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## Challenge: Skills and Applications

For use with pages 498-505

1. Figure $A B C$ is a triangle, and $D$ and $E$ are points on $\overleftrightarrow{A B}$ such that $\overrightarrow{C D}$ and $\overrightarrow{C E}$ bisect the interior and exterior angles at $C$, respectively.


Complete the following steps to prove that $\frac{A D}{B D}=\frac{A E}{B E}$.
a. Use the fact that $\overrightarrow{C D}$ is an angle bisector to write a proportion.
b. Copy the diagram. Then draw $\overline{B F}$ such that $F$ is on $\overleftrightarrow{A C}$ and $\overleftrightarrow{B F} \| \overleftrightarrow{C E}$

Use the fact that $\overleftrightarrow{B F} \| \overleftrightarrow{C E}$ to write a proportion involving $A E$ and $B E$.
c. Show that $\triangle F B C$ is an isosceles triangle. (Hint: Use the fact that $\overleftrightarrow{B F} \| \overleftrightarrow{C E}$.)
d. Use the results from parts (a) through (c) to show that $\frac{A D}{B D}=\frac{A E}{B E}$.
2. If $A C=B C$, is the theorem given in Exercise 1 still true? Explain.

## In Exercises 3-11, refer to the diagram in Exercise 1.

3. If $A D=10, B D=8$, and $A C=15$, what are $B C$ and $B E$ ?
4. If $B E=9, B D=6$, and $B C=8$, what are $A C$ and $A D$ ?
5. If $A C=9, B C=6$, and $A E=30$, what are $A D$ and $B E$ ?
6. If $A B=11, B C=10$, and $B D=5$, what are $A C$ and $B E$ ?
7. If $A C=10, A D=5$, and $B E=12$, what are $B C$ and $A E$ ?
8. If $A D=3 x, A C=4 x, B C=x+1$, and $B D=x$, what is $x$ ?
9. If $A D=x+4, B C=x+2, B D=x$, and $A C=B E$, what is $x$ ?
10. If $C D=3$ and $C E=8$, what is $D E$ ?
11. If $m \angle C A B=40^{\circ}$ and $m \angle A D C=110^{\circ}$, what are $m \angle A B C$ and $m \angle B C E$ ?
