

8.7

Dilations

What you should learn

GOAL 1 Identify dilations.

GOAL 2 Use properties of dilations to create a **real-life** perspective drawing in Ex. 34.

Why you should learn it

▼ To solve **real-life** problems, such as estimating the height of the shadow of a shadow puppet in Example 3.



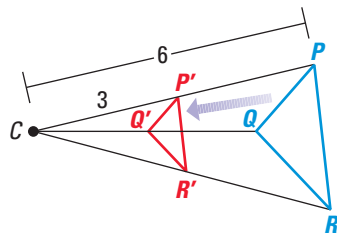
GOAL 1 IDENTIFYING DILATIONS

In Chapter 7, you studied rigid transformations, in which the image and preimage of a figure are *congruent*. In this lesson, you will study a type of nonrigid transformation called a *dilation*, in which the image and preimage of a figure are *similar*.

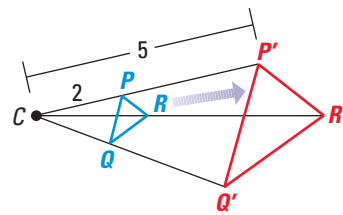
A **dilation** with center C and scale factor k is a transformation that maps every point P in the plane to a point P' so that the following properties are true.

1. If P is not the center point C , then the image point P' lies on \overrightarrow{CP} . The scale factor k is a positive number such that $k = \frac{CP'}{CP}$, and $k \neq 1$.
2. If P is the center point C , then $P = P'$.

The dilation is a **reduction** if $0 < k < 1$ and it is an **enlargement** if $k > 1$.



$$\text{Reduction: } k = \frac{CP'}{CP} = \frac{3}{6} = \frac{1}{2}$$

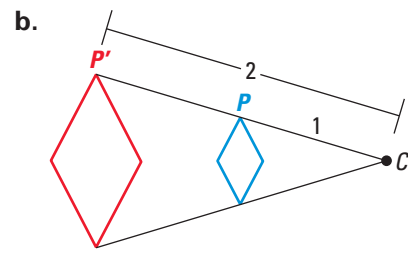
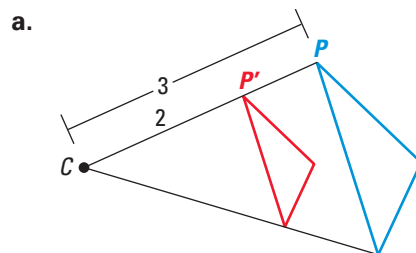


$$\text{Enlargement: } k = \frac{CP'}{CP} = \frac{5}{2}$$

Because $\triangle PQR \sim \triangle P'Q'R'$, $\frac{P'Q'}{PQ}$ is equal to the scale factor of the dilation.

EXAMPLE 1 Identifying Dilations

Identify the dilation and find its scale factor.



SOLUTION

a. Because $\frac{CP'}{CP} = \frac{2}{3}$, the scale factor is $k = \frac{2}{3}$. This is a reduction.

b. Because $\frac{CP'}{CP} = \frac{2}{1}$, the scale factor is $k = 2$. This is an enlargement.

STUDENT HELP

Look Back

For help with the blue to red color scheme used in transformations, see p. 396.

In a coordinate plane, dilations whose centers are the origin have the property that the image of $P(x, y)$ is $P'(kx, ky)$.

EXAMPLE 2 *Dilation in a Coordinate Plane*

Draw a dilation of rectangle $ABCD$ with $A(2, 2)$, $B(6, 2)$, $C(6, 4)$, and $D(2, 4)$. Use the origin as the center and use a scale factor of $\frac{1}{2}$. How does the perimeter of the preimage compare to the perimeter of the image?

SOLUTION

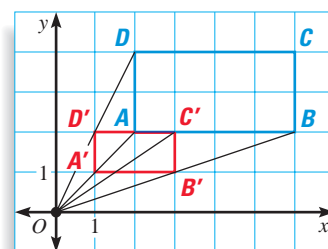
Because the center of the dilation is the origin, you can find the image of each vertex by multiplying its coordinates by the scale factor.

$$A(2, 2) \rightarrow A'(1, 1)$$

$$B(6, 2) \rightarrow B'(3, 1)$$

$$C(6, 4) \rightarrow C'(3, 2)$$

$$D(2, 4) \rightarrow D'(1, 2)$$



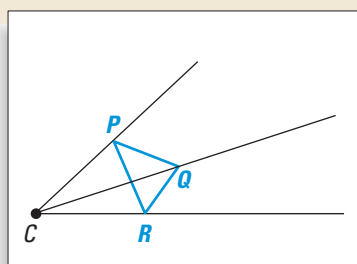
From the graph, you can see that the preimage has a perimeter of 12 and the image has a perimeter of 6. A preimage and its image after a dilation are similar figures. Therefore, the ratio of the perimeters of a preimage and its image is equal to the scale factor of the dilation.

