9.1 Similar Right Triangles

What you should learn

GOAL 1 Solve problems involving similar right triangles formed by the altitude drawn to the hypotenuse of a right triangle.

GOAL 2 Use a geometric mean to solve problems, such as estimating a climbing distance in **Ex. 32**.

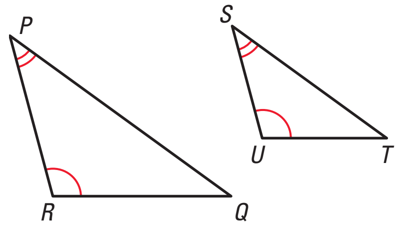
Why you should learn it

▼ You can use right triangles and a geometric mean to help you estimate distances, such as finding the approximate height of a monorail track in **Example 3**.

REAL LIFE

GOAL 1 PROPORTIONS IN RIGHT TRIANGLES

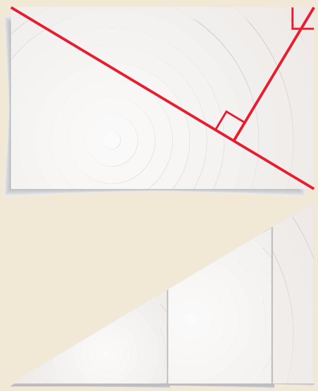
In Lesson 8.4, you learned that two triangles are similar if two of their corresponding angles are congruent. For example, $\triangle PQR \sim \triangle STU$. Recall that the corresponding side lengths of similar triangles are in proportion.



In the activity, you will see how a right triangle can be divided into two similar right triangles.

ACTIVITY Developing Concepts — Investigating Similar Right Triangles

1. Cut an index card along one of its diagonals.
2. On one of the right triangles, draw an altitude from the right angle to the hypotenuse. Cut along the altitude to form two right triangles.
3. You should now have three right triangles. Compare the triangles. What special property do they share? Explain.

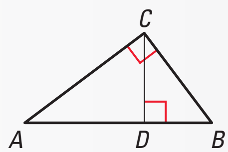


In the activity, you may have discovered the following theorem. A plan for proving the theorem appears on page 528, and you are asked to prove it in Exercise 34 on page 533.

THEOREM

THEOREM 9.1

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

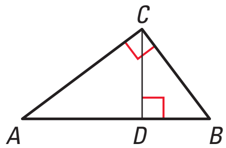


$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$

A plan for proving Theorem 9.1 is shown below.

GIVEN ▶ $\triangle ABC$ is a right triangle; altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

PROVE ▶ $\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.



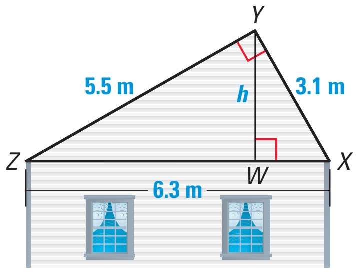
Plan for Proof First prove that $\triangle CBD \sim \triangle ABC$. Each triangle has a right angle, and each includes $\angle B$. The triangles are similar by the AA Similarity Postulate. You can use similar reasoning to show that $\triangle ACD \sim \triangle ABC$. To show that $\triangle CBD \sim \triangle ACD$, begin by showing that $\angle ACD \cong \angle B$ because they are both complementary to $\angle DCB$. Then you can use the AA Similarity Postulate.

EXAMPLE 1 *Finding the Height of a Roof*

REAL LIFE **ROOF HEIGHT** A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section.

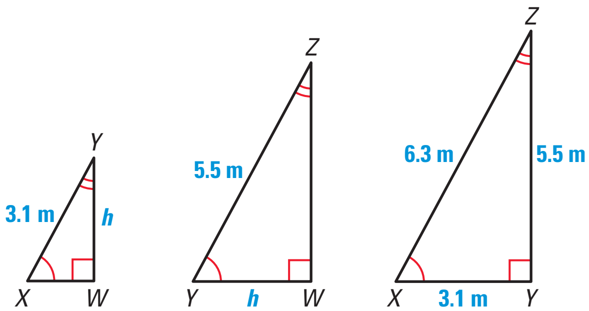
a. Identify the similar triangles.

b. Find the height h of the roof.



