9.6 Solving Right Triangles

What you should learn

GOAL 1 Solve a right triangle.

GOAL 2 Use right triangles to solve **real-life** problems, such as finding the glide angle and altitude of a space shuttle in **Example 3**.

Why you should learn it

▼ To solve **real-life** problems such as determining the correct dimensions of a wheel-chair ramp in **Exs. 39–41**.

REAL LIFE

GOAL 1 SOLVING A RIGHT TRIANGLE

Every right triangle has one right angle, two acute angles, one hypotenuse, and two legs. To **solve a right triangle** means to determine the measures of all six parts. You can solve a right triangle if you know either of the following:

- Two side lengths
- One side length and one acute angle measure

As you learned in Lesson 9.5, you can use the side lengths of a right triangle to find trigonometric ratios for the acute angles of the triangle. As you will see in this lesson, once you know the sine, the cosine, or the tangent of an acute angle, you can use a calculator to find the measure of the angle.

In general, for an acute angle A:

if $\sin A = x$, then $\sin^{-1} x = m\angle A$. ← **The expression $\sin^{-1} x$ is read as "the inverse sine of x."**

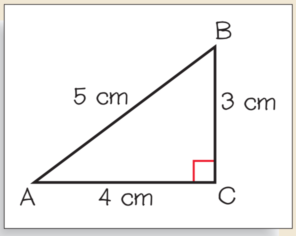
if $\cos A = y$, then $\cos^{-1} y = m\angle A$.

if $\tan A = z$, then $\tan^{-1} z = m\angle A$.

ACTIVITY
Developing Concepts

Finding Angles in Right Triangles

1. Carefully draw right $\triangle ABC$ with side lengths of 3 centimeters, 4 centimeters, and 5 centimeters, as shown.



2. Use trigonometric ratios to find the sine, the cosine, and the tangent of $\angle A$. Express the ratios in decimal form.

3. In Step 2, you found that $\sin A = \frac{3}{5} = 0.6$. You can use a calculator to find $\sin^{-1} 0.6$. Most calculators use one of the keystroke sequences below.

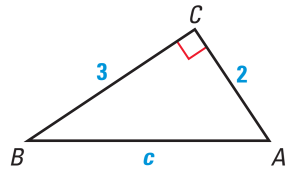
   $\underbrace{\text{2nd} \quad \text{SIN}}_{\sin^{-1}}$ 0.6 ENTER or 0.6 $\underbrace{\text{2nd} \quad \text{SIN}}_{\sin^{-1}}$

   Make sure your calculator is in degree mode. Then use each of the trigonometric ratios you found in Step 2 to approximate the measure of $\angle A$ to the nearest tenth of a degree.

4. Use a protractor to measure $\angle A$. How does the measured value compare with your calculated values?

EXAMPLE 1 *Solving a Right Triangle*

Solve the right triangle. Round decimals to the nearest tenth.



SOLUTION

Begin by using the Pythagorean Theorem to find the length of the hypotenuse.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Pythagorean Theorem
$c^2 = 3^2 + 2^2$	Substitute.
$c^2 = 13$	Simplify.
$c = \sqrt{13}$	Find the positive square root.
$c \approx 3.6$	Use a calculator to approximate.

Then use a calculator to find the measure of $\angle B$:

(2 ÷ 3) 2nd TAN $\approx 33.7°$

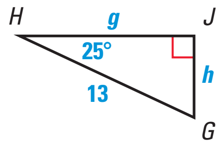
Finally, because $\angle A$ and $\angle B$ are complements, you can write

$$m\angle A = 90° - m\angle B \approx 90° - 33.7° = 56.3°.$$

▶ The side lengths of the triangle are 2, 3, and $\sqrt{13}$, or about 3.6. The triangle has one right angle and two acute angles whose measures are about 33.7° and 56.3°.

EXAMPLE 2 *Solving a Right Triangle*

Solve the right triangle. Round decimals to the nearest tenth.



SOLUTION

Use trigonometric ratios to find the values of g and h.