ACTIVITY

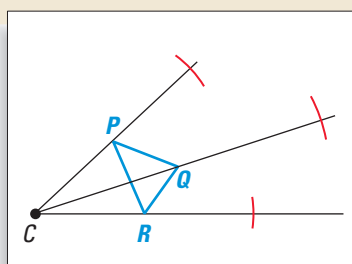
Construction

Drawing a Dilation

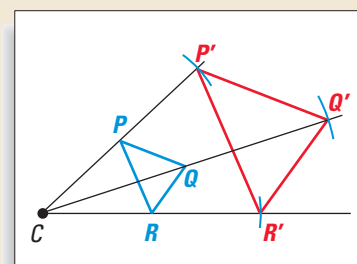
Use the following steps to construct a dilation ($k = 2$) of a triangle using a straightedge and a compass.



- 1 Draw $\triangle PQR$ and choose the center of the dilation C . Use a straightedge to draw lines from C through the vertices of the triangle.



- 2 Use the compass to locate P' on \overrightarrow{CP} so that $CP' = 2(CP)$. Locate Q' and R' in the same way.



- 3 Connect the points P' , Q' , and R' .

In the construction above, notice that $\triangle PQR \sim \triangle P'Q'R'$. You can prove this by using the SAS and SSS Similarity Theorems.

GOAL 2 USING DILATIONS IN REAL LIFE

EXAMPLE 3 Finding the Scale Factor

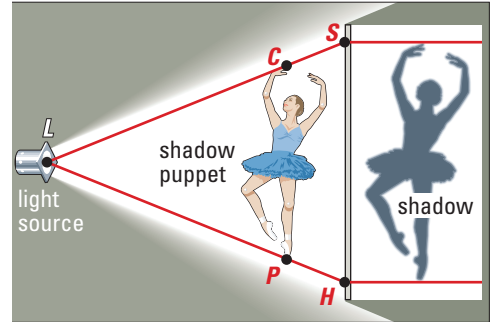
FOCUS ON APPLICATIONS



REAL LIFE SHADOW PUPPETS

Some experienced shadowmaster puppeteers can manipulate over 20 carved leather puppets at the same time.

SHADOW PUPPETS Shadow puppets have been used in many countries for hundreds of years. A flat figure is held between a light and a screen. The audience on the other side of the screen sees the puppet's shadow. The shadow is a dilation, or enlargement, of the shadow puppet. When looking at a cross sectional view, $\triangle LCP \sim \triangle LSH$.



The shadow puppet shown is 12 inches tall (CP in the diagram). Find the height of the shadow, SH , for each distance from the screen. In each case, by what percent is the shadow larger than the puppet?

- $LC = LP = 59$ in.; $LS = LH = 74$ in.
- $LC = LP = 66$ in.; $LS = LH = 74$ in.

SOLUTION

$$\text{a. } \frac{59}{74} = \frac{12}{SH} \qquad \frac{LC}{LS} = \frac{CP}{SH}$$

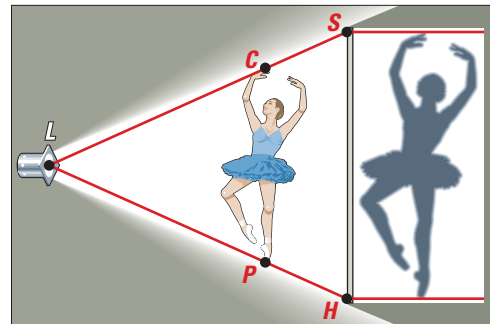
$$59(SH) = 888$$

$$SH \approx 15 \text{ inches}$$

To find the percent of size increase, use the scale factor of the dilation.

