# 9.7 Vectors

## What you should learn

**GOAL 1** Find the magnitude and the direction of a vector.

**GOAL 2** Add vectors.

## Why you should learn it

▼ To solve **real-life** problems, such as describing the velocity of a skydiver in **Exs. 41–45**.

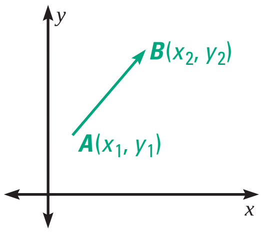
REAL LIFE

## GOAL 1 FINDING THE MAGNITUDE OF A VECTOR

As defined in Lesson 7.4, a *vector* is a quantity that has both magnitude and direction. In this lesson, you will learn how to find the *magnitude of a vector* and the *direction of a vector*. You will also learn how to add vectors.

The **magnitude of a vector** $\overrightarrow{AB}$ is the distance from the initial point $A$ to the terminal point $B$, and is written $|\overrightarrow{AB}|$. If a vector is drawn in a coordinate plane, you can use the Distance Formula to find its magnitude.

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$




### EXAMPLE 1 *Finding the Magnitude of a Vector*

Points $P$ and $Q$ are the initial and terminal points of the vector $\overrightarrow{PQ}$. Draw $\overrightarrow{PQ}$ in a coordinate plane. Write the component form of the vector and find its magnitude.

**a.** $P(0, 0)$, $Q(-6, 3)$ **b.** $P(0, 2)$, $Q(5, 4)$ **c.** $P(3, 4)$, $Q(-2, -1)$

**SOLUTION**

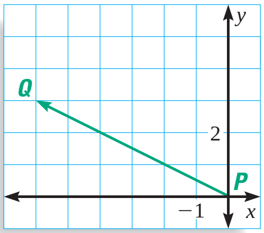
**a.** Component form $= \langle x_2 - x_1, y_2 - y_1 \rangle$

$$\overrightarrow{PQ} = \langle -6 - 0, 3 - 0 \rangle$$

$$= \langle -6, 3 \rangle$$

Use the Distance Formula to find the magnitude.

$$|\overrightarrow{PQ}| = \sqrt{(-6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$



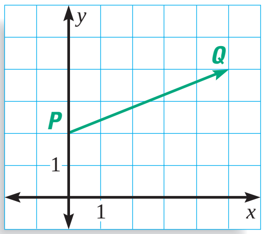

**b.** Component form $= \langle x_2 - x_1, y_2 - y_1 \rangle$

$$\overrightarrow{PQ} = \langle 5 - 0, 4 - 2 \rangle$$

$$= \langle 5, 2 \rangle$$

Use the Distance Formula to find the magnitude.

$$|\overrightarrow{PQ}| = \sqrt{(5 - 0)^2 + (4 - 2)^2} = \sqrt{29} \approx 5.4$$



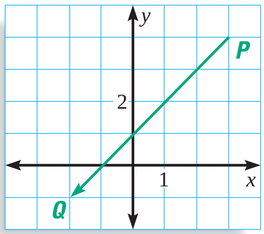

**c.** Component form $= \langle x_2 - x_1, y_2 - y_1 \rangle$

$$\overrightarrow{PQ} = \langle -2 - 3, -1 - 4 \rangle$$

$$= \langle -5, -5 \rangle$$

Use the Distance Formula to find the magnitude.

$$|\overrightarrow{PQ}| = \sqrt{(-2 - 3)^2 + (-1 - 4)^2} = \sqrt{50} \approx 7.1$$




**STUDENT HELP**

**Look Back**
For help with the component form of a vector, see p. 423.

**FOCUS ON APPLICATIONS**

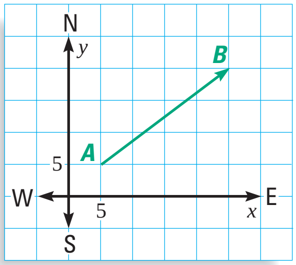
**REAL LIFE**

**NAVIGATION** One of the most common vector quantities in real life is the velocity of a moving object. A velocity vector is used in navigation to describe both the speed and the direction of a moving object.

