8.4 Similar Triangles

What you should learn

GOAL 1 Identify similar triangles.

GOAL 2 Use similar triangles in **real-life** problems, such as using shadows to determine the height of the Great Pyramid in **Ex. 55**.

Why you should learn it

▼ To solve **real-life** problems, such as using similar triangles to understand aerial photography in **Example 4**.

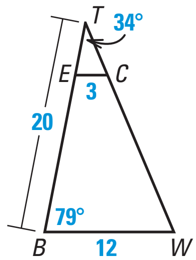
REAL LIFE

GOAL 1 IDENTIFYING SIMILAR TRIANGLES

In this lesson, you will continue the study of similar polygons by looking at properties of similar triangles. The activity that follows Example 1 allows you to explore one of these properties.

EXAMPLE 1 *Writing Proportionality Statements*

In the diagram, $\triangle BTW \sim \triangle ETC$.



a. Write the statement of proportionality.

b. Find $m\angle TEC$.

c. Find ET and BE.

SOLUTION

a. $\dfrac{ET}{BT} = \dfrac{TC}{TW} = \dfrac{CE}{WB}$

b. $\angle B \cong \angle TEC$, so $m\angle TEC = 79°$.

c. $\dfrac{CE}{WB} = \dfrac{ET}{BT}$ — Write proportion.

$\dfrac{3}{12} = \dfrac{ET}{20}$ — Substitute.

$\dfrac{3(20)}{12} = ET$ — Multiply each side by 20.

$5 = ET$ — Simplify.

Because $BE = BT - ET$, $BE = 20 - 5 = 15$.

▶ So, ET is 5 units and BE is 15 units.

ACTIVITY Developing Concepts — Investigating Similar Triangles

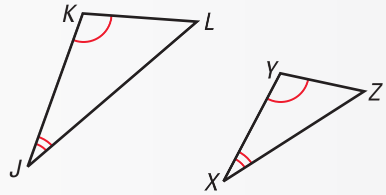
Use a protractor and a ruler to draw two noncongruent triangles so that each triangle has a 40° angle and a 60° angle. Check your drawing by measuring the third angle of each triangle—it should be 80°. Why? Measure the lengths of the sides of the triangles and compute the ratios of the lengths of corresponding sides. Are the triangles similar?

POSTULATE

POSTULATE 25 ***Angle-Angle (AA) Similarity Postulate***

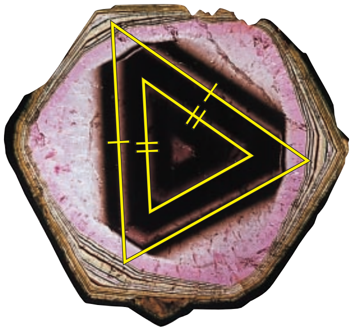
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If $\angle JKL \cong \angle XYZ$ and $\angle KJL \cong \angle YXZ$, then $\triangle JKL \sim \triangle XYZ$.



EXAMPLE 2 *Proving that Two Triangles are Similar*

Color variations in the tourmaline crystal shown lie along the sides of isosceles triangles. In the triangles each vertex angle measures 52°. Explain why the triangles are similar.



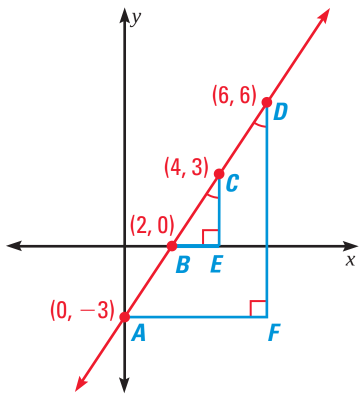
SOLUTION

Because the triangles are isosceles, you can determine that each base angle is 64°. Using the AA Similarity Postulate, you can conclude that the triangles are similar.

EXAMPLE 3 *Why a Line Has Only One Slope*

Using Algebra

Use properties of similar triangles to explain why any two points on a line can be used to calculate the slope. Find the slope of the line using both pairs of points shown.



STUDENT HELP

Look Back
For help with finding slope, see p. 165.

SOLUTION

By the AA Similarity Postulate $\triangle BEC \sim \triangle AFD$, so the ratios of corresponding sides are the same. In particular, $\frac{CE}{DF} = \frac{BE}{AF}$.