$$\begin{aligned} \text{scale factor} &= \frac{SH}{CP} \\ \frac{15}{12} &= 1.25 \end{aligned}$$

► So, the shadow is 25% larger than the puppet.



$$\text{b. } \frac{66}{74} = \frac{12}{SH}$$

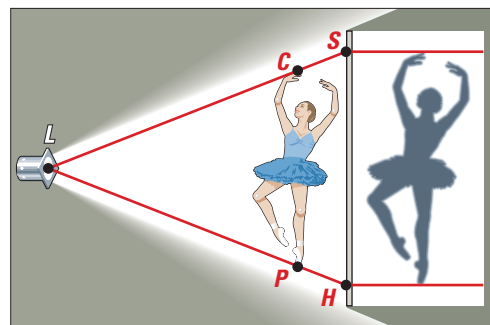
$$66(SH) = 888$$

$$SH \approx 13.45 \text{ inches}$$

Use the scale factor again to find the percent of size increase.

$$\begin{aligned} \text{scale factor} &= \frac{SH}{CP} \\ \frac{13.45}{12} &\approx 1.12 \end{aligned}$$

► So, the shadow is about 12% larger than the puppet.



Notice that as the puppet moves closer to the screen, the shadow height decreases.

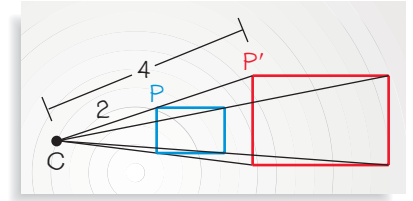
GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

Skill Check ✓

- In a *dilation* every image is ? to its preimage.
- ERROR ANALYSIS** Katie found the scale factor of the dilation shown to be $\frac{1}{2}$. What did Katie do wrong?
- Is the dilation shown a reduction or an enlargement? How do you know?

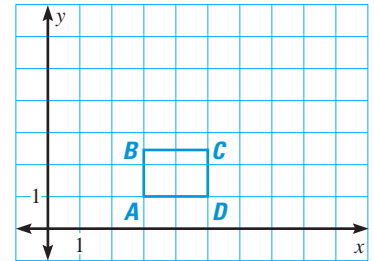


$\triangle PQR$ is mapped onto $\triangle P'Q'R'$ by a dilation with center C . Complete the statement.

- $\triangle PQR$ is (similar, congruent) to $\triangle P'Q'R'$.
- If $\frac{CP'}{CP} = \frac{4}{3}$, then $\triangle P'Q'R'$ is (larger, smaller) than $\triangle PQR$, and the dilation is (a reduction, an enlargement).

Use the following information to draw a dilation of rectangle $ABCD$.

- Draw a dilation of rectangle $ABCD$ on a coordinate plane, with $A(3, 1)$, $B(3, 2.5)$, $C(5, 2.5)$, and $D(5, 1)$. Use the origin as the center and use a scale factor of 2.
- Is $ABCD \sim A'B'C'D'$? Explain your answer.



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 818.

IDENTIFYING DILATIONS Identify the dilation and find its scale factor.

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FINDING SCALE FACTORS Identify the dilation, and find its scale factor. Then, find the values of the variables.

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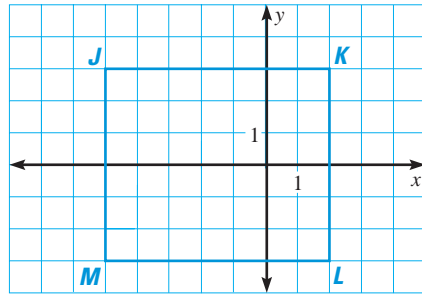
STUDENT HELP

HOMEWORK HELP

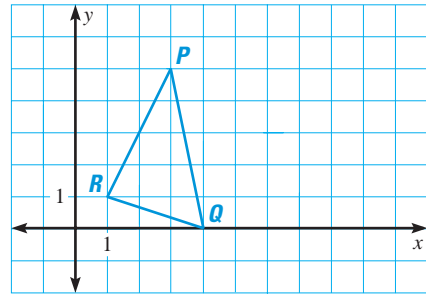
- Example 1:** Exs. 8–11, 20–23
Example 2: Exs. 12–15
Example 3: Exs. 24–26, 33

DILATIONS IN A COORDINATE PLANE Use the origin as the center of the dilation and the given scale factor to find the coordinates of the vertices of the image of the polygon.