The **direction of a vector** is determined by the angle it makes with a horizontal line. In real-life applications, the direction angle is described relative to the directions north, east, south, and west. In a coordinate plane, the *x*-axis represents an east-west line. The *y*-axis represents a north-south line.

## EXAMPLE 2 *Describing the Direction of a Vector*

The vector $\overrightarrow{AB}$ describes the velocity of a moving ship. The scale on each axis is in miles per hour.




**a.** Find the speed of the ship.

**b.** Find the direction it is traveling relative to east.

**SOLUTION**

**a.** The magnitude of the vector $\overrightarrow{AB}$ represents the ship's speed. Use the Distance Formula.

$$\begin{aligned} |\overrightarrow{AB}| &= \sqrt{(25-5)^2 + (20-5)^2} \\ &= \sqrt{20^2 + 15^2} \\ &= 25 \end{aligned}$$

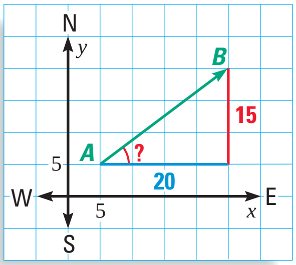
▶ The speed of the ship is 25 miles per hour.

**STUDENT HELP**

**Look Back** For help with using trigonometric ratios to find angle measures, see pp. 567 and 568.

**b.** The tangent of the angle formed by the vector and a line drawn parallel to the *x*-axis is $\frac{15}{20}$, or 0.75. Use a calculator to find the angle measure.

0.75 [2nd] [TAN] $\approx 36.9°$




▶ The ship is traveling in a direction about 37° north of east.

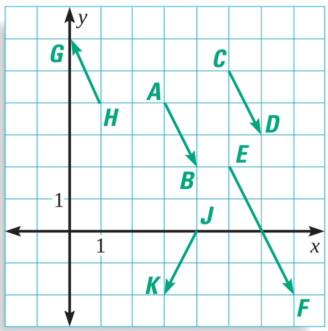
Two vectors are **equal** if they have the same magnitude and direction. They do *not* have to have the same initial and terminal points. Two vectors are **parallel** if they have the same or opposite directions.

## EXAMPLE 3 *Identifying Equal and Parallel Vectors*

In the diagram, these vectors have the same direction: $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{EF}$.

These vectors are equal: $\overrightarrow{AB}$, $\overrightarrow{CD}$.

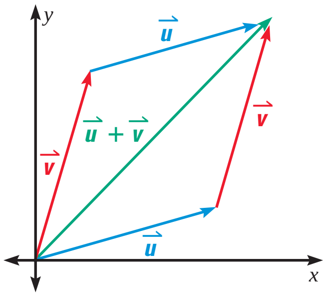
These vectors are parallel: $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{EF}$, $\overrightarrow{HG}$.



## GOAL 2 ADDING VECTORS

> **STUDENT HELP**
>
> **Study Tip**
> A single letter with an arrow over it, such as $\vec{u}$, can be used to denote a vector.

Two vectors can be added to form a new vector. To add $\vec{u}$ and $\vec{v}$ geometrically, place the initial point of $\vec{v}$ on the terminal point of $\vec{u}$, (or place the initial point of $\vec{u}$ on the terminal point of $\vec{v}$). The sum is the vector that joins the initial point of the first vector and the terminal point of the second vector.




This method of adding vectors is often called the *parallelogram rule* because the sum vector is the diagonal of a parallelogram. You can also add vectors algebraically.

### ADDING VECTORS

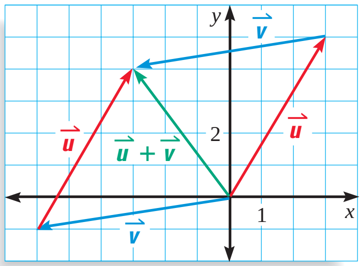
**SUM OF TWO VECTORS**

The **sum** of $\vec{u} = \langle a_1, b_1\rangle$ and $\vec{v} = \langle a_2, b_2\rangle$ is $\vec{u} + \vec{v} = \langle a_1 + a_2, b_1 + b_2\rangle$.

