8.2 Problem Solving in Geometry with Proportions

What you should learn

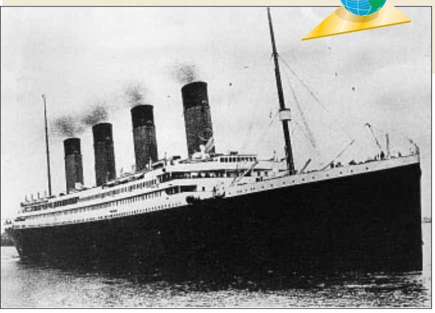
GOAL 1 Use properties of proportions.

GOAL 2 Use proportions to solve **real-life** problems, such as using the scale of a map in **Exs. 41 and 42**.

Why you should learn it

▼ To solve **real-life** problems, such as using a scale model to calculate the dimensions of the Titanic in **Example 4**.

REAL LIFE



GOAL 1 Using Properties of Proportions

In Lesson 8.1, you studied the reciprocal property and the cross product property. Two more properties of proportions, which are especially useful in geometry, are given below.

You can use the cross product property and the reciprocal property to help prove these properties in Exercises 36 and 37.

ADDITIONAL PROPERTIES OF PROPORTIONS

3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

EXAMPLE 1 *Using Properties of Proportions*

Tell whether the statement is true.

a. If $\frac{p}{6} = \frac{r}{10}$, then $\frac{p}{r} = \frac{3}{5}$.

b. If $\frac{a}{3} = \frac{c}{4}$, then $\frac{a+3}{3} = \frac{c+3}{4}$.

SOLUTION

a. $\frac{p}{6} = \frac{r}{10}$ — Given

$\frac{p}{r} = \frac{6}{10}$ — If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

$\frac{p}{r} = \frac{3}{5}$ — Simplify.

▶ The statement is true.

b. $\frac{a}{3} = \frac{c}{4}$ — Given

$\frac{a+3}{3} = \frac{c+4}{4}$ — If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

Because $\frac{c+4}{4} \neq \frac{c+3}{4}$, the conclusions are not equivalent.

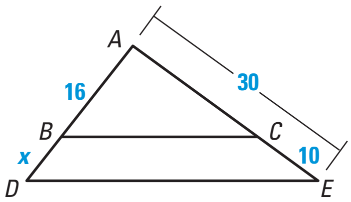
▶ The statement is false.

North Carolina Standards and Assessment / Competency Goals
MA.B.2.4.1, 2.03, MA.C.3.4.1

Using Algebra

EXAMPLE 2 *Using Properties of Proportions*

In the diagram $\frac{AB}{BD} = \frac{AC}{CE}$. Find the length of $\overline{BD}$.



SOLUTION

$\frac{AB}{BD} = \frac{AC}{CE}$	Given
$\frac{16}{x} = \frac{30-10}{10}$	Substitute.
$\frac{16}{x} = \frac{20}{10}$	Simplify.
$20x = 160$	Cross product property
$x = 8$	Divide each side by 20.

▶ So, the length of $\overline{BD}$ is 8.

STUDENT HELP

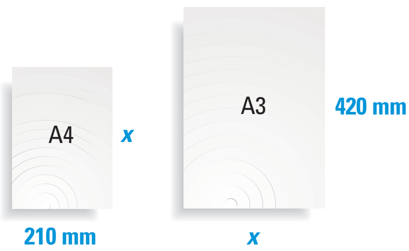
INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

The **geometric mean** of two positive numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. If you solve this proportion for x, you find that $x = \sqrt{a \cdot b}$, which is a positive number.

For example, the geometric mean of 8 and 18 is **12**, because $\frac{8}{12} = \frac{12}{18}$, and also because $\sqrt{8 \cdot 18} = \sqrt{144} = 12$.

EXAMPLE 3 *Using a Geometric Mean*

REAL LIFE **PAPER SIZES** International standard paper sizes are commonly used all over the world. The various sizes all have the same width-to-length ratios. Two sizes of paper are shown, called A4 and A3. The distance labeled x is the geometric mean of 210 mm and 420 mm. Find the value of x.



STUDENT HELP

Skills Review
For help with simplifying square roots, see p. 799.

SOLUTION

$\frac{210}{x} = \frac{x}{420}$	Write proportion.
$x^2 = 210 \cdot 420$	Cross product property
$x = \sqrt{210 \cdot 420}$	Simplify.
$x = \sqrt{210 \cdot 210 \cdot 2}$	Factor.
$x = 210\sqrt{2}$	Simplify.