SOLUTION

a. You may find it helpful to sketch the three similar right triangles so that the corresponding angles and sides have the same orientation. Mark the congruent angles. Notice that some sides appear in more than one triangle. For instance, $\overline{XY}$ is the hypotenuse in $\triangle XYW$ and the shorter leg in $\triangle XZY$.



▶ $\triangle XYW \sim \triangle YZW \sim \triangle XZY$

b. Use the fact that $\triangle XYW \sim \triangle XZY$ to write a proportion.

$\dfrac{YW}{ZY} = \dfrac{XY}{XZ}$	**Corresponding side lengths are in proportion.**
$\dfrac{h}{5.5} = \dfrac{3.1}{6.3}$	**Substitute.**
$6.3h = 5.5(3.1)$	**Cross product property**
$h \approx 2.7$	**Solve for *h*.**

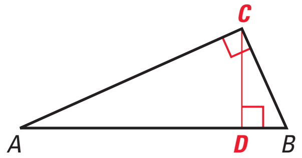
▶ The height of the roof is about 2.7 meters.

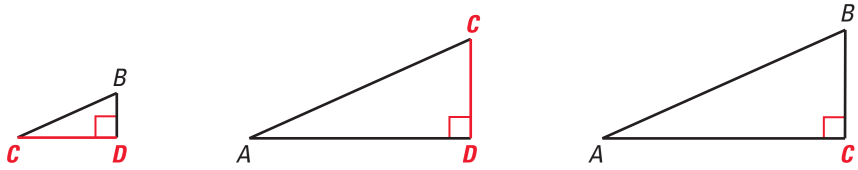
STUDENT HELP

Study Tip
In Example 1, all the side lengths of $\triangle XZY$ are given. This makes it a good choice for setting up a proportion to find an unknown side length of $\triangle XYW$.

GOAL 2 USING A GEOMETRIC MEAN TO SOLVE PROBLEMS

In right $\triangle ABC$, altitude $\overline{CD}$ is drawn to the hypotenuse, forming two smaller right triangles that are similar to $\triangle ABC$. From Theorem 9.1, you know that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.





STUDENT HELP

Look Back
The geometric mean of two numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. For more help with finding a geometric mean, see p. 466.

Notice that $\overline{CD}$ is the longer leg of $\triangle CBD$ and the shorter leg of $\triangle ACD$. When you write a proportion comparing the leg lengths of $\triangle CBD$ and $\triangle ACD$, you can see that CD is the *geometric mean* of BD and AD.

$$\frac{\text{shorter leg of } \triangle CBD}{\text{shorter leg of } \triangle ACD} \quad \frac{BD}{CD} = \frac{CD}{AD} \quad \frac{\text{longer leg of } \triangle CBD}{\text{longer leg of } \triangle ACD}$$

Sides $\overline{CB}$ and $\overline{AC}$ also appear in more than one triangle. Their side lengths are also geometric means, as shown by the proportions below:

$$\frac{\text{hypotenuse of } \triangle ABC}{\text{hypotenuse of } \triangle CBD} \quad \frac{AB}{CB} = \frac{CB}{DB} \quad \frac{\text{shorter leg of } \triangle ABC}{\text{shorter leg of } \triangle CBD}$$

$$\frac{\text{hypotenuse of } \triangle ABC}{\text{hypotenuse of } \triangle ACD} \quad \frac{AB}{AC} = \frac{AC}{AD} \quad \frac{\text{longer leg of } \triangle ABC}{\text{longer leg of } \triangle ACD}$$

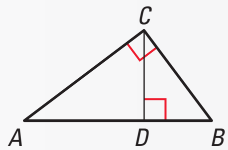
These results are expressed in the theorems below. You are asked to prove the theorems in Exercises 35 and 36.

