# 8.3 Similar Polygons

**What you should learn**

GOAL 1 Identify similar polygons.

GOAL 2 Use similar polygons to solve **real-life** problems, such as making an enlargement similar to an original photo in **Example 3**.

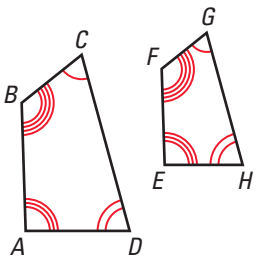
**Why you should learn it**

▼ To solve **real-life** problems, such as comparing television screen sizes in **Exs. 43 and 44**.


## GOAL 1 IDENTIFYING SIMILAR POLYGONS

When there is a correspondence between two polygons such that their corresponding angles are congruent and the lengths of corresponding sides are proportional the two polygons are called **similar polygons**.

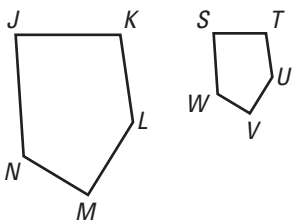
In the diagram, $ABCD$ is similar to $EFGH$. The symbol $\sim$ is used to indicate similarity. So, $ABCD \sim EFGH$.




$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

### EXAMPLE 1 Writing Similarity Statements

Pentagons $JKLMN$ and $STUVW$ are similar. List all the pairs of congruent angles. Write the ratios of the corresponding sides in a statement of proportionality.




**SOLUTION**

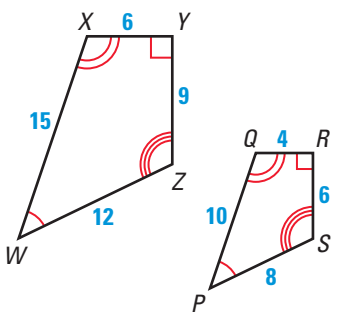
Because $JKLMN \sim STUVW$, you can write $\angle J \cong \angle S$, $\angle K \cong \angle T$, $\angle L \cong \angle U$, $\angle M \cong \angle V$, and $\angle N \cong \angle W$.

You can write the statement of proportionality as follows:

$$\frac{JK}{ST} = \frac{KL}{TU} = \frac{LM}{UV} = \frac{MN}{VW} = \frac{NJ}{WS}.$$

### EXAMPLE 2 Comparing Similar Polygons

Decide whether the figures are similar. If they are similar, write a similarity statement.




**SOLUTION**

As shown, the corresponding angles of $WXYZ$ and $PQRS$ are congruent. Also, the corresponding side lengths are proportional.

$$\frac{WX}{PQ} = \frac{15}{10} = \frac{3}{2} \qquad \frac{XY}{QR} = \frac{6}{4} = \frac{3}{2}$$

$$\frac{YZ}{RS} = \frac{9}{6} = \frac{3}{2} \qquad \frac{ZW}{SP} = \frac{12}{8} = \frac{3}{2}$$

▶ So, the two figures are similar and you can write $WXYZ \sim PQRS$.

**STUDENT HELP**

**Study Tip**

When you refer to similar polygons, their corresponding vertices must be listed in the same order.

## GOAL 2 USING SIMILAR POLYGONS IN REAL LIFE

### EXAMPLE 3 *Comparing Photographic Enlargements*

**POSTER DESIGN** You have been asked to create a poster to advertise a field trip to see the Liberty Bell. You have a 3.5 inch by 5 inch photo that you want to enlarge. You want the enlargement to be 16 inches wide. How long will it be?

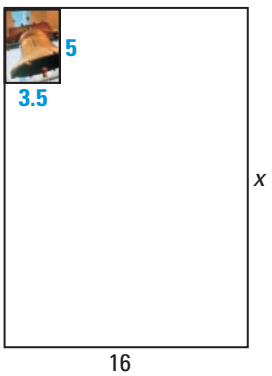
**SOLUTION**

To find the length of the enlargement, you can compare the enlargement to the original measurements of the photo.

$$\frac{16 \text{ in.}}{3.5 \text{ in.}} = \frac{x \text{ in.}}{5 \text{ in.}}$$

$$x = \frac{16}{3.5} \cdot 5$$

$$x \approx 22.9 \text{ inches}$$





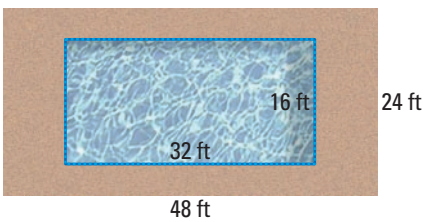
▶ The length of the enlargement will be about 23 inches.

