8.1 Ratio and Proportion

What you should learn

GOAL 1 Find and simplify the ratio of two numbers.

GOAL 2 Use proportions to solve **real-life** problems, such as computing the width of a painting in **Example 6**.

Why you should learn it

▼ To solve **real-life** problems, such as using a scale model to determine the dimensions of a sculpture like the baseball glove below and the baseball bat in **Exs. 51–53**.

REAL LIFE

GOAL 1 COMPUTING RATIOS

If a and b are two quantities that are measured in the *same* units, then the **ratio of a to b** is $\frac{a}{b}$. The ratio of a to b can also be written as $a:b$. Because a ratio is a quotient, its denominator cannot be zero.

Ratios are usually expressed in simplified form. For instance, the ratio of 6:8 is usually simplified as 3:4.

EXAMPLE 1 *Simplifying Ratios*

Simplify the ratios.

a. $\frac{12\text{ cm}}{4\text{ m}}$ **b.** $\frac{6\text{ ft}}{18\text{ in.}}$

SOLUTION

To simplify ratios with unlike units, convert to like units so that the units divide out. Then simplify the fraction, if possible.

a. $\frac{12\text{ cm}}{4\text{ m}} = \frac{12\text{ cm}}{4 \cdot \mathbf{100\text{ cm}}} = \frac{12}{400} = \frac{3}{100}$ **b.** $\frac{6\text{ ft}}{18\text{ in.}} = \frac{6 \cdot \mathbf{12\text{ in.}}}{18\text{ in.}} = \frac{72}{18} = \frac{4}{1}$

ACTIVITY Developing Concepts — Investigating Ratios

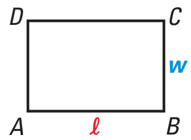
1. Use a tape measure to measure the circumference of the base of your thumb, the circumference of your wrist, and the circumference of your neck. Record the results in a table.
2. Compute the ratio of your wrist measurement to your thumb measurement. Then, compute the ratio of your neck measurement to your wrist measurement.
3. Compare the two ratios.
4. Compare your ratios to those of others in the class.
5. Does it matter whether you record your measurements all in inches or all in centimeters? Explain.

STUDENT HELP

Look Back
For help with perimeter, see p. 51.

EXAMPLE 2 *Using Ratios*

The perimeter of rectangle $ABCD$ is 60 centimeters. The ratio of $AB:BC$ is 3:2. Find the length and width of the rectangle.



SOLUTION

Because the ratio of $AB:BC$ is 3:2, you can represent the length AB as $3x$ and the width BC as $2x$.

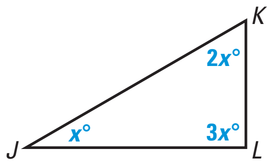
$2\ell + 2w = P$	Formula for perimeter of rectangle
$2(3x) + 2(2x) = 60$	Substitute for ℓ, w, and P.
$6x + 4x = 60$	Multiply.
$10x = 60$	Combine like terms.
$x = 6$	Divide each side by 10.

▶ So, $ABCD$ has a length of 18 centimeters and a width of 12 centimeters.

Using Algebra

EXAMPLE 3 *Using Extended Ratios*

The measure of the angles in $\triangle JKL$ are in the *extended ratio* of 1:2:3. Find the measures of the angles.



SOLUTION

Begin by sketching a triangle. Then use the extended ratio of 1:2:3 to label the measures of the angles as $x°$, $2x°$, and $3x°$.

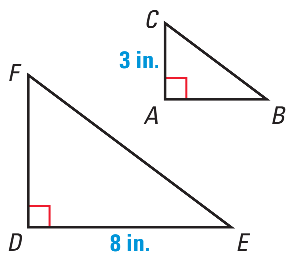
$x° + 2x° + 3x° = 180°$	Triangle Sum Theorem
$6x = 180$	Combine like terms.
$x = 30$	Divide each side by 6.

