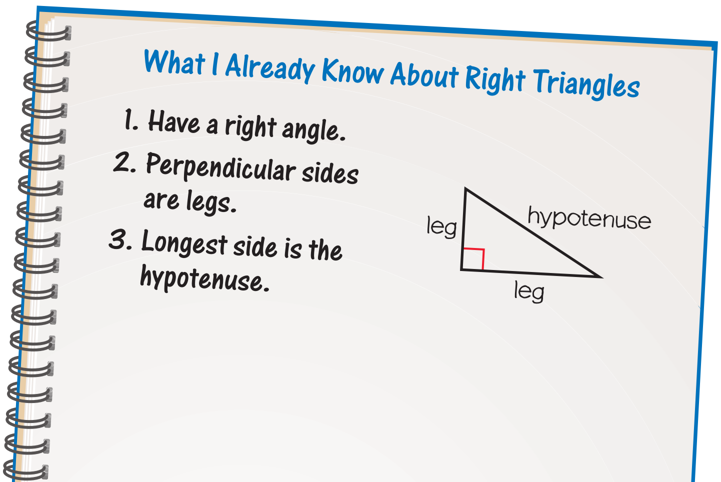
How does Chapter 9 fit into the BIGGER PICTURE of geometry?

In this chapter, you studied two of the most important theorems in mathematics—the Pythagorean Theorem and its converse. You were also introduced to a branch of mathematics called *trigonometry*. Properties of right triangles allow you to estimate distances and angle measures that cannot be measured directly. These properties are important tools in areas such as surveying, construction, and navigation.

STUDY STRATEGY

What did you learn about right triangles?

Your lists about what you knew and what you expected to learn about right triangles, following the study strategy on page 526, may resemble this one.



CHAPTER 9

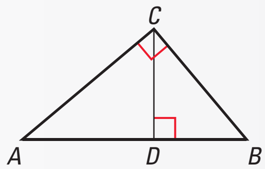
Chapter Review

- Pythagorean triple, p. 536
- special right triangles, p. 551
- trigonometric ratio, p. 558
- sine, p. 558
- cosine, p. 558
- tangent, p. 558
- angle of elevation, p. 561
- solve a right triangle, p. 567
- magnitude of a vector, p. 573
- direction of a vector, p. 574
- equal vectors, p. 574
- parallel vectors, p. 574
- sum of two vectors, p. 575

9.1 SIMILAR RIGHT TRIANGLES

Examples on pp. 528–530

EXAMPLES



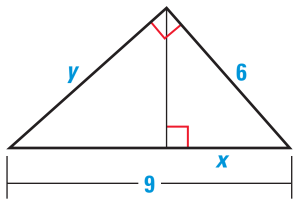
$\triangle ACB \sim \triangle CDB$, so $\frac{DB}{CB} = \frac{CB}{AB}$. CB is the geometric mean of DB and AB.

$\triangle ADC \sim \triangle ACB$, so $\frac{AD}{AC} = \frac{AC}{AB}$. AC is the geometric mean of AD and AB.

$\triangle CDB \sim \triangle ADC$, so $\frac{DA}{DC} = \frac{DC}{DB}$. DC is the geometric mean of DA and DB.

Find the value of each variable.

1.



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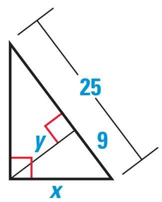

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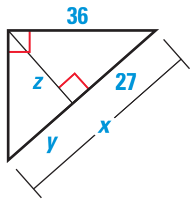


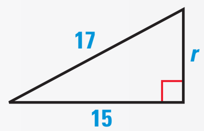
9.2 THE PYTHAGOREAN THEOREM

Examples on pp. 536–537

EXAMPLE You can use the Pythagorean Theorem to find the value of r.

$17^2 = r^2 + 15^2$, or $289 = r^2 + 225$. Then $64 = r^2$, so $r = 8$.

The side lengths 8, 15, and 17 form a Pythagorean triple because they are integers.



The variables *r* and *s* represent the lengths of the legs of a right triangle, and *t* represents the length of the hypotenuse. Find the unknown value. Then tell whether the lengths form a Pythagorean triple.

4. $r = 12, s = 16$ **5.** $r = 8, t = 12$ **6.** $s = 16, t = 34$ **7.** $r = 4, s = 6$

9.3 THE CONVERSE OF THE PYTHAGOREAN THEOREM

Examples on pp. 543–545

EXAMPLES You can use side lengths to classify a triangle by its angle measures. Let a, b, and c represent the side lengths of a triangle, with c as the length of the longest side.