▶ The geometric mean of 210 and 420 is $210\sqrt{2}$, or about 297. So, the distance labeled x in the diagram is about 297 mm.

GOAL 2 USING PROPORTIONS IN REAL LIFE

In general, when solving word problems that involve proportions, there is more than one correct way to set up the proportion.

EXAMPLE 4 *Solving a Proportion*

REAL LIFE **MODEL BUILDING** A scale model of the Titanic is 107.5 inches long and 11.25 inches wide. The Titanic itself was 882.75 feet long. How wide was it?

SOLUTION

One way to solve this problem is to set up a proportion that compares the measurements of the Titanic to the measurements of the scale model.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\frac{\text{Width of Titanic}}{\text{Width of model ship}} = \frac{\text{Length of Titanic}}{\text{Length of model ship}}$$

LABELS

Width of Titanic $= x$ (feet)

Width of model ship $= 11.25$ (inches)

Length of Titanic $= 882.75$ (feet)

Length of model ship $= 107.5$ (inches)

REASONING

$$\frac{x \text{ ft}}{11.25 \text{ in.}} = \frac{882.75 \text{ ft}}{107.5 \text{ in.}}$$ **Substitute.**

$$x = \frac{11.25 \cdot (882.75)}{107.5}$$ **Multiply each side by 11.25.**

$$x \approx 92.4$$ **Use a calculator.**

▶ So, the Titanic was about 92.4 feet wide.

Notice that the proportion in Example 4 contains measurements that are not in the same units. When writing a proportion with unlike units, the numerators should have the same units and the denominators should have the same units.

GUIDED PRACTICE

Vocabulary Check ✓

1. If x is the *geometric mean* of two positive numbers a and b, write a proportion that relates a, b, and x. $\frac{a}{x} = \frac{x}{b}$

Concept Check ✓

2. If $\frac{x}{4} = \frac{y}{5}$, then $\frac{x+4}{4} = \frac{?}{5}$. $y + 5$

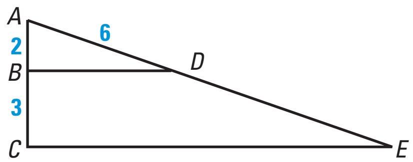
3. If $\frac{b}{6} = \frac{c}{2}$, then $\frac{b}{c} = \frac{?}{?}$. $\frac{6}{2}$; or 3

Skill Check ✓

4. Decide whether the statement is *true* or *false*. true

If $\frac{r}{s} = \frac{6}{15}$, then $\frac{15}{s} = \frac{6}{r}$.

5. Find the geometric mean of 3 and 12. 6



6. In the diagram $\frac{AB}{BC} = \frac{AD}{DE}$. Substitute the known values into the proportion and solve for DE. 9

7. **UNITED STATES FLAG** The official height-to-width ratio of the United States flag is 1 : 1.9. If a United States flag is 6 feet high, how wide is it? 11.4 ft

8. **UNITED STATES FLAG** The blue portion of the United States flag is called the union. What is the ratio of the height of the union to the height of the flag? $\frac{7}{13}$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 817.