GEOMETRIC MEAN THEOREMS

THEOREM 9.2

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.



$$\frac{BD}{CD} = \frac{CD}{AD}$$

THEOREM 9.3

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

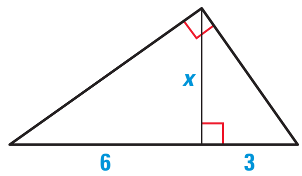
The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

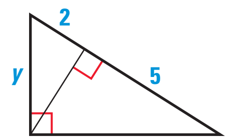
$$\frac{AB}{CB} = \frac{CB}{DB}$$

$$\frac{AB}{AC} = \frac{AC}{AD}$$

EXAMPLE 2 *Using a Geometric Mean*

Find the value of each variable.





STUDENT HELP

Skills Review
For help with simplifying radicals, see p. 799.

Study Tip
In part (a) of Example 2, the equation $18 = x^2$ has two solutions, $+\sqrt{18}$ and $-\sqrt{18}$. Because you are finding a length, you use the *positive* square root.

SOLUTION

a. Apply Theorem 9.2.

$$\frac{6}{x} = \frac{x}{3}$$

$$18 = x^2$$

$$\sqrt{18} = x$$

$$\sqrt{9} \cdot \sqrt{2} = x$$

$$3\sqrt{2} = x$$

b. Apply Theorem 9.3.

$$\frac{5+2}{y} = \frac{y}{2}$$

$$\frac{7}{y} = \frac{y}{2}$$

$$14 = y^2$$

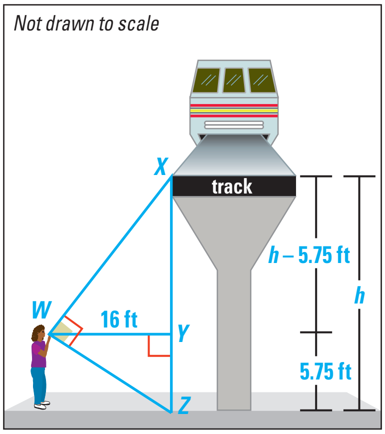
$$\sqrt{14} = y$$

EXAMPLE 3 *Using Indirect Measurement*

FOCUS ON APPLICATIONS

REAL LIFE **MONORAILS** are railways that have only one rail. The Jacksonville monorail, shown in the photo above, can travel up to 35 miles per hour and can carry about 56 passengers per car.

MONORAIL TRACK To estimate the height of a monorail track, your friend holds a cardboard square at eye level. Your friend lines up the top edge of the square with the track and the bottom edge with the ground. You measure the distance from the ground to your friend's eye and the distance from your friend to the track.



In the diagram, $XY = h - 5.75$ is the difference between the track height h and your friend's eye level. Use Theorem 9.2 to write a proportion involving XY. Then you can solve for h.

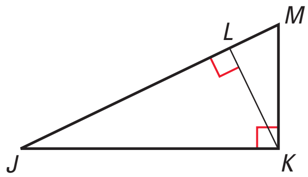
$\frac{XY}{WY} = \frac{WY}{ZY}$	Geometric Mean Theorem 9.2
$\frac{h - 5.75}{16} = \frac{16}{5.75}$	Substitute.
$5.75(h - 5.75) = 16^2$	Cross product property
$5.75h - 33.0625 = 256$	Distributive property
$5.75h = 289.0625$	Add 33.0625 to each side.
$h \approx 50$	Divide each side by 5.75.

▶ The height of the track is about 50 feet.

GUIDED PRACTICE

Vocabulary Check ✓

In Exercises 1–3, use the diagram at the right.



1. In the diagram, KL is the ___?___ of ML and JL.

Concept Check ✓

2. Complete the following statement:
$\triangle JKL \sim \triangle$ ___?___ $\sim \triangle$ ___?___ .

3. Which segment's length is the geometric mean of ML and MJ?

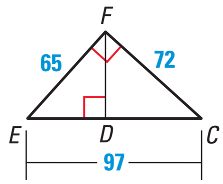
Skill Check ✓

In Exercises 4–9, use the diagram above. Complete the proportion.