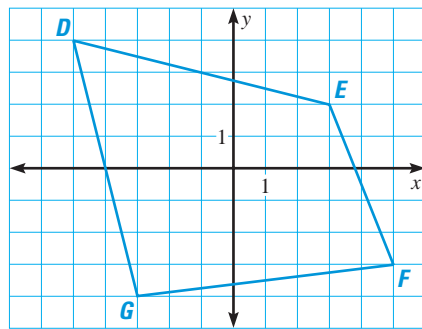
12. $k = \frac{1}{2}$



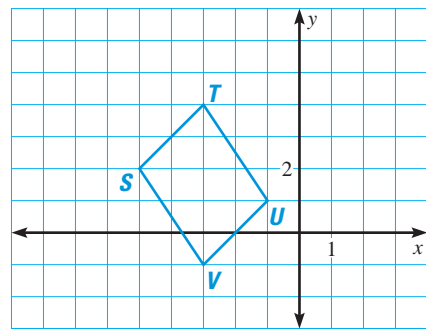
13. $k = 2$



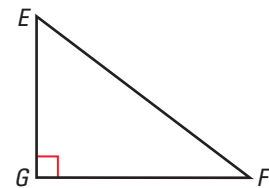
14. $k = \frac{1}{3}$



15. $k = 4$



16. **COMPARING RATIOS** Use the triangle shown. Let P and Q be the midpoints of the sides \overline{EG} and \overline{FG} , respectively. Find the scale factor and the center of the dilation that enlarges $\triangle PQG$ to $\triangle EFG$. Find the ratio of EF to PQ . How does this ratio compare to the scale factor?

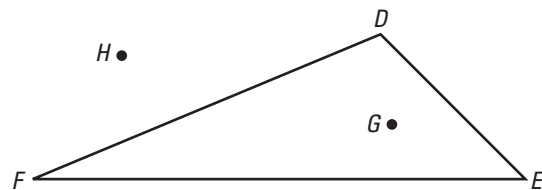


 **CONSTRUCTION** Copy $\triangle DEF$ and points G and H as shown. Then, use a straightedge and a compass to construct the dilation.

17. $k = 3$; Center: G

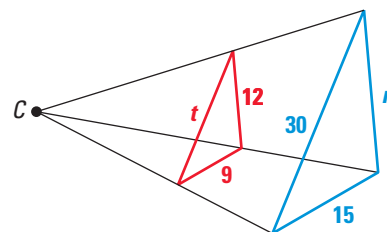
18. $k = \frac{1}{2}$; Center: H

19. $k = 2$; Center: E

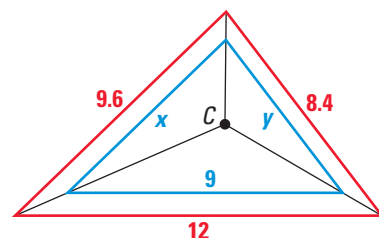


SIMILAR TRIANGLES The red triangle is the image of the blue triangle after a dilation. Find the values of the variables. Then find the ratio of their perimeters.

20.



21.



STUDENT HELP



HOMEWORK HELP

Visit our Web site
www.mcdougallittell.com
for help with problem
solving in Exs. 22 and 23.

IDENTIFYING DILATIONS $\triangle ABC$ is mapped onto $\triangle A'B'C'$ by a dilation. Use the given information to sketch the dilation, identify it as a reduction or an enlargement, and find the scale factor. Then find the missing lengths.

22. In $\triangle ABC$, $AB = 6$, $BC = 9$, and $AC = 12$. In $\triangle A'B'C'$, $A'B' = 2$. Find the lengths of $\overline{B'C'}$ and $\overline{A'C'}$.
23. In $\triangle ABC$, $AB = 5$ and $BC = 7$. In $\triangle A'B'C'$, $A'B' = 20$ and $A'C' = 36$. Find the lengths of \overline{AC} and $\overline{B'C}$.

FLASHLIGHT IMAGE In Exercises 24–26, use the following information.