$\sin H = \frac{\text{opp.}}{\text{hyp.}}$	$\cos H = \frac{\text{adj.}}{\text{hyp.}}$
$\sin 25° = \frac{h}{13}$	$\cos 25° = \frac{g}{13}$
$13 \sin 25° = h$	$13 \cos 25° = g$
$13(0.4226) \approx h$	$13(0.9063) \approx g$
$5.5 \approx h$	$11.8 \approx g$

Because $\angle H$ and $\angle G$ are complements, you can write

$$m\angle G = 90° - m\angle H = 90° - 25° = 65°.$$

▶ The side lengths of the triangle are about 5.5, 11.8, and 13. The triangle has one right angle and two acute angles whose measures are 65° and 25°.

STUDENT HELP

Study Tip
There are other ways to find the side lengths in Examples 1 and 2. For instance, in Example 2, you can use a trigonometric ratio to find one side length, and then use the Pythagorean Theorem to find the other side length.

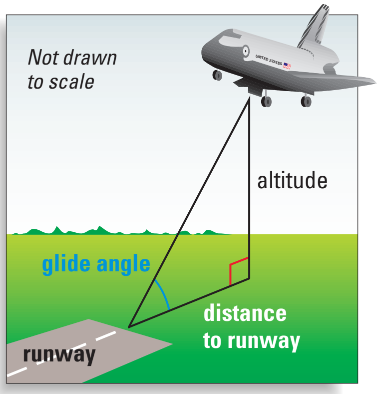
GOAL 2 USING RIGHT TRIANGLES IN REAL LIFE

EXAMPLE 3 *Solving a Right Triangle*

SPACE SHUTTLE During its approach to Earth, the space shuttle's glide angle changes.

a. When the shuttle's altitude is about 15.7 miles, its horizontal distance to the runway is about 59 miles. What is its glide angle? Round your answer to the nearest tenth.

b. When the space shuttle is 5 miles from the runway, its glide angle is about 19°. Find the shuttle's altitude at this point in its descent. Round your answer to the nearest tenth.



SOLUTION

a. Sketch a right triangle to model the situation. Let $x°$ = the measure of the shuttle's glide angle. You can use the tangent ratio and a calculator to find the approximate value of x.

$$\tan x° = \frac{\text{opp.}}{\text{adj.}}$$

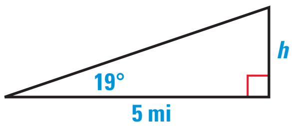
$$\tan x° = \frac{15.7}{59}$$ **Substitute.**

$x =$ (15.7 ÷ 59) 2nd TAN **Use a calculator to find $\tan^{-1}\left(\frac{15.7}{59}\right)$.**

$$x \approx 14.9$$

▶ When the space shuttle's altitude is about 15.7 miles, the glide angle is about 14.9°.

b. Sketch a right triangle to model the situation. Let h = the altitude of the shuttle. You can use the tangent ratio and a calculator to find the approximate value of h.



$$\tan 19° = \frac{\text{opp.}}{\text{adj.}}$$

$$\tan 19° = \frac{h}{5}$$ **Substitute.**

$$0.3443 \approx \frac{h}{5}$$ **Use a calculator.**

$$1.7 \approx h$$ **Multiply each side by 5.**

▶ The shuttle's altitude is about 1.7 miles.

FOCUS ON CAREERS

REAL LIFE **ASTRONAUT** Some astronauts are pilots who are qualified to fly the space shuttle. Some shuttle astronauts are mission specialists whose responsibilities include conducting scientific experiments in space. All astronauts need to have a strong background in science and mathematics.

INTERNET **CAREER LINK** www.mcdougallittell.com

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for extra examples.

Guided Practice

Vocabulary Check ✓

1. Explain what is meant by *solving* a right triangle.

Concept Check ✓

Tell whether the statement is *true* or *false*.

2. You can solve a right triangle if you are given the lengths of any two sides.

3. You can solve a right triangle if you know only the measure of one acute angle.

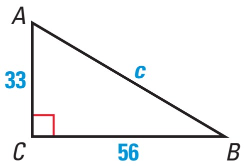
Skill Check ✓

CALCULATOR **In Exercises 4–7, $\angle A$ is an acute angle. Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

4. $\tan A = 0.7$ **5.** $\tan A = 5.4$ **6.** $\sin A = 0.9$ **7.** $\cos A = 0.1$

Solve the right triangle. Round decimals to the nearest tenth.