By a property of proportions, $\frac{CE}{BE} = \frac{DF}{AF}$.

The slope of a line is the ratio of the change in y to the corresponding change in x. The ratios $\frac{CE}{BE}$ and $\frac{DF}{AF}$ represent the slopes of $\overline{BC}$ and $\overline{AD}$, respectively.

Because the two slopes are equal, any two points on a line can be used to calculate its slope. You can verify this with specific values from the diagram.

$$\text{slope of } \overline{BC} = \frac{3-0}{4-2} = \frac{3}{2}$$

$$\text{slope of } \overline{AD} = \frac{6-(-3)}{6-0} = \frac{9}{6} = \frac{3}{2}$$

FOCUS ON CAREERS

REAL LIFE

AERIAL PHOTOGRAPHER

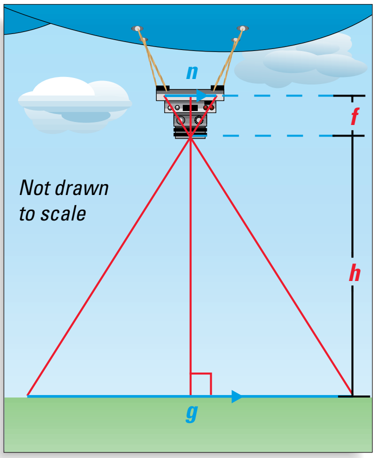
An aerial photographer can take photos from a plane or using a remote-controlled blimp as discussed in Example 4.

INTERNET CAREER LINK
www.mcdougallittell.com

GOAL 2 USING SIMILAR TRIANGLES IN REAL LIFE

EXAMPLE 4 *Using Similar Triangles*

AERIAL PHOTOGRAPHY Low-level aerial photos can be taken using a remote-controlled camera suspended from a blimp. You want to take an aerial photo that covers a ground distance g of 50 meters. Use the proportion $\frac{f}{h} = \frac{n}{g}$ to estimate the altitude h that the blimp should fly at to take the photo. In the proportion, use $f = 8$ cm and $n = 3$ cm. These two variables are determined by the type of camera used.



SOLUTION

$$\frac{f}{h} = \frac{n}{g}$$ Write proportion.

$$\frac{8 \text{ cm}}{h} = \frac{3 \text{ cm}}{50 \text{ m}}$$ Substitute.

$$3h = 400$$ Cross product property

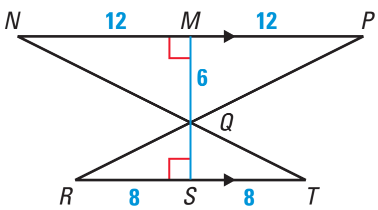
$$h \approx 133$$ Divide each side by 3.

▶ The blimp should fly at an altitude of about 133 meters to take a photo that covers a ground distance of 50 meters.

In Lesson 8.3, you learned that the perimeters of similar polygons are in the same ratio as the lengths of the corresponding sides. This concept can be generalized as follows. If two polygons are similar, then the ratio of *any two corresponding lengths* (such as altitudes, medians, angle bisector segments, and diagonals) is equal to the scale factor of the similar polygons.

EXAMPLE 5 *Using Scale Factors*

Find the length of the altitude $\overline{QS}$.



SOLUTION

Find the scale factor of $\triangle NQP$ to $\triangle TQR$.

$$\frac{NP}{TR} = \frac{12 + 12}{8 + 8} = \frac{24}{16} = \frac{3}{2}$$

Now, because the ratio of the lengths of the altitudes is equal to the scale factor, you can write the following equation.

$$\frac{QM}{QS} = \frac{3}{2}$$

▶ Substitute 6 for QM and solve for QS to show that $QS = 4$.

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

GUIDED PRACTICE

Vocabulary Check ✓

1. If $\triangle ABC \sim \triangle XYZ$, $AB = 6$, and $XY = 4$, what is the *scale factor* of the triangles?

Concept Check ✓

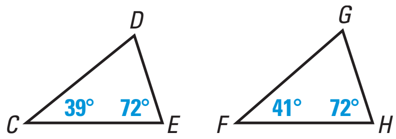
2. The points $A(2, 3)$, $B(-1, 6)$, $C(4, 1)$, and $D(0, 5)$ lie on a line. Which two points could be used to calculate the slope of the line? Explain.

3. Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent?

Skill Check ✓

Determine whether $\triangle CDE \sim \triangle FGH$.

4.



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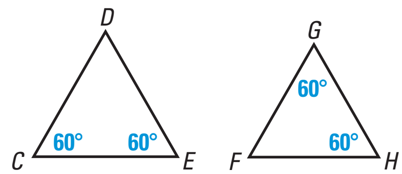
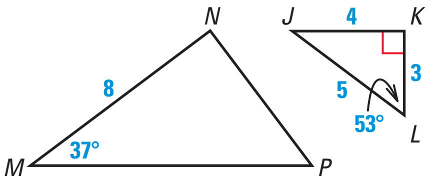


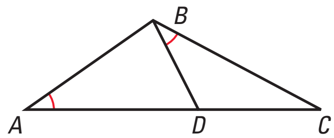
In the diagram shown $\triangle JKL \sim \triangle MNP$.

6. Find $m\angle J$, $m\angle N$, and $m\angle P$.

7. Find MP and PN.



8. Given that $\angle CAB \cong \angle CBD$, how do you know that $\triangle ABC \sim \triangle BDC$? Explain your answer.



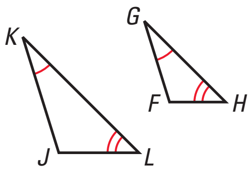
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 818.

USING SIMILARITY STATEMENTS **The triangles shown are similar. List all the pairs of congruent angles and write the statement of proportionality.**

9.



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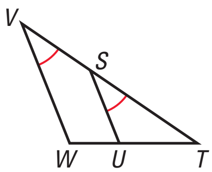

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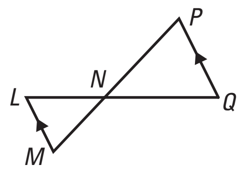


LOGICAL REASONING **Use the diagram to complete the following.**

12. $\triangle PQR \sim$ ___?___

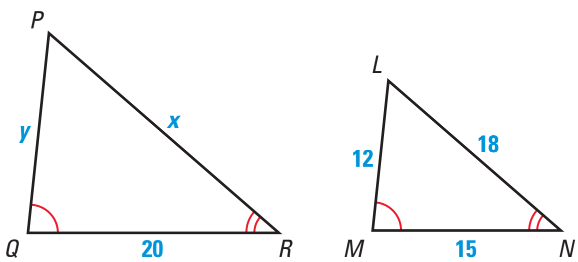
13. $\dfrac{PQ}{?} = \dfrac{QR}{?} = \dfrac{RP}{?}$

14. $\dfrac{20}{?} = \dfrac{?}{12}$

15. $\dfrac{?}{20} = \dfrac{18}{?}$

16. $y =$ ___?___

17. $x =$ ___?___



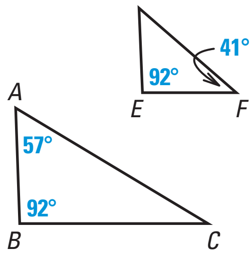
STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 9–17, 33–38
Example 2: Exs. 18–26
Example 3: Exs. 27–32
Example 4: Exs. 39–44, 53, 55, 56
Example 5: Exs. 45–47

DETERMINING SIMILARITY **Determine whether the triangles can be proved similar. If they are similar, write a similarity statement. If they are not similar, explain why.**

18.



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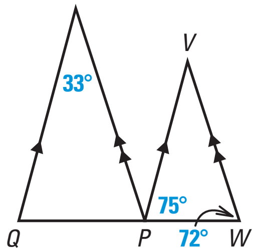

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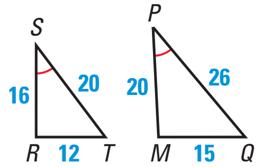

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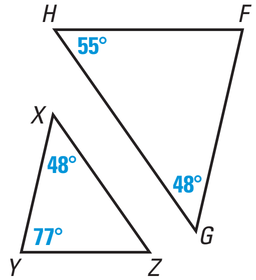

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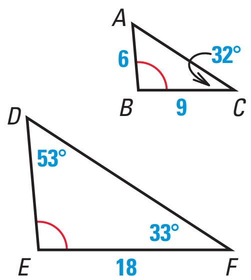