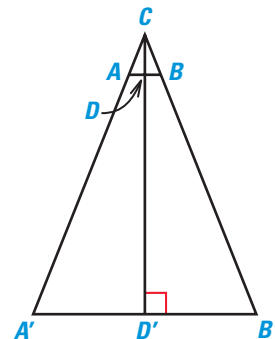
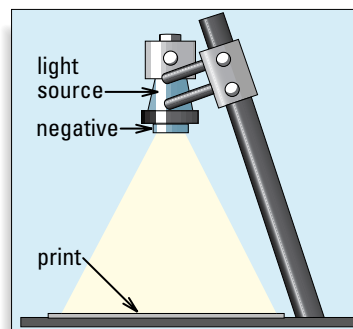
You are projecting images onto a wall with a flashlight. The lamp of the flashlight is 8.3 centimeters away from the wall. The preimage is imprinted onto a clear cap that fits over the end of the flashlight. This cap has a diameter of 3 centimeters. The preimage has a height of 2 centimeters, and the lamp of the flashlight is located 2.7 centimeters from the preimage.

24. Sketch a diagram of the dilation.
25. Find the diameter of the circle of light projected onto the wall from the flashlight.
26. Find the height of the image projected onto the wall.



ENLARGEMENTS In Exercises 27 and 28, use the following information.

By adjusting the distance between the negative and the enlarged print in the photographic enlarger shown, you can make prints of different sizes. In the diagram shown, you want the enlarged print to be 7 inches wide ($A'B'$). The negative is 1 inch wide (AB), and the distance between the light source and the negative is 1.25 inches (CD).



27. What is the scale factor of the enlargement?
28. What is the distance between the light source and the enlarged print?

DIMENSIONS OF PHOTOS Use the diagram from Exercise 27 to determine the missing information.

	CD	CD'	AB	$A'B'$
29.	1.2 in.	7.2 in.	0.8 in.	?
30.	?	14 cm	2 cm	12 cm
31.	2 in.	10 in.	?	8.5 in.

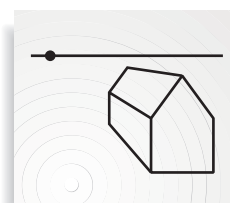
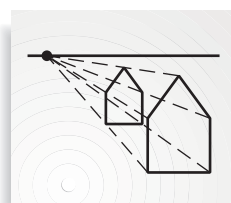
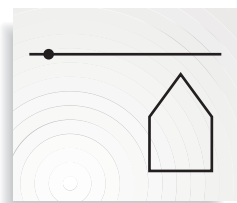


REAL LIFE ARCHITECTURAL RENDERING

An architectural renderer uses techniques like the one shown in Exercise 34 to create three dimensional drawings of buildings and other structures.

CAREER LINK
www.mcdougallittell.com

32. **LOGICAL REASONING** Draw any triangle, and label it $\triangle PQR$. Using a scale factor of 2, draw the image of $\triangle PQR$ after a dilation with a center outside the triangle, with a center inside the triangle, and with a center on the triangle. Explain the relationship between the three images created.
33. **Writing** Use the information about shadow puppet theaters from Example 3, page 508. Explain how you could use a shadow puppet theater to help another student understand the terms *image*, *preimage*, *center of dilation*, and *dilation*. Draw a diagram and label the terms on the diagram.
34. **PERSPECTIVE DRAWING** Create a perspective drawing by following the given steps.

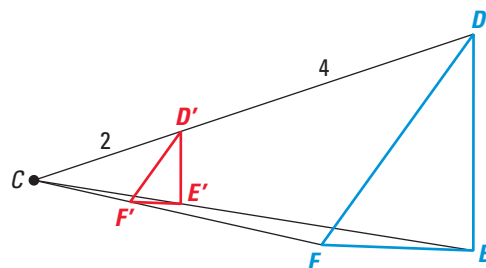


- 1 Draw a horizontal line across the paper, and choose a point on this line to be the center of the dilation, also called the *vanishing point*. Next, draw a polygon.
- 2 Draw rays from the vanishing point to all vertices of the polygon. Draw a reduction of the polygon by locating image points on the rays.
- 3 Connect the preimage to the image by darkening the segments between them. Erase all hidden lines.

Test Preparation

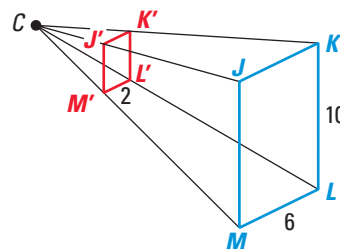
35. **MULTIPLE CHOICE** Identify the dilation shown as an enlargement or reduction and find its scale factor.