▶ So, the angle measures are 30°, 2(30°) = 60°, and 3(30°) = 90°.

Logical Reasoning

EXAMPLE 4 *Using Ratios*

The ratios of the side lengths of $\triangle DEF$ to the corresponding side lengths of $\triangle ABC$ are 2:1. Find the unknown lengths.



SOLUTION

- DE is twice AB and $DE = 8$, so $AB = \frac{1}{2}(8) = 4$.
- Using the Pythagorean Theorem, you can determine that $BC = 5$.
- DF is twice AC and $AC = 3$, so $DF = 2(3) = 6$.
- EF is twice BC and $BC = 5$, so $EF = 2(5) = 10$.

STUDENT HELP

Look Back
For help with the Pythagorean Theorem, see p. 20.

GOAL 2 USING PROPORTIONS

An equation that equates two ratios is a **proportion**. For instance, if the ratio $\frac{a}{b}$ is equal to the ratio $\frac{c}{d}$, then the following proportion can be written:

Means Extremes

$$\frac{a}{b} = \frac{c}{d}$$

The numbers a and d are the **extremes** of the proportion. The numbers b and c are the **means** of the proportion.

PROPERTIES OF PROPORTIONS

1. **CROSS PRODUCT PROPERTY** The product of the extremes equals the product of the means.

   If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

2. **RECIPROCAL PROPERTY** If two ratios are equal, then their reciprocals are also equal.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

STUDENT HELP

Skills Review
For help with reciprocals, see p. 788.

To *solve the proportion* you find the value of the variable.

Using Algebra

EXAMPLE 5 *Solving Proportions*

Solve the proportions.

a. $\frac{4}{x} = \frac{5}{7}$ **b.** $\frac{3}{y+2} = \frac{2}{y}$

SOLUTION

a. $\frac{4}{x} = \frac{5}{7}$ Write original proportion.

$\frac{x}{4} = \frac{7}{5}$ Reciprocal property

$x = 4\left(\frac{7}{5}\right)$ Multiply each side by 4.

$x = \frac{28}{5}$ Simplify.

b. $\frac{3}{y+2} = \frac{2}{y}$ Write original proportion.

$3y = 2(y + 2)$ Cross product property

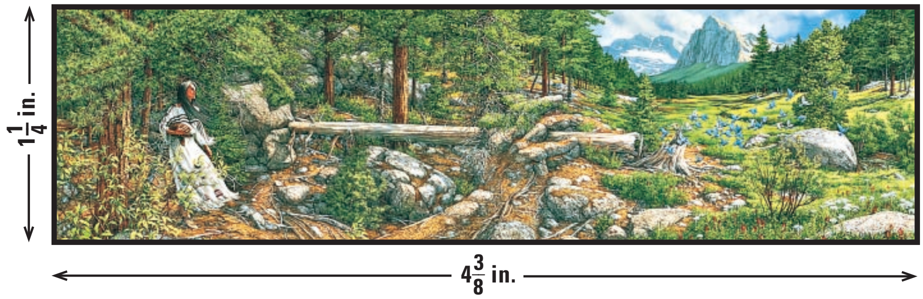
$3y = 2y + 4$ Distributive property

$y = 4$ Subtract $2y$ from each side.

▶ The solution is 4. Check this by substituting in the original proportion.

EXAMPLE 6 *Solving a Proportion*

PAINTING The photo shows Bev Dolittle's painting *Music in the Wind*. Her actual painting is 12 inches high. How wide is it?