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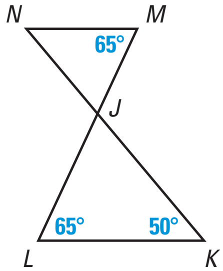

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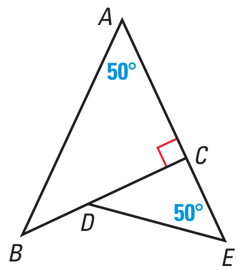

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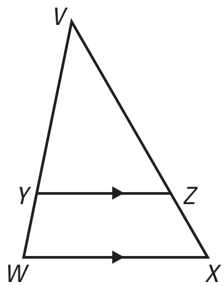

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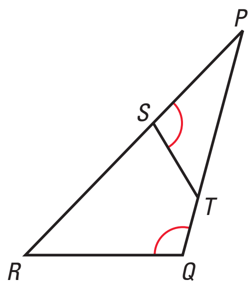
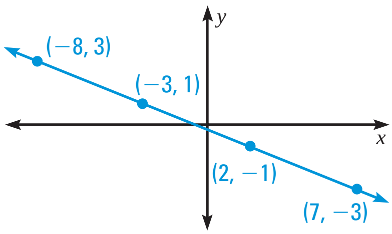
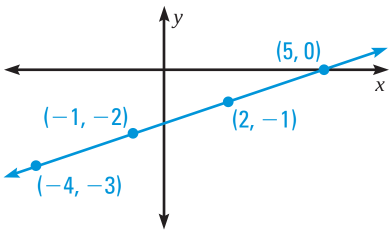


USING ALGEBRA **Using the labeled points, find the slope of the line. To verify your answer, choose another pair of points and find the slope using the new points. Compare the results.**

27.



28.



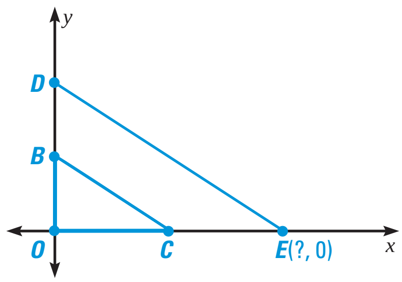
USING ALGEBRA **Find coordinates for point *E* so that $\triangle OBC \sim \triangle ODE$.**

29. $O(0, 0)$, $B(0, 3)$, $C(6, 0)$, $D(0, 5)$

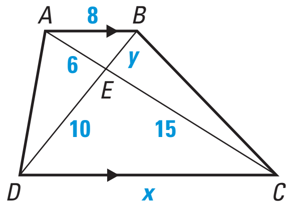
30. $O(0, 0)$, $B(0, 4)$, $C(3, 0)$, $D(0, 7)$

31. $O(0, 0)$, $B(0, 1)$, $C(5, 0)$, $D(0, 6)$

32. $O(0, 0)$, $B(0, 8)$, $C(4, 0)$, $D(0, 9)$



USING ALGEBRA You are given that *ABCD* is a trapezoid, *AB* = 8, *AE* = 6, *EC* = 15, and *DE* = 10.



33. $\triangle ABE \sim \triangle$ _?_

34. $\frac{AB}{?} = \frac{AE}{?} = \frac{BE}{?}$

35. $\frac{6}{?} = \frac{8}{?}$

36. $\frac{15}{?} = \frac{10}{?}$

37. $x =$ _?_

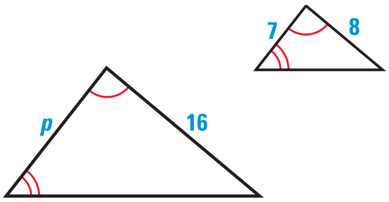
38. $y =$ _?_

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 39–44.

SIMILAR TRIANGLES The triangles are similar. Find the value of the variable.

39.



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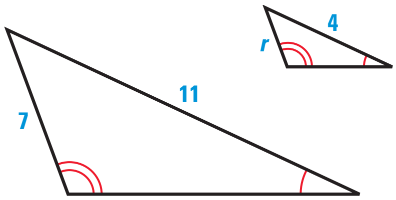

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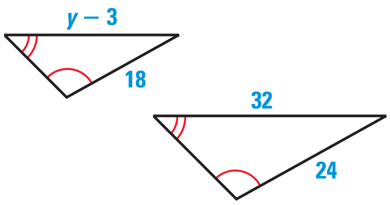
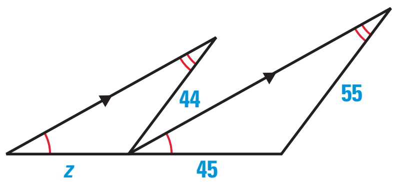


42.