8.



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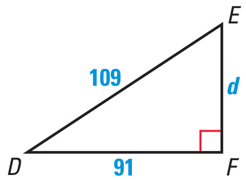

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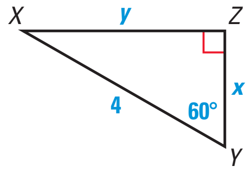


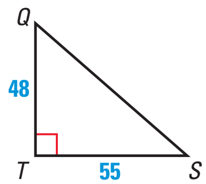
Practice and Applications

STUDENT HELP

Extra Practice to help you master skills is on p. 820.

FINDING MEASUREMENTS **Use the diagram to find the indicated measurement. Round your answer to the nearest tenth.**

11. QS **12.** $m\angle Q$ **13.** $m\angle S$



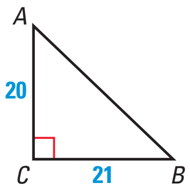
CALCULATOR **In Exercises 14–21, $\angle A$ is an acute angle. Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

14. $\tan A = 0.5$ **15.** $\tan A = 1.0$ **16.** $\sin A = 0.5$ **17.** $\sin A = 0.35$

18. $\cos A = 0.15$ **19.** $\cos A = 0.64$ **20.** $\tan A = 2.2$ **21.** $\sin A = 0.11$

SOLVING RIGHT TRIANGLES **Solve the right triangle. Round decimals to the nearest tenth.**

22.



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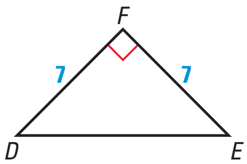

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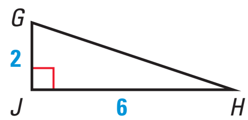

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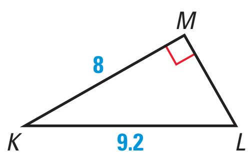

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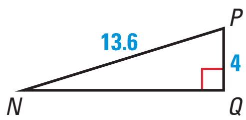

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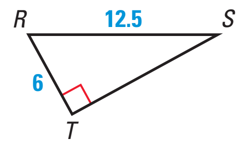


STUDENT HELP

HOMEWORK HELP

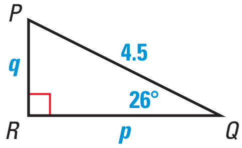
Example 1: Exs. 11–27, 34–37

Example 2: Exs. 28–33

Example 3: Exs. 38–41

SOLVING RIGHT TRIANGLES **Solve the right triangle. Round decimals to the nearest tenth.**

28.



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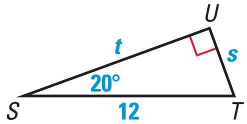

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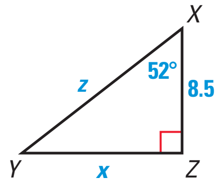

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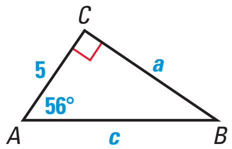

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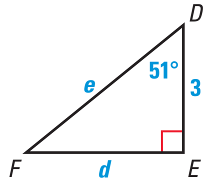

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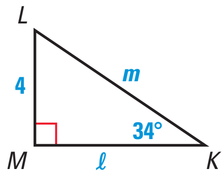


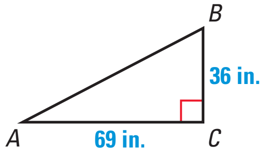
NATIONAL AQUARIUM **Use the diagram of one of the triangular windowpanes at the National Aquarium in Baltimore, Maryland, to find the indicated value.**

34. $\tan B \approx$ ___?___

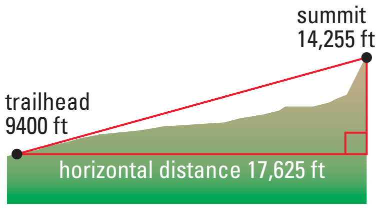
35. $m\angle B \approx$ ___?___

36. $AB \approx$ ___?___

37. $\sin A \approx$ ___?___



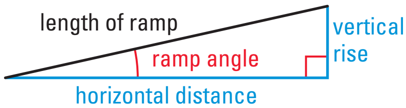
38. **HIKING** You are hiking up a mountain peak. You begin hiking at a trailhead whose elevation is about 9400 feet. The trail ends near the summit at 14,255 feet. The horizontal distance between these two points is about 17,625 feet. Estimate the angle of elevation from the trailhead to the summit.