- (A) enlargement; $k = 2$
- (B) enlargement; $k = \frac{1}{3}$
- (C) reduction; $k = \frac{1}{3}$
- (D) reduction; $k = \frac{1}{2}$
- (E) reduction; $k = 3$



36. **MULTIPLE CHOICE** In the diagram shown, the center of the dilation of $\square JKLM$ is point C . The length of a side of $\square J'K'L'M'$ is what percent of the length of the corresponding side of $\square JKLM$?

- (A) 3%
- (B) 12%
- (C) 20%
- (D) $33\frac{1}{3}\%$
- (E) 300%

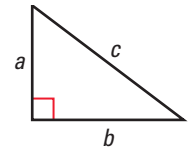


★ Challenge

37. **CREATING NEW IMAGES** A polygon is reduced by a dilation with center C and scale factor $\frac{1}{k}$. The image is then enlarged by a dilation with center C and scale factor k . Describe the size and shape of this new image.

MIXED REVIEW

USING THE PYTHAGOREAN THEOREM Refer to the triangle shown to find the length of the missing side by using the Pythagorean Theorem.



(Review 1.3 for 9.1)

38. $a = 5, b = 12$

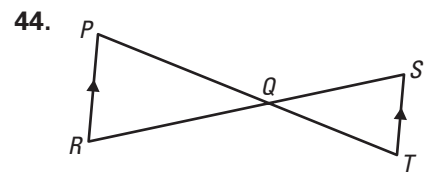
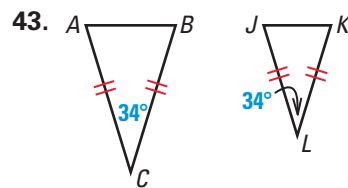
39. $a = 8, c = 2\sqrt{65}$

40. $b = 2, c = 5\sqrt{5}$

41. $b = 1, c = \sqrt{50}$

42. Find the geometric mean of 11 and 44. (Review 8.2 for 9.1)

DETERMINING SIMILARITY Determine whether the triangles can be proved similar or not. Explain your reasoning. (Review 8.4 and 8.5)



QUIZ 3

Self-Test for Lessons 8.6 and 8.7

Use the figure to complete the proportion.

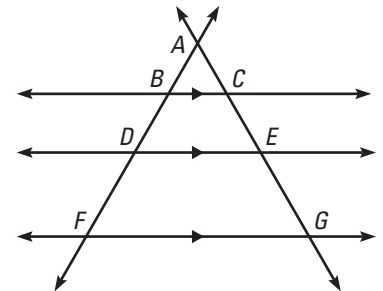
(Lesson 8.6)

1. $\frac{AC}{CE} = \frac{AB}{?}$

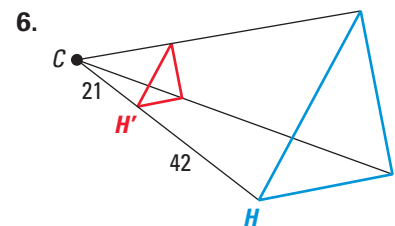
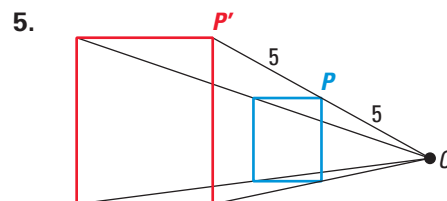
2. $\frac{BD}{BF} = \frac{?}{CG}$

3. $\frac{EG}{AG} = \frac{DF}{?}$

4. $\frac{GA}{EA} = \frac{?}{DA}$



In Exercises 5 and 6, identify the dilation and find its scale factor. (Lesson 8.7)



7. $\triangle JKL$ is mapped onto $\triangle J'K'L'$ by a dilation, with center C . If $\frac{CJ'}{CJ} = \frac{5}{6}$, then the dilation is (a reduction, an enlargement) and $\triangle JKL$ is (larger, smaller) than $\triangle J'K'L'$. (Lesson 8.7)

8. **ENLARGING PHOTOS** An 8 inch by 10 inch photo is enlarged to produce an 18 inch by $22\frac{1}{2}$ inch photo. What is the scale factor? (Lesson 8.7)