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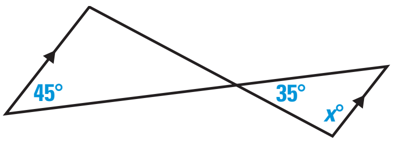

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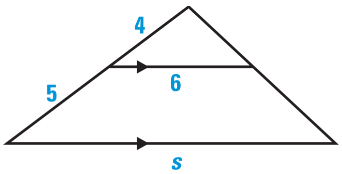
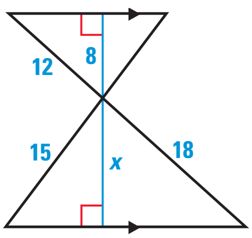


SIMILAR TRIANGLES The segments in blue are special segments in the similar triangles. Find the value of the variable.

45.



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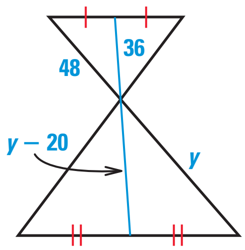
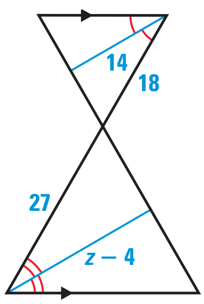


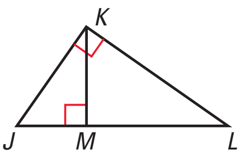
47.



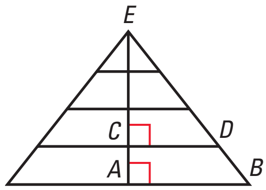
48. **PROOF** Write a paragraph or two-column proof.

GIVEN ▶ $\overline{KM} \perp \overline{JL}, \overline{JK} \perp \overline{KL}$

PROVE ▶ $\triangle JKL \sim \triangle JMK$



49. **PROOF** Write a paragraph proof or a two-column proof. The National Humanities Center is located in Research Triangle Park in North Carolina. Some of its windows consist of nested right triangles, as shown in the diagram. Prove that $\triangle ABE \sim \triangle CDE$.



GIVEN ▶ $\angle ECD$ is a right angle, $\angle EAB$ is a right angle.

PROVE ▶ $\triangle ABE \sim \triangle CDE$

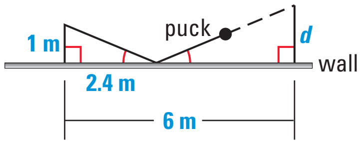
LOGICAL REASONING In Exercises 50–52, decide whether the statement is *true* or *false*. Explain your reasoning.

50. If an acute angle of a right triangle is congruent to an acute angle of another right triangle, then the triangles are similar.

51. Some equilateral triangles are not similar.

52. All isosceles triangles with a 40° vertex angle are similar.

53. **ICE HOCKEY** A hockey player passes the puck to a teammate by bouncing the puck off the wall of the rink as shown. From physics, the angles that the path of the puck makes with the wall are congruent. How far from the wall will the pass be picked up by his teammate?

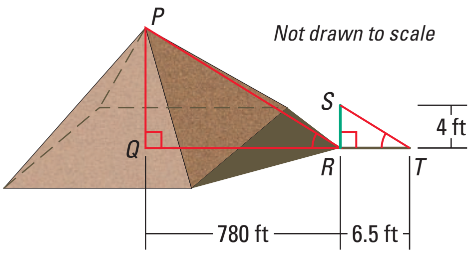


STUDENT HELP

INTERNET SOFTWARE HELP Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

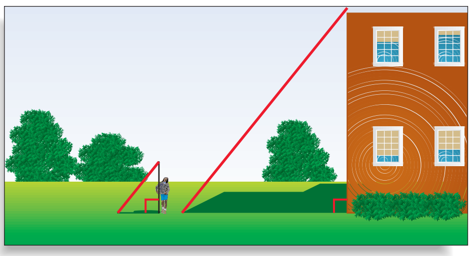
54. **TECHNOLOGY** Use geometry software to verify that any two points on a line can be used to calculate the slope of the line. Draw a line k with a negative slope in a coordinate plane. Draw two right triangles of different size whose hypotenuses lie along line k and whose other sides are parallel to the x- and y-axes. Calculate the slope of each triangle by finding the ratio of the vertical side length to the horizontal side length. Are the slopes equal?