### EXAMPLE 4 *Finding the Sum of Two Vectors*

Let $\vec{u} = \langle 3, 5\rangle$ and $\vec{v} = \langle -6, -1\rangle$. To find the sum vector $\vec{u} + \vec{v}$, add the horizontal components and add the vertical components of $\vec{u}$ and $\vec{v}$.

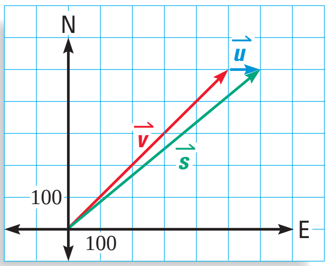
$$\vec{u} + \vec{v} = \langle 3 + (-6), 5 + (-1)\rangle$$
$$= \langle -3, 4\rangle$$




### EXAMPLE 5 *Velocity of a Jet*

**AVIATION** A jet is flying northeast at about 707 miles per hour. Its velocity is represented by the vector $\vec{v} = \langle 500, 500\rangle$.

The jet encounters a wind blowing from the west at 100 miles per hour. The wind velocity is represented by $\vec{u} = \langle 100, 0\rangle$. The jet's new velocity vector $\vec{s}$ is the sum of its original velocity vector and the wind's velocity vector.




$$\vec{s} = \vec{v} + \vec{u}$$
$$= \langle 500 + 100, 500 + 0\rangle$$
$$= \langle 600, 500\rangle$$

The magnitude of the sum vector $\vec{s}$ represents the new speed of the jet.

$$\text{New speed} = |\vec{s}| = \sqrt{(600 - 0)^2 + (500 - 0)^2} \approx 781 \text{ mi/h}$$

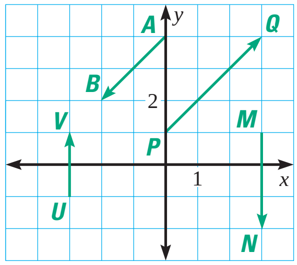
## GUIDED PRACTICE

**Vocabulary Check ✓**

**1.** What is meant by the *magnitude of a vector* and the *direction of a vector*?

**Concept Check ✓**

**In Exercises 2–4, use the diagram.**




**2.** Write the component form of each vector.

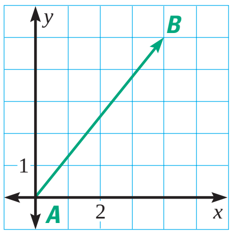
**3.** Identify any parallel vectors.

**4.** Vectors $\overrightarrow{PQ}$ and $\overrightarrow{ST}$ are equal vectors. Although $\overrightarrow{ST}$ is not shown, the coordinates of its initial point are $(-1, -1)$. Give the coordinates of its terminal point.

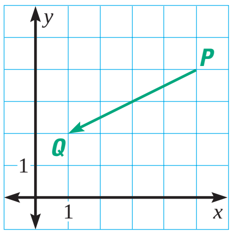
**Skill Check ✓**

**Write the vector in component form. Find the magnitude of the vector. Round your answer to the nearest tenth.**

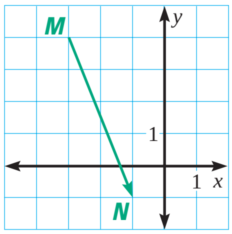
**5.**




**6.**




**7.**




**8.** Use the vector in Exercise 5. Find the direction of the vector relative to east.

**9.** Find the sum of the vectors in Exercises 5 and 6.

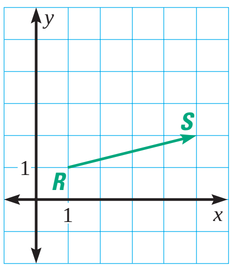
## PRACTICE AND APPLICATIONS

**STUDENT HELP**

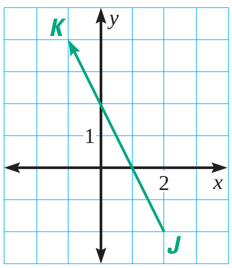
**Extra Practice** to help you master skills is on p. 820.

