PROJECT
for Chapters 8 and 9

Investigating Fractals

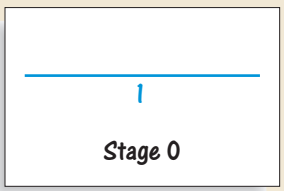
OBJECTIVE **Learn how fractals are formed and explore the properties of fractals.**

Materials: ruler, protractor, graphing calculator, graph paper

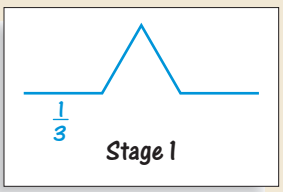
A *fractal* is a mathematical object that has the property of *self-similarity*: a part of the shape resembles the whole shape.

Many fractals are formed by a repetitive process called *iteration*. The stages that lead to a simple fractal called a *Koch curve* are shown below.

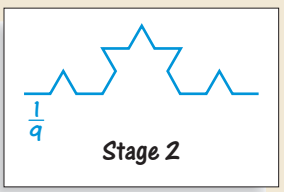
STAGES OF A KOCH CURVE



For stage 0, start with a segment of length 1.



For stage 1, replace the middle third of the segment in stage 0 with two congruent segments of length $\frac{1}{3}$.



For stage 2, replace the middle third of every segment of stage 1 with two congruent segments of length $\frac{1}{9}$.

For later stages, replace the middle third of every segment of the previous stage with two congruent segments of the appropriate length. The fractal is the shape that would result if you could continue this process infinitely many times.

INVESTIGATION

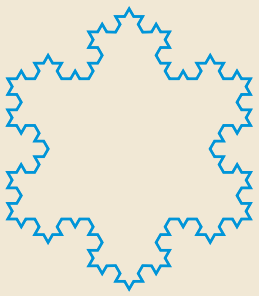
Copy the table below. Then answer the exercises.

Stage, n	0	1	2	3	4
Length, L	1	?	?	?	?

1. Use the diagrams above to fill in the lengths for stages 1 and 2.
2. Look for a pattern in the lengths at stages 0, 1, and 2. Use the pattern to predict the lengths of the Koch curve at stages 3 and 4.
3. Predict what happens to the length if you repeat this iteration many times. Explain your reasoning.
4. The length L of the fractal can be written as a function of stage n using this formula: $L = \left(\frac{4}{3}\right)^n$. Use a graphing calculator to sketch a graph of this function. Does the graph support your answer to Exercise 3? Explain.

STAGES OF A KOCH SNOWFLAKE

If you join three Koch curves, you can create a closed shape known as a *Koch snowflake*.



5. Sketch stage 0, stage 1, and stage 2 of the Koch snowflake.
6. Make a table like the one on the previous page to find the perimeter of the Koch snowflake at stage 3 and stage 4.
7. Write a formula for the perimeter P of the Koch snowflake at stage n.

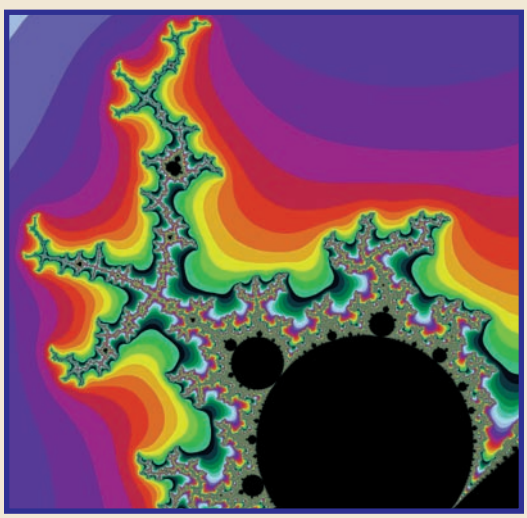
PRESENT YOUR RESULTS

Write a report or make a poster of your results.

- Include your drawing of the stages of the Koch snowflake fractal.
- Include the tables you made and the answers to Exercises 1–7.
- Describe what you learned about fractals.

EXTENSIONS

- Research the Sierpinski Triangle or the Cantor Set. Sketch the first four stages of these fractals.
- Create a fractal of your own. Start with a geometric shape and perform some alteration or transformation. Show the first three stages of your fractal.
- Suppose a Koch snowflake has an area of 1 at stage 0. Find the area of the snowflake at stage 1 and stage 2.
- Research the Mandelbrot Set and find some images of it. Many Web sites on the Internet feature images of this fractal.



The Mandelbrot Set reveals dramatic images as you zoom in to see details.