If $c^2 = a^2 + b^2$, the triangle is a right triangle: $8^2 = (2\sqrt{7})^2 + 6^2$, so $2\sqrt{7}$, 6, and 8 are the side lengths of a right triangle.

If $c^2 < a^2 + b^2$, the triangle is an acute triangle: $12^2 < 8^2 + 9^2$, so 8, 9, and 12 are the side lengths of an acute triangle.

If $c^2 > a^2 + b^2$, the triangle is an obtuse triangle: $8^2 > 5^2 + 6^2$, so 5, 6, and 8 are the side lengths of an obtuse triangle.

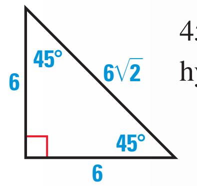
Decide whether the numbers can represent the side lengths of a triangle. If they can, classify the triangle as *acute, right,* or *obtuse*.

8. 6, 7, 10 **9.** 9, 40, 41 **10.** 8, 12, 20 **11.** 3, $4\sqrt{5}$, 9

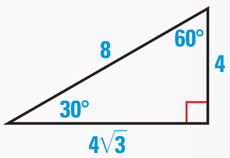
9.4 SPECIAL RIGHT TRIANGLES

Examples on pp. 551–553

EXAMPLES Triangles whose angle measures are 45°-45°-90° or 30°-60°-90° are called *special right triangles*.



45°-45°-90° triangle
hypotenuse $= \sqrt{2} \cdot$ leg



30°-60°-90° triangle
hypotenuse $= 2 \cdot$ shorter leg
longer leg $= \sqrt{3} \cdot$ shorter leg

12. An isosceles right triangle has legs of length $3\sqrt{2}$. Find the length of the hypotenuse.

13. A diagonal of a square is 6 inches long. Find its perimeter and its area.

14. A 30°-60°-90° triangle has a hypotenuse of length 12 inches. What are the lengths of the legs?

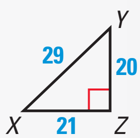
15. An equilateral triangle has sides of length 18 centimeters. Find the length of an altitude of the triangle. Then find the area of the triangle.

9.5 TRIGONOMETRIC RATIOS

Examples on pp. 558–561

EXAMPLE A trigonometric ratio is a ratio of the lengths of two sides of a right triangle.

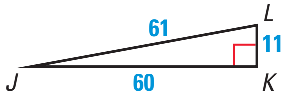
$\sin X = \frac{\text{opp.}}{\text{hyp.}} = \frac{20}{29}$ $\cos X = \frac{\text{adj.}}{\text{hyp.}} = \frac{21}{29}$ $\tan X = \frac{\text{opp.}}{\text{adj.}} = \frac{20}{21}$



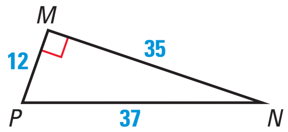
9.5 continued

Find the sine, the cosine, and the tangent of the acute angles of the triangle. Express each value as a decimal rounded to four places.

16.



17.



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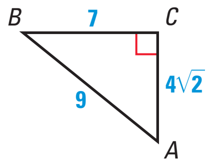


9.6 Solving Right Triangles

Examples on pp. 568–569

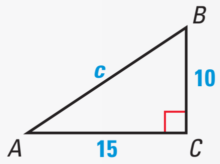
EXAMPLE To solve $\triangle ABC$, begin by using the Pythagorean Theorem to find the length of the hypotenuse.

$c^2 = 10^2 + 15^2 = 325$. So, $c = \sqrt{325} = 5\sqrt{13}$.

Then find $m\angle A$ and $m\angle B$.

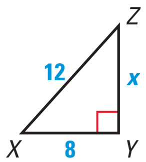
$\tan A = \frac{10}{15} = \frac{2}{3}$. Use a calculator to find that $m\angle A \approx 33.7°$.

Then $m\angle B = 90° - m\angle A \approx 90° - 33.7° = 56.3°$.



Solve the right triangle. Round decimals to the nearest tenth.

19.