**FINDING MAGNITUDE** **Write the vector in component form. Find the magnitude of the vector. Round your answer to the nearest tenth.**

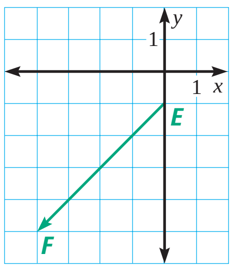
**10.**




**11.**




**12.**




**STUDENT HELP**

**HOMEWORK HELP**
**Example 1:** Exs. 10–20
**Example 2:** Exs. 21–24
**Example 3:** Exs. 25–29
**Example 4:** Exs. 31–40
**Example 5:** Exs. 41–45

**FINDING MAGNITUDE** **Draw vector $\overrightarrow{PQ}$ in a coordinate plane. Write the component form of the vector and find its magnitude. Round your answer to the nearest tenth.**

**13.** $P(0, 0)$, $Q(2, 7)$

**14.** $P(5, 1)$, $Q(2, 6)$

**15.** $P(-3, 2)$, $Q(7, 6)$

**16.** $P(-4, -3)$, $Q(2, -7)$

**17.** $P(5, 0)$, $Q(-1, -4)$

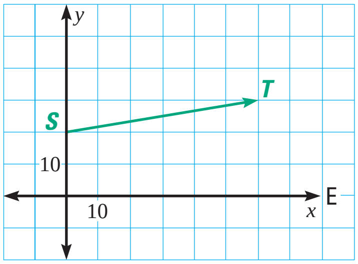
**18.** $P(6, 3)$, $Q(-2, 1)$

**19.** $P(-6, 0)$, $Q(-5, -4)$

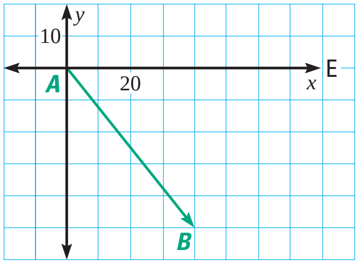
**20.** $P(0, 5)$, $Q(3, 5)$

**NAVIGATION** **The given vector represents the velocity of a ship at sea. Find the ship's speed, rounded to the nearest mile per hour. Then find the direction the ship is traveling relative to the given direction.**

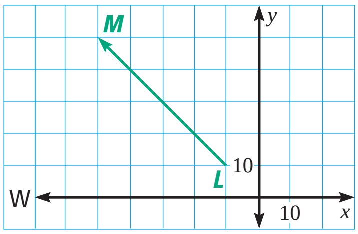
**21.** Find direction relative to east.



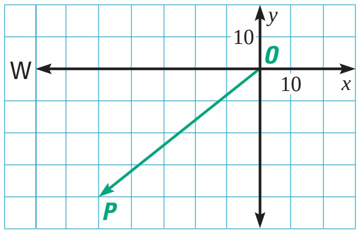

**22.** Find direction relative to east.



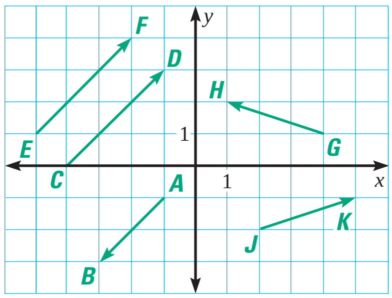

**23.** Find direction relative to west.




**24.** Find direction relative to west.




**PARALLEL AND EQUAL VECTORS** **In Exercises 25–28, use the diagram shown at the right.**




**25.** Which vectors are parallel?

**26.** Which vectors have the same direction?

**27.** Which vectors are equal?

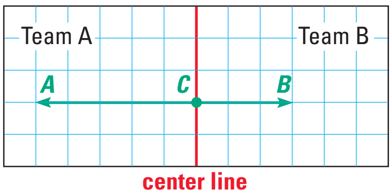
**28.** Name two vectors that have the same magnitude but different directions.