. . . . . .

If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**. In Example 2 on the previous page, the common ratio of $\frac{3}{2}$ is the scale factor of $WXYZ$ to $PQRS$.

### EXAMPLE 4 *Using Similar Polygons*

The rectangular patio around a pool is similar to the pool as shown. Calculate the scale factor of the patio to the pool, and find the ratio of their perimeters.




**SOLUTION**

Because the rectangles are similar, the scale factor of the patio to the pool is 48 ft : 32 ft, which is 3 : 2 in simplified form.

The perimeter of the patio is $2(24) + 2(48) = 144$ feet and the perimeter of the pool is $2(16) + 2(32) = 96$ feet. The ratio of the perimeters is $\frac{144}{96}$, or $\frac{3}{2}$.

. . . . . .

Notice in Example 4 that the ratio of the perimeters is the same as the scale factor of the rectangles. This observation is generalized in the following theorem. You are asked to prove Theorem 8.1 for two similar rectangles in Exercise 45.

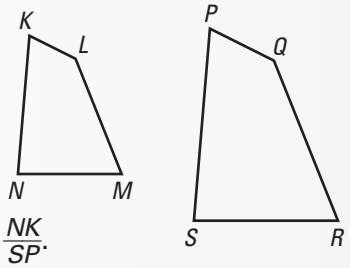
**THEOREM**

**THEOREM 8.1**

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

If $KLMN \sim PQRS$, then

$$\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}.$$

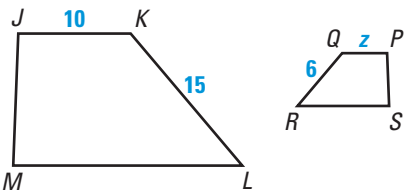



Using Algebra

**EXAMPLE 5** *Using Similar Polygons*

Quadrilateral $JKLM$ is similar to quadrilateral $PQRS$.

Find the value of $z$.




**SOLUTION**

Set up a proportion that contains $PQ$.

$\frac{KL}{QR} = \frac{JK}{PQ}$ **Write proportion.**

$\frac{15}{6} = \frac{10}{z}$ **Substitute.**

$z = 4$ **Cross multiply and divide by 15.**

# GUIDED PRACTICE

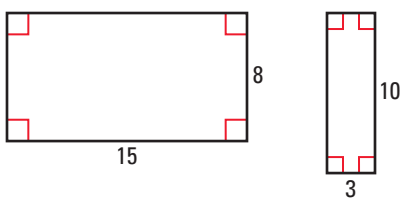
**Vocabulary Check** ✓

**1.** If two polygons are similar, must they also be congruent? Explain.

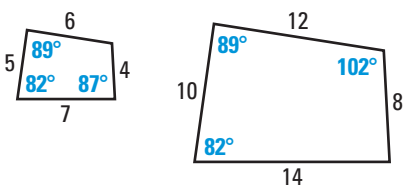
**Concept Check** ✓

**Decide whether the figures are similar. Explain your reasoning.**

**2.**

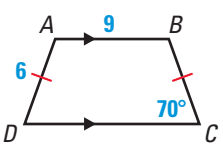



**3.**




**Skill Check** ✓

**In the diagram, $TUVW \sim ABCD$.**

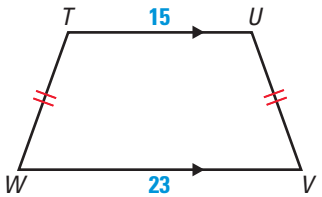



**4.** List all pairs of congruent angles and write the statement of proportionality for the polygons.

**5.** Find the scale factor of $TUVW$ to $ABCD$.

**6.** Find the length of $\overline{TW}$.

**7.** Find the measure of $\angle TUV$.



# PRACTICE AND APPLICATIONS

**STUDENT HELP**

**Extra Practice** to help you master skills is on p. 817.

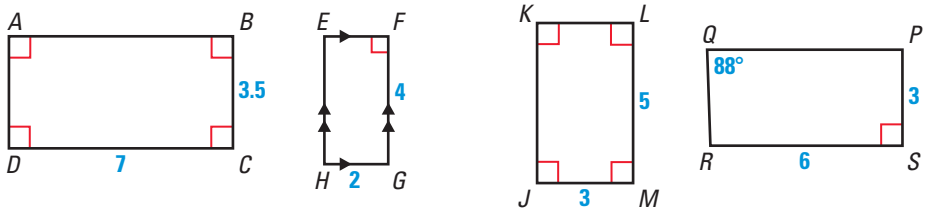
**WRITING SIMILARITY STATEMENTS** **Use the information given to list all pairs of congruent angles and write the statement of proportionality for the figures.**