LOGICAL REASONING **Complete the sentence.**

9. If $\frac{2}{x} = \frac{7}{y}$, then $\frac{2}{7} = \frac{?}{?}$. $\frac{x}{y}$

10. If $\frac{x}{6} = \frac{y}{34}$, then $\frac{x}{y} = \frac{?}{?}$. $\frac{6}{34}$ or $\frac{3}{17}$

11. If $\frac{x}{5} = \frac{y}{12}$, then $\frac{x+5}{5} = \frac{?}{?}$. $\frac{y+12}{12}$

12. If $\frac{13}{7} = \frac{x}{y}$, then $\frac{20}{7} = \frac{?}{?}$. $\frac{x+y}{y}$

LOGICAL REASONING **Decide whether the statement is *true* or *false*.**

13. If $\frac{7}{a} = \frac{b}{2}$, then $\frac{7+a}{a} = \frac{b+2}{2}$. true

14. If $\frac{3}{4} = \frac{p}{r}$, then $\frac{4}{3} = \frac{p}{r}$. false

15. If $\frac{c}{6} = \frac{d+2}{10}$, then $\frac{c}{d+2} = \frac{6}{10}$. true

16. If $\frac{12+m}{12} = \frac{3+n}{n}$, then $\frac{m}{12} = \frac{3}{n}$. true

GEOMETRIC MEAN **Find the geometric mean of the two numbers.**

17. 3 and 27 9

18. 4 and 16 8

19. 7 and 28 14

20. 2 and 40 $4\sqrt{5}$

21. 8 and 20 $4\sqrt{10}$

22. 5 and 15 $5\sqrt{3}$

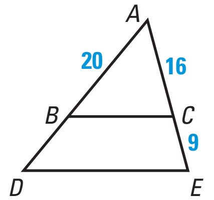
STUDENT HELP

HOMEWORK HELP

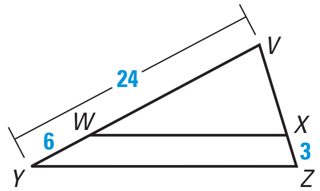
Example 1: Exs. 9–16
Example 2: Exs. 23–28
Example 3: Exs. 17–22, 43
Example 4: Exs. 29–32, 38–42

PROPERTIES OF PROPORTIONS **Use the diagram and the given information to find the unknown length.**

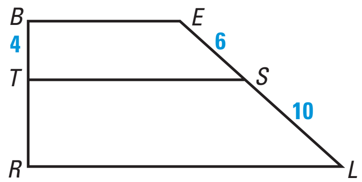
23. GIVEN ▶ $\frac{AB}{BD} = \frac{AC}{CE}$, find BD. **11.25**



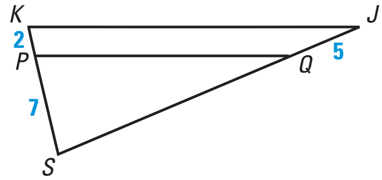
24. GIVEN ▶ $\frac{VW}{WY} = \frac{VX}{XZ}$, find VX. **9**



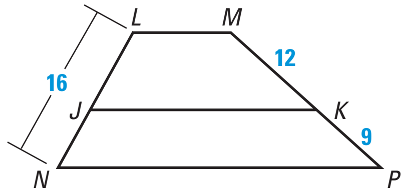
25. GIVEN ▶ $\frac{BT}{TR} = \frac{ES}{SL}$, find TR. $6\frac{2}{3}$



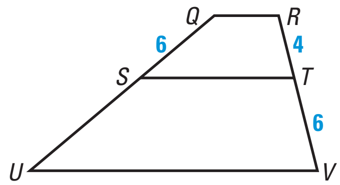
26. GIVEN ▶ $\frac{SP}{SK} = \frac{SQ}{SJ}$, find SQ. $17\frac{1}{2}$



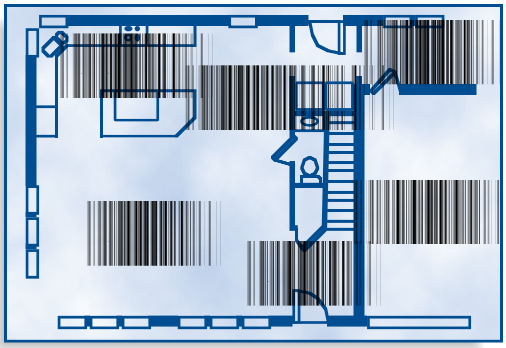
27. GIVEN ▶ $\frac{LJ}{JN} = \frac{MK}{KP}$, find JN. $6\frac{6}{7}$



28. GIVEN ▶ $\frac{QU}{QS} = \frac{RV}{RT}$, find SU. **9**



BLUEPRINTS **In Exercises 29 and 30, use the blueprint of the house in which $\frac{1}{16}$ inch = 1 foot. Use a ruler to approximate the dimension.**



29. Find the approximate width of the house to the nearest 5 feet.
about 25 ft

30. Find the approximate length of the house to the nearest 5 feet.
about 40 ft

31. **BATTING AVERAGE** The batting average of a baseball player is the ratio of the number of hits to the number of official at-bats. In 1998, Sammy Sosa of the Chicago Cubs had 643 official at-bats and a batting average of .308. Use the following verbal model to find the number of hits Sammy Sosa got.
198 hits

$$\frac{\textit{Number of hits}}{\textit{Number of at-bats}} = \frac{\textit{Batting average}}{1.000}$$

32. **CURRENCY EXCHANGE** Natalie has relatives in Russia. She decides to take a trip to Russia to visit them. She took 500 U.S. dollars to the bank to exchange for Russian rubles. The exchange rate on that day was 22.76 rubles per U.S. dollar. How many rubles did she get in exchange for the 500 U.S. dollars? ▶ Source: Russia Today **11,380 rubles**

FOCUS ON PEOPLE

REAL LIFE **SAMMY SOSA** was the National League Most Valuable Player in 1998. He hit 66 home runs to finish second to Mark McGwire who hit 70, a new record.