**FOCUS ON APPLICATIONS**

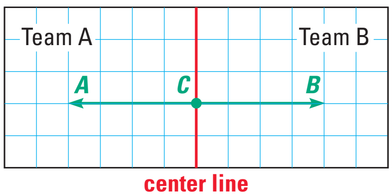
**REAL LIFE** **TUG-OF-WAR** In the game *tug-of-war*, two teams pull on opposite ends of a rope. The team that succeeds in pulling a member of the other team across a center line wins.

**TUG-OF-WAR GAME** **In Exercises 29 and 30, use the information below.** The forces applied in a game of *tug-of-war* can be represented by vectors. The magnitude of the vector represents the amount of force with which the rope is pulled. The direction of the vector represents the direction of the pull. The diagrams below show the forces applied in two different rounds of tug-of-war.

**Round 1**




**Round 2**

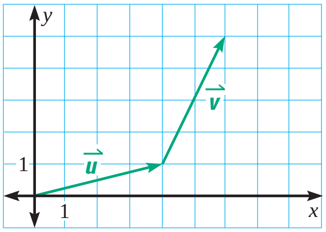



**29.** In Round 2, are $\overrightarrow{CA}$ and $\overrightarrow{CB}$ parallel vectors? Are they equal vectors?

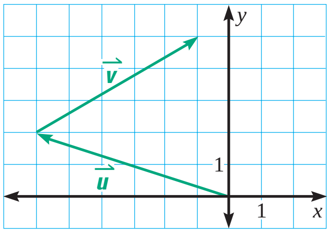
**30.** In which round was the outcome a tie? How do you know? Describe the outcome in the other round. Explain your reasoning.

**PARALLELOGRAM RULE** **Copy the vectors $\vec{u}$ and $\vec{v}$. Write the component form of each vector. Then find the sum $\vec{u} + \vec{v}$ and draw the vector $\vec{u} + \vec{v}$.**

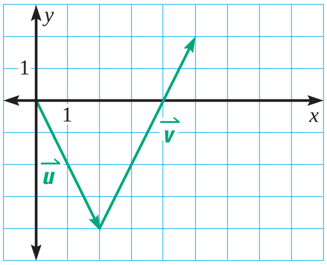
**31.**



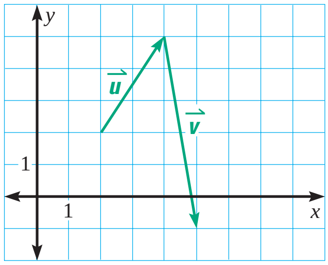

**32.**




**33.**




**34.**




**ADDING VECTORS** **Let $\vec{u} = \langle 7, 3\rangle$, $\vec{v} = \langle 1, 4\rangle$, $\vec{w} = \langle 3, 7\rangle$, and $\vec{z} = \langle -3, -7\rangle$. Find the given sum.**

**35.** $\vec{v} + \vec{w}$

**36.** $\vec{u} + \vec{v}$

**37.** $\vec{u} + \vec{w}$

**38.** $\vec{v} + \vec{z}$

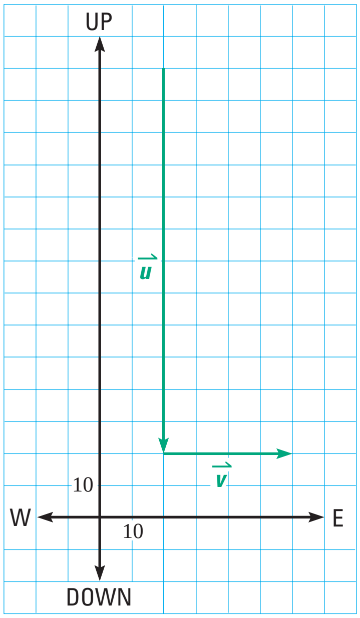
**39.** $\vec{u} + \vec{z}$

**40.** $\vec{w} + \vec{z}$

**FOCUS ON APPLICATIONS**

**REAL LIFE** **SKYDIVING** A skydiver who has not yet opened his or her parachute is in *free fall*. During free fall, the skydiver accelerates at first. Air resistance eventually stops this acceleration, and the skydiver falls at *terminal velocity*.