**8.** $\triangle DEF \sim \triangle PQR$

**9.** $\square JKLM \sim \square WXYZ$

**10.** $QRSTU \sim ABCDE$

**DETERMINING SIMILARITY** **Decide whether the quadrilaterals are similar. Explain your reasoning.**




**11.** *ABCD* and *FGHE*

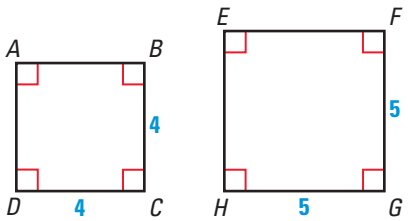
**12.** *ABCD* and *JKLM*

**13.** *ABCD* and *PQRS*

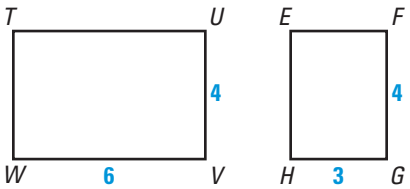
**14.** *JKLM* and *PQRS*

**DETERMINING SIMILARITY** **Decide whether the polygons are similar. If so, write a similarity statement.**

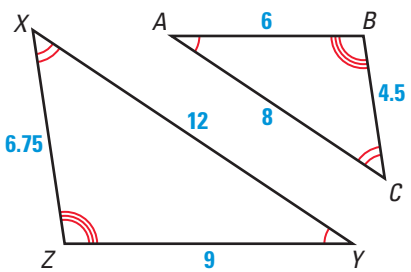
**15.**



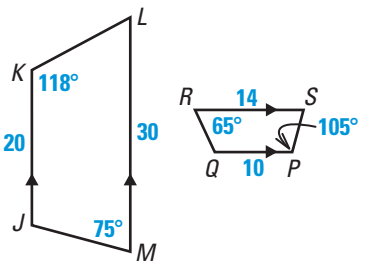

**16.**




**17.**




**18.**




**STUDENT HELP**

**HOMEWORK HELP**

**Example 1:** Exs. 8–10
**Example 2:** Exs. 11–18
**Example 3:** Exs. 19–30, 43, 44
**Example 4:** Exs. 19–30, 46–48
**Example 5:** Exs. 39–42

**USING SIMILAR POLYGONS** ***PQRS*** $\sim$ ***JKLM.***

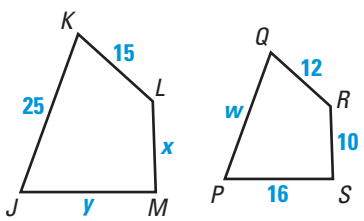
**19.** Find the scale factor of *PQRS* to *JKLM*.

**20.** Find the scale factor of *JKLM* to *PQRS*.

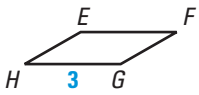
**21.** Find the values of $w$, $x$, and $y$.

**22.** Find the perimeter of each polygon.

**23.** Find the ratio of the perimeter of *PQRS* to the perimeter of *JKLM*.



**USING SIMILAR POLYGONS** $\square ABCD \sim \square EFGH$.

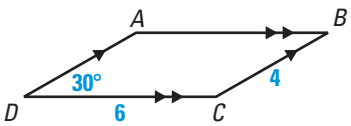



**24.** Find the scale factor of $\square ABCD$ to $\square EFGH$.

**25.** Find the length of $\overline{EH}$.

**26.** Find the measure of $\angle G$.

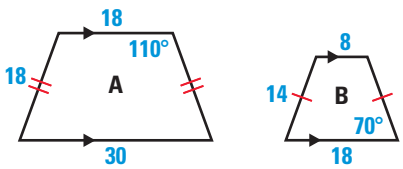
**27.** Find the perimeter of $\square EFGH$.



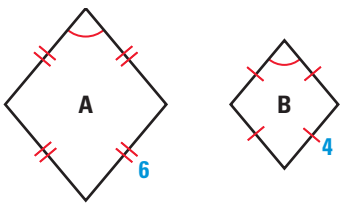

**28.** Find the ratio of the perimeter of $\square EFGH$ to the perimeter of $\square ABCD$.

**DETERMINING SIMILARITY** **Decide whether the polygons are similar. If so, find the scale factor of Figure A to Figure B.**