INTERNET **APPLICATION LINK**
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35. Each side of the equation represents the slope of the line through two of the points; if the points are collinear, the slopes are the same.

40. *Sample answer:* Construct a ramp consisting of two ramps in opposite directions, each 18 ft long. The first should be 3 ft high at its beginning and $1\frac{1}{2}$ ft high at its end, for a rise:run ratio of $\frac{1}{12}$. The second would be $1\frac{1}{2}$ ft high at its beginning and ground level at its end. The second ramp also has a rise:run ratio of $\frac{1}{12}$.

43. If the two sizes share a dimension, the shorter dimension of A5 paper must be the longer dimension of A6 paper. That is, the length of A6 paper must be 148 mm. Let x be the width of A6 paper; 148 is the geometric mean of x and 210. Then $\frac{x}{148} = \frac{148}{210}$ and $x \approx 104$ mm.

33. **COORDINATE GEOMETRY** The points $(-4, -1)$, $(1, 1)$, and $(x, 5)$ are collinear. Find the value of x by solving the proportion below. **11**

$$\frac{1-(-1)}{1-(-4)} = \frac{5-1}{x-1}$$

34. **COORDINATE GEOMETRY** The points $(2, 8)$, $(6, 18)$, and $(8, y)$ are collinear. Find the value of y by solving the proportion below. **23**

$$\frac{18-8}{6-2} = \frac{y-18}{8-6}$$

35. **CRITICAL THINKING** Explain why the method used in Exercises 33 and 34 is a correct way to express that three given points are collinear. **See margin.**

36. **PROOF** Prove property 3 of proportions (see page 465). **See margin.**

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a}{c} = \frac{b}{d}.$$

37. **PROOF** Prove property 4 of proportions (see page 465). **See margin.**

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{b} = \frac{c+d}{d}.$$

RAMP DESIGN **Assume that a wheelchair ramp has a slope of $\frac{1}{12}$, which is the maximum slope recommended for a wheelchair ramp.**

38. A wheelchair ramp has a 15 foot run. What is its rise? **$1\frac{1}{4}$ ft**

39. A wheelchair ramp rises 2 feet. What is its run? **24 ft**

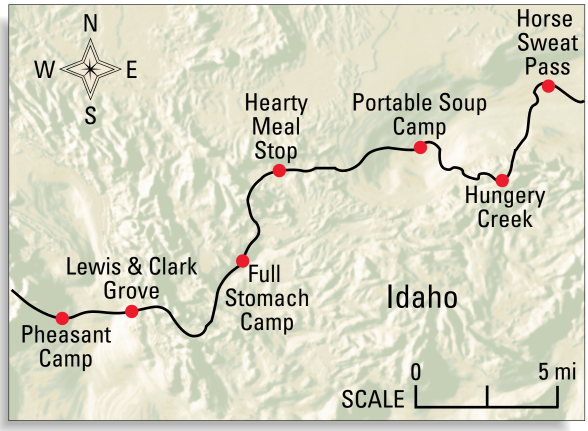
40. You are constructing a wheelchair ramp that must rise 3 feet. Because of space limitations, you cannot build a continuous ramp with a length greater than 21 feet. Design a ramp that solves this problem. **See margin.**

FOCUS ON PEOPLE

REAL LIFE

SACAGAWEA Representing liberty on the new dollar coin is Sacagawea, who played a crucial role in the Lewis and Clark expedition. She acted as an interpreter and guide, and is now given credit for much of the mission's success.

HISTORY CONNECTION **Part of the Lewis and Clark Trail on which Sacagawea acted as guide is now known as the Lolo Trail. The map, which shows a portion of the trail, has a scale of 1 inch = 6.7 miles.**



41. Use a ruler to estimate the distance (measured in a straight line) between Lewis and Clark Grove and Pheasant Camp. Then calculate the actual distance in miles. **about $\frac{3}{8}$ in.; about $2\frac{1}{2}$ mi**

42. Estimate the distance along the trail between Portable Soup Camp and Full Stomach Camp. Then calculate the actual distance in miles. **about $1\frac{1}{4}$ in.; about $8\frac{3}{8}$ mi**

43. ***Writing*** Size A5 paper has a width of 148 mm and a length of 210 mm. Size A6, which is the next smaller size, shares a dimension with size A5. Use the proportional relationship stated in Example 3 and geometric mean to explain how to determine the length and width of size A6 paper. **See margin.**