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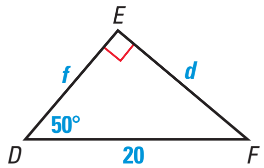

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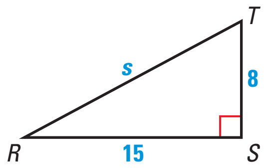


9.7 Vectors

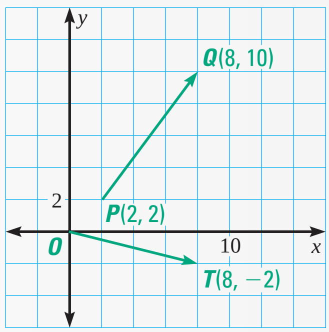
Examples on pp. 573–575

EXAMPLES You can use the Distance Formula to find the magnitude of $\overrightarrow{PQ}$.

$$|\overrightarrow{PQ}| = \sqrt{(8-2)^2 + (10-2)^2} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$$

To add vectors, find the sum of their horizontal components and the sum of their vertical components.

$$\overrightarrow{PQ} + \overrightarrow{OT} = \langle 6, 8\rangle + \langle 8, -2\rangle = \langle 6+8, 8+(-2)\rangle = \langle 14, 6\rangle$$



Draw vector $\overrightarrow{PQ}$ in a coordinate plane. Write the component form of the vector and find its magnitude. Round decimals to the nearest tenth.

22. $P(2, 3)$, $Q(1, -1)$

23. $P(-6, 3)$, $Q(6, -2)$

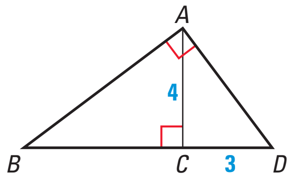
24. $P(-2, 0)$, $Q(1, 2)$

25. Let $\vec{u} = \langle 1, 2\rangle$ and $\vec{v} = \langle 13, 7\rangle$. Find $\vec{u} + \vec{v}$. Find the magnitude of the sum vector and its direction relative to east.

CHAPTER 9

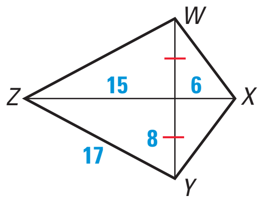
Chapter Test

Use the diagram at the right to match the angle or segment with its measure. (Some measures are rounded to two decimal places.)



1. $\overline{AB}$	**A.** 5.33
2. $\overline{BC}$	**B.** 36.87°
3. $\overline{AD}$	**C.** 5
4. $\angle BAC$	**D.** 53.13°
5. $\angle CAD$	**E.** 6.67

6. Refer to the diagram above. Complete the following statement: $\triangle ABC \sim \triangle$ __?__ $\sim \triangle$ __?__.



7. Classify quadrilateral $WXYZ$ in the diagram at the right. Explain your reasoning.

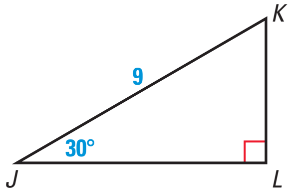
8. The vertices of $\triangle PQR$ are $P(-2, 3)$, $Q(3, 1)$, and $R(0, -3)$. Decide whether $\triangle PQR$ is *right*, *acute*, or *obtuse*.

9. Complete the following statement: 15, __?__, and 113 form a Pythagorean triple.

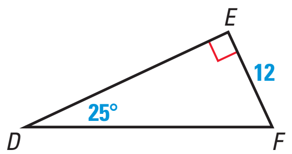
10. The measure of one angle of a rhombus is 60°. The perimeter of the rhombus is 24 inches. Sketch the rhombus and give its side lengths. Then find its area.

Solve the right triangle. Round decimals to the nearest tenth.

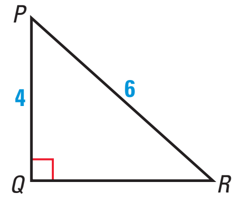
11.



12.

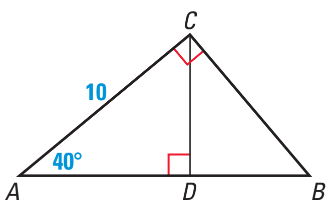


13.

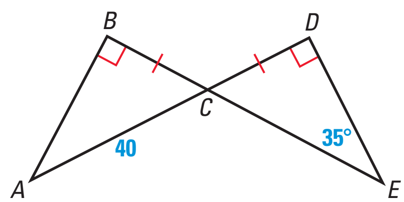


14. $L = (3, 7)$ and $M = (7, 4)$ are the initial and the terminal points of $\overrightarrow{LM}$. Draw $\overrightarrow{LM}$ in a coordinate plane. Write the component form of the vector. Then find its magnitude and direction relative to east.

15. Find the lengths of $\overline{CD}$ and $\overline{AB}$.



16. Find the measure of $\angle BCA$ and the length of $\overline{DE}$.



Let $\vec{u} = \langle 0, -5 \rangle$, $\vec{v} = \langle -2, -3 \rangle$, and $\vec{w} = \langle 4, 6 \rangle$. Find the given sum.

17. $\vec{u} + \vec{v}$

18. $\vec{u} + \vec{w}$

19. $\vec{v} + \vec{w}$