**29.**




**30.**




**LOGICAL REASONING** **Tell whether the polygons are *always*, *sometimes*, or *never* similar.**

**31.** Two isosceles triangles

**32.** Two regular polygons

**33.** Two isosceles trapezoids

**34.** Two rhombuses

**35.** Two squares

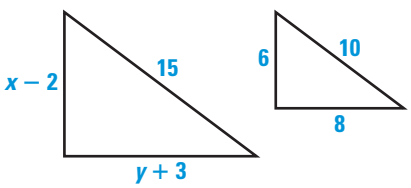
**36.** An isosceles and a scalene triangle

**37.** Two equilateral triangles

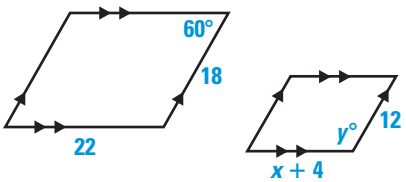
**38.** A right and an isosceles triangle

**USING ALGEBRA** **The two polygons are similar. Find the values of *x* and *y*.**

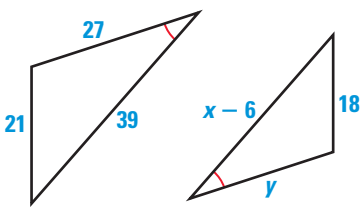
**39.**



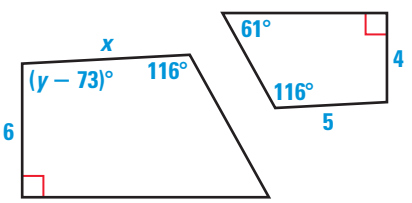

**40.**




**41.**




**42.**




**TV SCREENS** **In Exercises 43 and 44, use the following information.**

Television screen sizes are based on the length of the diagonal of the screen. The *aspect ratio* refers to the length to width ratio of the screen. A standard 27 inch analog television screen has an aspect ratio of 4:3. A 27 inch digital television screen has an aspect ratio of 16:9.

**43.** Make a scale drawing of each television screen. Use proportions and the Pythagorean Theorem to calculate the lengths and widths of the screens in inches.

**44.** Are the television screens similar? Explain.

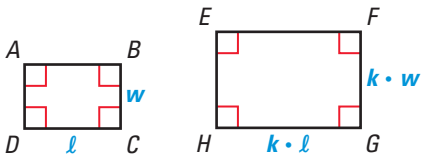
**STUDENT HELP**

**HOMEWORK HELP**
Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 31–38.

**FOCUS ON APPLICATIONS**

**DIGITAL TELEVISION** screens contain over 6 times as many pixels (the tiny dots that make up the picture) as standard analog screens.

**45.** **PROOF** Prove Theorem 8.1 for two similar rectangles.




**GIVEN** ▶ $ABCD \sim EFGH$

**PROVE** ▶ $\frac{\text{perimeter of } ABCD}{\text{perimeter of } EFGH} = \frac{AB}{EF}$

**46.** **SCALE** The ratio of the perimeter of $WXYZ$ to the perimeter of $QRST$ is 7.5 : 2. Find the scale factor of $QRST$ to $WXYZ$.

**47.** **SCALE** The ratio of one side of $\triangle CDE$ to the corresponding side of similar $\triangle FGH$ is 2 : 5. The perimeter of $\triangle FGH$ is 28 inches. Find the perimeter of $\triangle CDE$.

**48.** **SCALE** The perimeter of $\square PQRS$ is 94 centimeters. The perimeter of $\square JKLM$ is 18.8 centimeters, and $\square JKLM \sim \square PQRS$. The lengths of the sides of $\square PQRS$ are 15 centimeters and 32 centimeters. Find the scale factor of $\square PQRS$ to $\square JKLM$, and the lengths of the sides of $\square JKLM$.

## Test Preparation

**49.** **MULTI-STEP PROBLEM** Use the similar figures shown. The scale factor of Figure 1 to Figure 2 is 7 : 10.

**a.** Copy and complete the table.

| | $AB$ | $BC$ | $CD$ | $DE$ | $EA$ |
|---|---|---|---|---|---|
| Figure 1 | ? | ? | ? | ? | ? |
| Figure 2 | 6.0 | 3.0 | 5.0 | 2.0 | 4.0 |

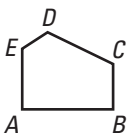


Figure 1

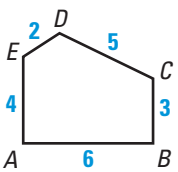


Figure 2

**b.** Graph the data in the table. Let $x$ represent the length of a side in Figure 1 and let $y$ represent the length of the corresponding side in Figure 2. Determine an equation that relates $x$ and $y$.

**c.** **ANALYZING DATA** The equation you obtained in part (b) should be linear. What is its slope? How does its slope compare to the scale factor?