4. $\frac{KM}{KL} = \frac{?}{JK}$ **5.** $\frac{JM}{?} = \frac{JK}{JL}$ **6.** $\frac{?}{LK} = \frac{LK}{LM}$

7. $\frac{JM}{?} = \frac{KM}{LM}$ **8.** $\frac{LK}{LM} = \frac{JK}{?}$ **9.** $\frac{?}{JK} = \frac{MK}{MJ}$

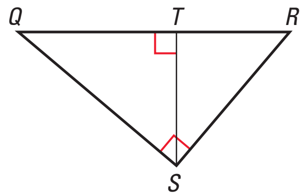
10. Use the diagram at the right. Find DC. Then find DF. Round decimals to the nearest tenth.



PRACTICE AND APPLICATIONS

STUDENT HELP
Extra Practice to help you master skills is on p. 819.

SIMILAR TRIANGLES **Use the diagram.**

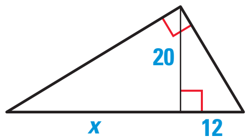


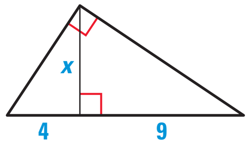
11. Sketch the three similar triangles in the diagram. Label the vertices.

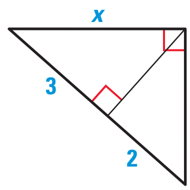
12. Write similarity statements for the three triangles.

USING PROPORTIONS **Complete and solve the proportion.**

13. $\frac{x}{20} = \frac{?}{12}$ **14.** $\frac{4}{x} = \frac{x}{?}$ **15.** $\frac{5}{x} = \frac{x}{?}$

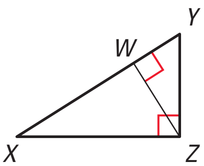


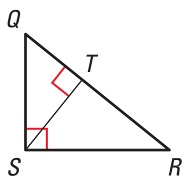


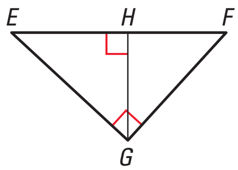


COMPLETING PROPORTIONS **Write similarity statements for the three similar triangles in the diagram. Then complete the proportion.**

16. $\frac{XW}{ZW} = \frac{?}{YW}$ **17.** $\frac{QT}{SQ} = \frac{SQ}{?}$ **18.** $\frac{?}{EG} = \frac{EG}{EF}$



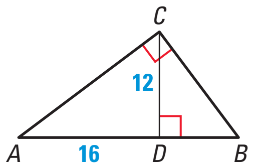




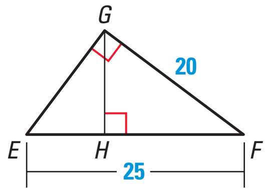
STUDENT HELP
HOMEWORK HELP
Example 1: Exs. 11–31
Example 2: Exs. 11–31
Example 3: Ex. 32

FINDING LENGTHS **Write similarity statements for three triangles in the diagram. Then find the given length. Round decimals to the nearest tenth.**

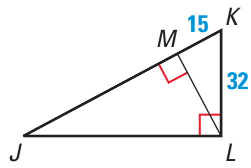
19. Find DB.



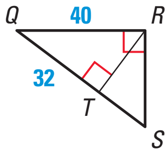
20. Find HF.



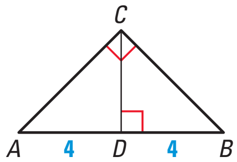
21. Find JK.



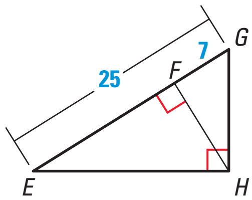
22. Find QS.



23. Find CD.

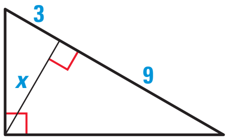


24. Find FH.



USING ALGEBRA **Find the value of each variable.**

25.