Test Preparation

44. MULTIPLE CHOICE There are 24 fish in an aquarium. If $\frac{1}{8}$ of the fish are tetras, and $\frac{2}{3}$ of the remaining fish are guppies, how many guppies are in the aquarium? **D**

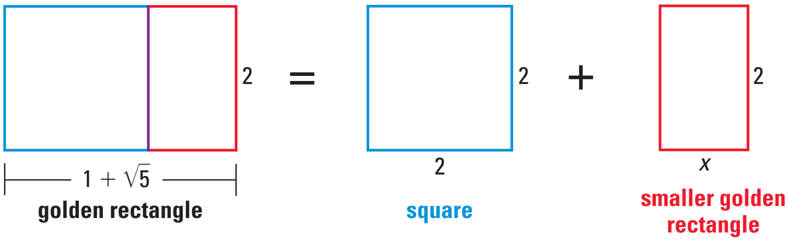
Ⓐ 2 Ⓑ 3 Ⓒ 10 Ⓓ 14 Ⓔ 16

45. MULTIPLE CHOICE A basketball team had a ratio of wins to losses of 3 : 1. After winning 6 games in a row, the team's ratio of wins to losses was 5 : 1. How many games had the team won before it won the 6 games in a row? **C**

Ⓐ 3 Ⓑ 6 Ⓒ 9 Ⓓ 15 Ⓔ 24

★ Challenge

46. GOLDEN RECTANGLE A golden rectangle has its length and width in the golden ratio $\frac{1 + \sqrt{5}}{2}$. If you cut a square away from a golden rectangle, the shape that remains is also a golden rectangle.



a. The diagram indicates that $1 + \sqrt{5} = 2 + x$. Find x. **$-1 + \sqrt{5}$**

b. To prove that the large and small rectangles are both golden rectangles, show that $\frac{1 + \sqrt{5}}{2} = \frac{2}{x}$.

46. b. $\frac{1 + \sqrt{5}}{2} = \frac{2}{x}$ if 2 is the geometric mean of $1 + \sqrt{5}$ and x. The geometric mean of $1 + \sqrt{5}$ and $-1 + \sqrt{5}$ is $\sqrt{(1 + \sqrt{5})(-1 + \sqrt{5})} = \sqrt{4} = 2$.

c. Give a decimal approximation for the golden ratio to six decimal places. **1.618034**

EXTRA CHALLENGE
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MIXED REVIEW

FINDING AREA **Find the area of the figure described.** (Review 1.7)

47. Rectangle: width = 3 m, length = 4 m **$12\ m^2$**

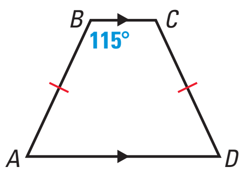
48. Square: side = 3 cm **$9\ cm^2$**

49. Triangle: base = 13 cm, height = 4 cm **$26\ cm^2$**

50. Circle: diameter = 11 ft **about $95\ ft^2$**

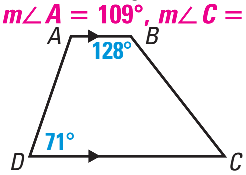
FINDING ANGLE MEASURES **Find the angle measures.** (Review 6.5 for 8.3)

51.



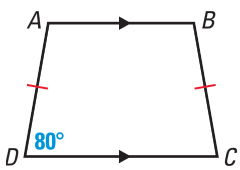
$m\angle C = 115°$, $m\angle A = m\angle D = 65°$

52.



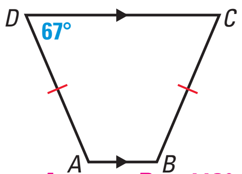
$m\angle A = 109°$, $m\angle C = 52°$

53.



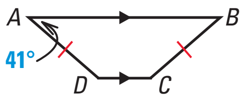
$m\angle A = m\angle B = 100°$, $m\angle C = 80°$

54.



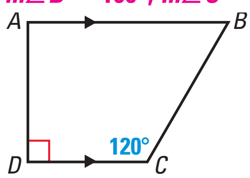
$m\angle A = m\angle B = 113°$, $m\angle C = 67°$

55.



$m\angle B = 41°$, $m\angle C = m\angle D = 139°$

56.



$m\angle A = 90°$, $m\angle B = 60°$

57. PENTAGON Describe any symmetry in a regular pentagon $ABCDE$. (Review 7.2, 7.3)

57. A regular pentagon has 5 lines of symmetry (one from each vertex to the midpoint of the opposite side) and rotational symmetries of 72° and 144°, clockwise and counterclockwise, about the center of the pentagon.