## Challenge

**TOTAL ECLIPSE** **Use the following information in Exercises 50–52.**

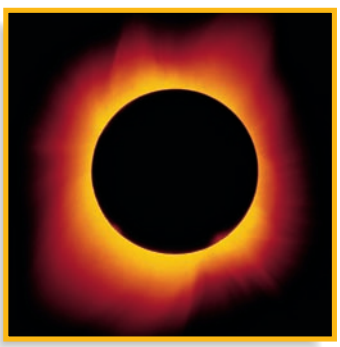
From your perspective on Earth during a total eclipse of the sun, the moon is directly in line with the sun and blocks the sun's rays. The ratio of the radius of the moon to its distance to Earth is about the same as the ratio of the radius of the sun to its distance to Earth.

Distance between Earth and the moon: 240,000 miles

Distance between Earth and the sun: 93,000,000 miles

Radius of the sun: 432,500 miles

**50.** Make a sketch of Earth, the moon, and the sun during a total eclipse of the sun. Include the given distances in your sketch.



**51.** Your sketch should contain some similar triangles. Use the similar triangles in your sketch to explain a total eclipse of the sun.

**52.** Write a statement of proportionality for the similar triangles. Then use the given distances to estimate the radius of the moon.

EXTRA CHALLENGE
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# MIXED REVIEW

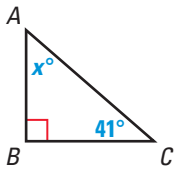
**FINDING SLOPE** **Find the slope of the line that passes through the given points.** (Review 3.6 for 8.4)

**53.** $A(-1, 4), B(3, 8)$ **54.** $P(0, -7), Q(-6, -3)$ **55.** $J(9, 4), K(2, 5)$

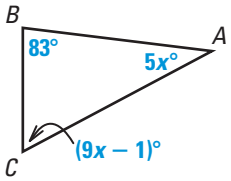
**56.** $L(-2, -3), M(1, 10)$ **57.** $S(-4, 5), T(2, -2)$ **58.** $Y(-1, 6), Z(5, -5)$

**FINDING ANGLE MEASURES** **Find the value of $x$.** (Review 4.1 for 8.4)

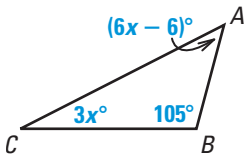
**59.**




**60.**




**61.**




**SOLVING PROPORTIONS** **Solve the proportion.** (Review 8.1)

**62.** $\frac{x}{9} = \frac{6}{27}$ **63.** $\frac{4}{y} = \frac{2}{19}$ **64.** $\frac{5}{24} = \frac{25}{z}$

**65.** $\frac{4}{13} = \frac{b}{8}$ **66.** $\frac{11}{x+2} = \frac{9}{x}$ **67.** $\frac{3x+7}{5} = \frac{4x}{6}$

# QUIZ 1

***Self-Test for Lessons 8.1–8.3***

**Solve the proportions.** (Lesson 8.1)

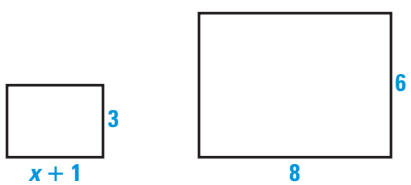
**1.** $\frac{p}{15} = \frac{2}{3}$ **2.** $\frac{5}{7} = \frac{20}{d}$ **3.** $\frac{4}{2x-6} = \frac{16}{x}$

**Find the geometric mean of the two numbers.** (Lesson 8.2)

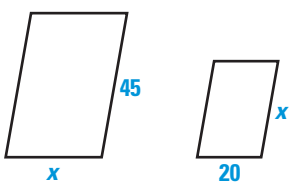
**4.** 7 and 63 **5.** 5 and 11 **6.** 10 and 7

**In Exercises 7 and 8, the two polygons are similar. Find the value of $x$. Then find the scale factor and the ratio of the perimeters.** (Lesson 8.3)

**7.**




**8.**




**COMPARING PHOTO SIZES** **Use the following information.** (Lesson 8.3)

You are ordering your school pictures. You decide to order one $8 \times 10$ (8 inches by 10 inches), two $5 \times 7$'s (5 inches by 7 inches), and 24 wallets $\left(2\frac{1}{4}\text{ inches by }3\frac{1}{4}\text{ inches}\right)$.

**9.** Are any of these sizes similar to each other?

**10.** Suppose you want the wallet photos to be similar to the $8 \times 10$ photo. If the wallet photo were $2\frac{1}{2}$ inches wide, how tall would it be?