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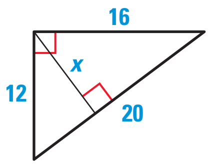

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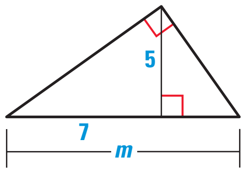
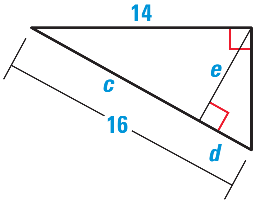


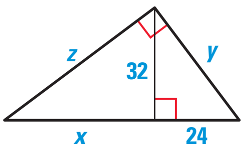
STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with Exs. 28–30.

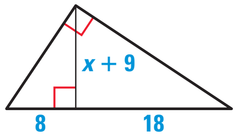
28.



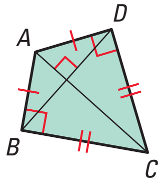
29.



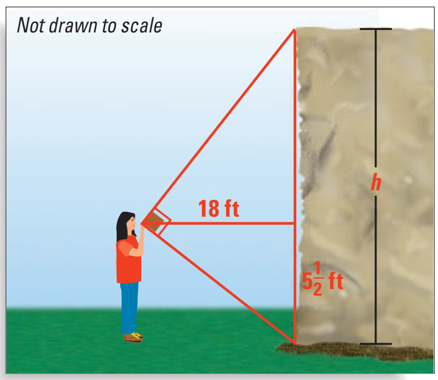
30.



31. **KITE DESIGN** You are designing a diamond-shaped kite. You know that $AD = 44.8$ centimeters, $DC = 72$ centimeters, and $AC = 84.8$ centimeters. You want to use a straight crossbar $\overline{BD}$. About how long should it be? Explain.



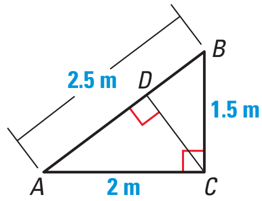
32. **ROCK CLIMBING** You and a friend want to know how much rope you need to climb a large rock. To estimate the height of the rock, you use the method from Example 3 on page 530. As shown at the right, your friend uses a square to line up the top and the bottom of the rock. You measure the vertical distance from the ground to your friend's eye and the distance from your friend to the rock. Estimate the height of the rock.



STUDENT HELP

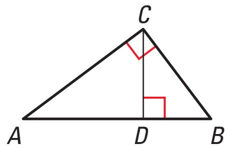
Look Back
For help with finding the area of a triangle, see p. 51.

33. FINDING AREA Write similarity statements for the three similar right triangles in the diagram. Then find the area of each triangle. Explain how you got your answers.



PROVING THEOREMS 9.1, 9.2, AND 9.3 In Exercises 34–36, use the diagram at the right.

34. Use the diagram to prove Theorem 9.1 on page 527. (*Hint:* Look back at the plan for proof on page 528.)



GIVEN ▶ $\triangle ABC$ is a right triangle; altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

PROVE ▶ $\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.

35. Use the diagram to prove Theorem 9.2 on page 529.

GIVEN ▶ $\triangle ABC$ is a right triangle; altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

PROVE ▶ $\frac{BD}{CD} = \frac{CD}{AD}$

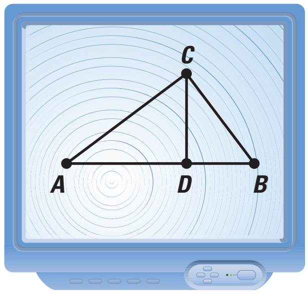
36. Use the diagram to prove Theorem 9.3 on page 529.

GIVEN ▶ $\triangle ABC$ is a right triangle; altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

PROVE ▶ $\frac{AB}{BC} = \frac{BC}{BD}$ and $\frac{AB}{AC} = \frac{AC}{AD}$

USING TECHNOLOGY In Exercises 37–40, use geometry software. You will demonstrate that Theorem 9.2 is true only for a *right* triangle. Follow the steps below to construct a triangle.

1. Draw a triangle and label its vertices A, B, and C. The triangle should *not* be a right triangle.
2. Draw altitude $\overline{CD}$ from point C to side $\overline{AB}$.
3. Measure $\angle C$. Then measure $\overline{AD}$, $\overline{CD}$, and $\overline{BD}$.