55. **THE GREAT PYRAMID** The Greek mathematician Thales (640–546 B.C.) calculated the height of the Great Pyramid in Egypt by placing a rod at the tip of the pyramid's shadow and using similar triangles.



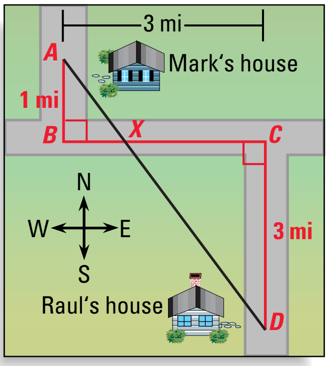
In the figure, $\overline{PQ} \perp \overline{QT}$, $\overline{SR} \perp \overline{QT}$, and $\overline{PR} \parallel \overline{ST}$. Write a paragraph proof to show that the height of the pyramid is 480 feet.

56. **ESTIMATING HEIGHT** On a sunny day, use a rod or pole to estimate the height of your school building. Use the method that Thales used to estimate the height of the Great Pyramid in Exercise 55.



Test Preparation

57. MULTI-STEP PROBLEM Use the following information.
Going from his own house to Raul's house, Mark drives due south one mile, due east three miles, and due south again three miles. What is the distance between the two houses as the crow flies?



a. Explain how to prove that $\triangle ABX \sim \triangle DCX$.

b. Use corresponding side lengths of the triangles to calculate BX.

c. Use the Pythagorean Theorem to calculate AX, and then DX. Then find AD.

d. *Writing* Using the properties of rectangles, explain a way that a point E could be added to the diagram so that $\overline{AD}$ would be the hypotenuse of $\triangle AED$, and $\overline{AE}$ and $\overline{ED}$ would be its legs of known length.

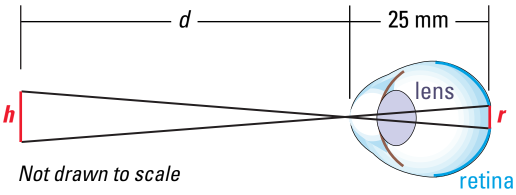
★ Challenge

HUMAN VISION In Exercises 58–60, use the following information.
The diagram shows how similar triangles relate to human vision. An image similar to a viewed object appears on the retina. The actual height of the object h is proportional to the size of the image as it appears on the retina r. In the same manner, the distances from the object to the lens of the eye d and from the lens to the retina, 25 mm in the diagram, are also proportional.

58. Write a proportion that relates r, d, h, and 25 mm.

59. An object that is 10 meters away appears on the retina as 1 mm tall. Find the height of the object.

60. An object that is 1 meter tall appears on the retina as 1 mm tall. How far away is the object?



EXTRA CHALLENGE
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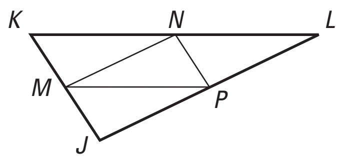
MIXED REVIEW

61. USING THE DISTANCE FORMULA Find the distance between the points $A(-17, 12)$ and $B(14, -21)$. **(Review 1.3)**

TRIANGLE MIDSEGMENTS *M, N,* and *P* are the midpoints of the sides of $\triangle JKL$. Complete the statement.
(Review 5.4 for 8.5)



62. $\overline{NP} \parallel$ ___?___

63. If $NP = 23$, then $KJ =$ ___?___.

64. If $KN = 16$, then $MP =$ ___?___.

65. If $JL = 24$, then $MN =$ ___?___.

PROPORTIONS Solve the proportion. (Review 8.1)

66. $\frac{x}{12} = \frac{3}{8}$

67. $\frac{3}{y} = \frac{12}{32}$

68. $\frac{17}{x} = \frac{11}{33}$

69. $\frac{34}{11} = \frac{x+6}{3}$

70. $\frac{23}{24} = \frac{x}{72}$

71. $\frac{8}{x} = \frac{x}{32}$