**SKYDIVING** **In Exercises 41–45, use the information and diagram below.** A skydiver is falling at a constant downward velocity of 120 miles per hour. In the diagram, vector $\vec{u}$ represents the skydiver's velocity. A steady breeze pushes the skydiver to the east at 40 miles per hour. Vector $\vec{v}$ represents the wind velocity. The scales on the axes of the graph are in miles per hour.




**41.** Write the vectors $\vec{u}$ and $\vec{v}$ in component form.

**42.** Let $\vec{s} = \vec{u} + \vec{v}$. Copy the diagram and draw vector $\vec{s}$.

**43.** Find the magnitude of $\vec{s}$. What information does the magnitude give you about the skydiver's fall?

**44.** If there were no wind, the skydiver would fall in a path that was straight down. At what angle to the ground is the path of the skydiver when the skydiver is affected by the 40 mile per hour wind from the west?

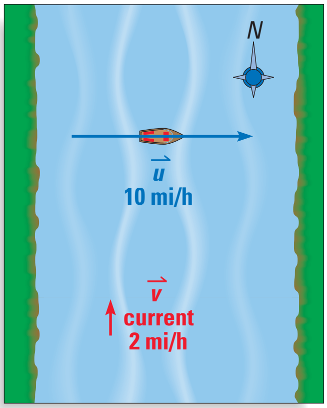
**45.** Suppose the skydiver was blown to the west at 30 miles per hour. Sketch a new diagram and find the skydiver's new velocity.

**46.** *Writing* Write the component form of a vector with the same magnitude as $\overrightarrow{JK} = \langle 1, 3 \rangle$ but a different direction. Explain how you found the vector.

**47.** **LOGICAL REASONING** Let vector $\vec{u} = \langle r, s \rangle$. Suppose the horizontal and the vertical components of $\vec{u}$ are multiplied by a constant $k$. The resulting vector is $\vec{v} = \langle kr, ks \rangle$. How are the magnitudes and the directions of $\vec{u}$ and $\vec{v}$ related when $k$ is positive? when $k$ is negative? Justify your answers.

**Test Preparation**

**48.** **MULTI-STEP PROBLEM** A motorboat heads due east across a river at a speed of 10 miles per hour. Vector $\vec{u} = \langle 10, 0 \rangle$ represents the velocity of the motorboat. The current of the river is flowing due north at a speed of 2 miles per hour. Vector $\vec{v} = \langle 0, 2 \rangle$ represents the velocity of the current.



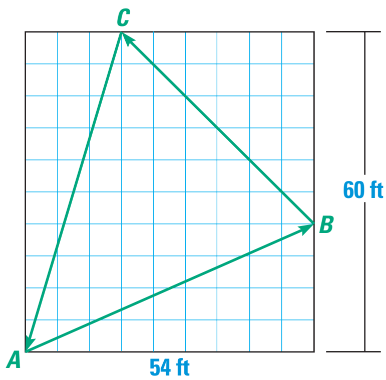

**a.** Let $\vec{s} = \vec{u} + \vec{v}$. Draw the vectors $\vec{u}$, $\vec{v}$, and $\vec{s}$ in a coordinate plane.

**b.** Find the speed and the direction of the motorboat as it is affected by the current.

**c.** Suppose the speed of the motorboat is greater than 10 miles per hour, and the speed of the current is less than 2 miles per hour. Describe one possible set of vectors $\vec{u}$ and $\vec{v}$ that could represent the velocity of the motorboat and the velocity of the current. Write and solve a word problem that can be solved by finding the sum of the two vectors.

**★ Challenge**

**BUMPER CARS** **In Exercises 49–52, use the information below.** As shown in the diagram below, a bumper car moves from point $A$ to point $B$ to point $C$ and back to point $A$. The car follows the path shown by the vectors. The magnitude of each vector represents the distance traveled by the car from the initial point to the terminal point.