FOCUS ON APPLICATIONS

REAL LIFE **BENCHMARKS** If you hike to the top of a mountain you may find a brass plate called a *benchmark*. A benchmark gives an official elevation for the point that can be used by surveyors as a reference for surveying elevations of other landmarks.

RAMPS **In Exercises 39–41, use the information about wheelchair ramps.** The Uniform Federal Accessibility Standards specify that the ramp angle used for a wheelchair ramp must be less than or equal to 4.76°.



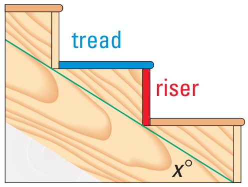
39. The length of one ramp is 20 feet. The vertical rise is 17 inches. Estimate the ramp's horizontal distance and its ramp angle.

40. You want to build a ramp with a vertical rise of 8 inches. You want to minimize the horizontal distance taken up by the ramp. Draw a sketch showing the approximate dimensions of your ramp.

41. *Writing* Measure the horizontal distance and the vertical rise of a ramp near your home or school. Find the ramp angle. Does the ramp meet the specifications described above? Explain.

Test Preparation

MULTI-STEP PROBLEM In Exercises 42–45, use the diagram and the information below.
The horizontal part of a step is called the *tread*. The vertical part is called the *riser*. The ratio of the riser length to the tread length affects the safety of a staircase. Traditionally, builders have used a riser-to-tread ratio of about $8\frac{1}{4}$ inches : 9 inches. A newly recommended ratio is 7 inches : 11 inches.



42. Find the value of x for stairs built using the new riser-to-tread ratio.

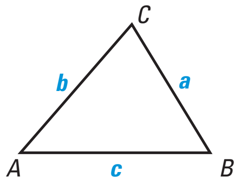
43. Find the value of x for stairs built using the old riser-to-tread ratio.

44. Suppose you want to build a stairway that is less steep than either of the ones in Exercises 42 and 43. Give an example of a riser-to-tread ratio that you could use. Find the value of x for your stairway.

45. *Writing* Explain how the riser-to-tread ratio that is used for a stairway could affect the safety of the stairway.

★ Challenge

46. **PROOF** Write a proof.



GIVEN ▶ $\angle A$ and $\angle B$ are acute angles.

PROVE ▶ $\frac{a}{\sin A} = \frac{b}{\sin B}$

(*Hint:* Draw an altitude from C to $\overline{AB}$. Label it h.)

EXTRA CHALLENGE
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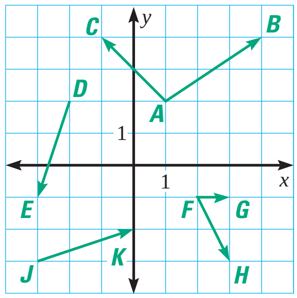
MIXED REVIEW

USING VECTORS Write the component form of the vector. (Review 7.4 for 9.7)



47. $\overrightarrow{AB}$ **48.** $\overrightarrow{AC}$

49. $\overrightarrow{DE}$ **50.** $\overrightarrow{FG}$

51. $\overrightarrow{FH}$ **52.** $\overrightarrow{JK}$

SOLVING PROPORTIONS Solve the proportion. (Review 8.1)

53. $\frac{x}{30} = \frac{5}{6}$ **54.** $\frac{7}{16} = \frac{49}{y}$ **55.** $\frac{3}{10} = \frac{g}{42}$

56. $\frac{7}{18} = \frac{84}{k}$ **57.** $\frac{m}{2} = \frac{7}{1}$ **58.** $\frac{8}{t} = \frac{4}{11}$

CLASSIFYING TRIANGLES Decide whether the numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right*, *acute*, or *obtuse*. (Review 9.3)

59. 18, 14, 2 **60.** 60, 228, 220 **61.** 8.5, 7.7, 3.6

62. 250, 263, 80 **63.** 113, 15, 112 **64.** 15, 75, 59