37. Calculate the values of the ratios $\frac{BD}{CD}$ and $\frac{CD}{AD}$. What does Theorem 9.2 say about the values of these ratios?

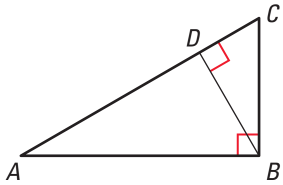
38. Drag point C until $m\angle C = 90°$. What happens to the values of the ratios $\frac{BD}{CD}$ and $\frac{CD}{AD}$?

39. Explain how your answers to Exercises 37 and 38 support the conclusion that Theorem 9.2 is true only for a right triangle.

40. Use the triangle you constructed to show that Theorem 9.3 is true only for a right triangle. Describe your procedure.

Test Preparation

41. MULTIPLE CHOICE Use the diagram at the right. Decide which proportions are true.



I. $\dfrac{DB}{DC} = \dfrac{DA}{DB}$ **II.** $\dfrac{BA}{CB} = \dfrac{CB}{BD}$

III. $\dfrac{CA}{BA} = \dfrac{BA}{CA}$ **IV.** $\dfrac{DB}{BC} = \dfrac{DA}{BA}$

Ⓐ I only Ⓑ II only Ⓒ I and II only Ⓓ I and IV only

42. MULTIPLE CHOICE In the diagram above, $AC = 24$ and $BC = 12$. Find AD. If necessary, round to the nearest hundredth.

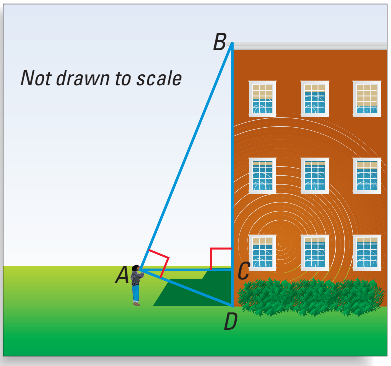
Ⓐ 6 Ⓑ 16.97 Ⓒ 18 Ⓓ 20.78

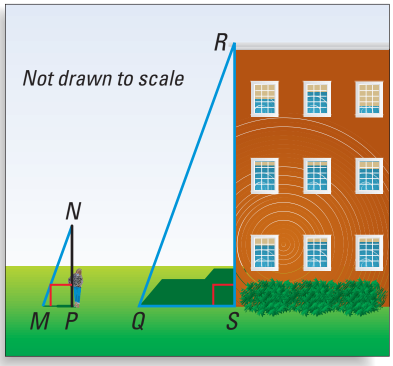
★ Challenge

43. *Writing* Two methods for indirectly measuring the height of a building are shown below. For each method, describe what distances need to be measured directly. Explain how to find the height of the building using these measurements. Describe one advantage and one disadvantage of each method. Copy and label the diagrams as part of your explanations.

Method 1 Use the method described in Example 3 on page 530.

Method 2 Use the method described in Exercises 55 and 56 on page 486.





EXTRA CHALLENGE

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MIXED REVIEW

SOLVING EQUATIONS **Solve the equation.** (Skills Review, p. 800, for 9.2)

44. $n^2 = 169$ **45.** $14 + x^2 = 78$ **46.** $d^2 + 18 = 99$

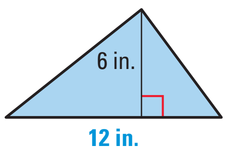
LOGICAL REASONING **Write the converse of the statement. Decide whether the converse is *true* or *false*.** (Review 2.1)

47. If a triangle is obtuse, then one of its angles is greater than 90°.

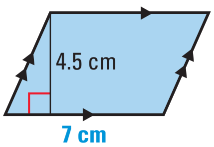
48. If two triangles are congruent, then their corresponding angles are congruent.

FINDING AREA **Find the area of the figure.** (Review 1.7, 6.7 for 9.2)

49.



50.



51.