**49.** Find the sum of $\overrightarrow{AB}$ and $\overrightarrow{BC}$. Write the sum vector in component form.

**50.** Add vector $\overrightarrow{CA}$ to the sum vector from Exercise 49.

**51.** Find the total distance traveled by the car.

**52.** Compare your answers to Exercises 50 and 51. Why are they different?

EXTRA CHALLENGE
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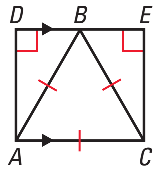
# MIXED REVIEW

**53.** **PROOF** Use the information and the diagram to write a proof. **(Review 4.5)**

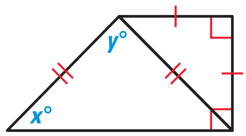



**GIVEN** ▸ $\angle D$ and $\angle E$ are right angles; $\triangle ABC$ is equilateral; $\overline{DE} \parallel \overline{AC}$

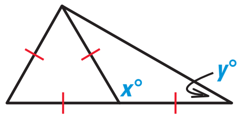
**PROVE** ▸ $B$ is the midpoint of $\overline{DE}$.

**USING ALGEBRA** **Find the values of *x* and *y*.** **(Review 4.6)**

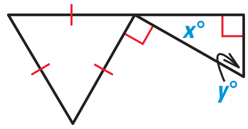
**54.**




**55.**




**56.**




**USING ALGEBRA** **Find the product.** **(Skills Review, p. 798, for 10.1)**

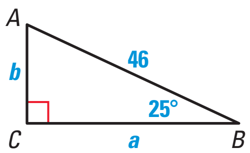
**57.** $(x + 1)^2$ **58.** $(x + 7)^2$ **59.** $(x + 11)^2$ **60.** $(7 + x)^2$

# QUIZ 3

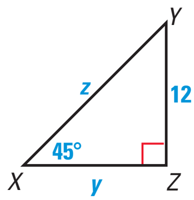
***Self-Test for Lessons 9.6 and 9.7***

**Solve the right triangle. Round decimals to the nearest tenth.** **(Lesson 9.6)**

**1.**




**2.**

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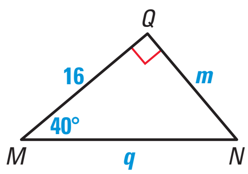


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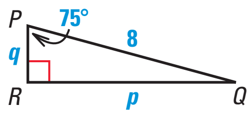


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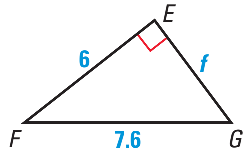


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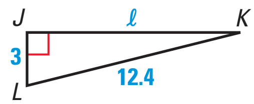


**Draw vector $\overrightarrow{PQ}$ in a coordinate plane. Write the component form of the vector and find its magnitude. Round your answer to the nearest tenth.** **(Lesson 9.7)**

**7.** $P(3, 4)$, $Q(-2, 3)$

**8.** $P(-2, 2)$, $Q(4, -3)$

**9.** $P(0, -1)$, $Q(3, 4)$

**10.** $P(2, 6)$, $Q(-5, -5)$

**11.** Vector $\overrightarrow{ST} = \langle 3, 8 \rangle$. Draw $\overrightarrow{ST}$ in a coordinate plane and find its direction relative to east. **(Lesson 9.7)**

**Let $\vec{u} = \langle 0, -5 \rangle$, $\vec{v} = \langle 4, 7 \rangle$, $\vec{w} = \langle -2, -3 \rangle$, and $\vec{z} = \langle 2, 6 \rangle$. Find the given sum.** **(Lesson 9.7)**

**12.** $\vec{u} + \vec{v}$ **13.** $\vec{v} + \vec{w}$ **14.** $\vec{u} + \vec{w}$

**15.** $\vec{u} + \vec{z}$ **16.** $\vec{v} + \vec{z}$ **17.** $\vec{w} + \vec{z}$