SOLUTION

You can reason that in the photograph all measurements of the artist's painting have been reduced by the same ratio. That is, the ratio of the actual width to the reduced width is equal to the ratio of the actual height to the reduced height. The photograph is $1\frac{1}{4}$ inches by $4\frac{3}{8}$ inches.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\frac{\boxed{\text{Width of painting}}}{\boxed{\text{Width of photo}}} = \frac{\boxed{\text{Height of painting}}}{\boxed{\text{Height of photo}}}$$

LABELS

Width of painting $= x$ Height of painting $= \mathbf{12}$ (inches)

Width of photo $= \mathbf{4.375}$ Height of photo $= \mathbf{1.25}$ (inches)

REASONING

$\dfrac{x}{4.375} = \dfrac{12}{1.25}$ **Substitute.**

$x = 4.375\left(\dfrac{12}{1.25}\right)$ **Multiply each side by 4.375.**

$x = 42$ **Use a calculator.**

▶ So, the actual painting is 42 inches wide.

EXAMPLE 7 *Solving a Proportion*

Estimate the length of the hidden flute in Bev Doolittle's actual painting.

SOLUTION

In the photo, the flute is about $1\frac{7}{8}$ inches long. Using the reasoning from above you can say that:

$$\frac{\text{Length of flute in painting}}{\text{Length of flute in photo}} = \frac{\text{Height of painting}}{\text{Height of photo}}.$$

$\dfrac{f}{1.875} = \dfrac{12}{1.25}$ **Substitute.**

$f = 18$ **Multiply each side by 1.875 and simplify.**

▶ So, the flute is about 18 inches long in the painting.

GUIDED PRACTICE

Vocabulary Check ✓

1. In the proportion $\frac{r}{s} = \frac{p}{q}$, the variables s and p are the __?__ of the proportion and r and q are the __?__ of the proportion.

Concept Check ✓

ERROR ANALYSIS **In Exercises 2 and 3, find and correct the errors.**

2. A table is 18 inches wide and 3 feet long. The ratio of length to width is 1 : 6.

3.
$$\frac{10}{x+6} = \frac{4}{x}$$
$$10x = 4x + 6$$
$$6x = 6$$
$$x = 1$$

Skill Check ✓

Given that the track team won 8 meets and lost 2, find the ratios.

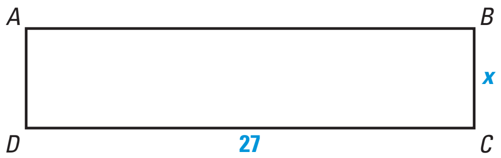
4. What is the ratio of wins to losses? What is the ratio of losses to wins?

5. What is the ratio of wins to the total number of track meets?

In Exercises 6–8, solve the proportion.

6. $\frac{2}{x} = \frac{3}{9}$ **7.** $\frac{5}{8} = \frac{6}{z}$ **8.** $\frac{2}{b+3} = \frac{4}{b}$

9. The ratio $BC:DC$ is $2:9$. Find the value of x.



PRACTICE AND APPLICATIONS

STUDENT HELP

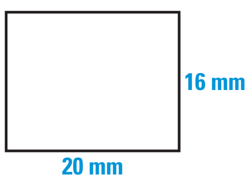
Extra Practice to help you master skills is on p. 817.

SIMPLIFYING RATIOS **Simplify the ratio.**

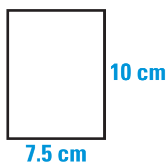
10. $\frac{16 \text{ students}}{24 \text{ students}}$ **11.** $\frac{48 \text{ marbles}}{8 \text{ marbles}}$ **12.** $\frac{22 \text{ feet}}{52 \text{ feet}}$ **13.** $\frac{6 \text{ meters}}{9 \text{ meters}}$

WRITING RATIOS **Find the width to length ratio of each rectangle. Then simplify the ratio.**

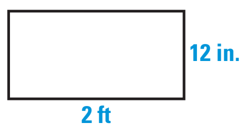
14.



15.



16.



STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 10–24
Example 2: Exs. 29, 30
Example 3: Exs. 31, 32
Example 4: Exs. 57, 58

continued on p. 462

CONVERTING UNITS **Rewrite the fraction so that the numerator and denominator have the same units. Then simplify.**

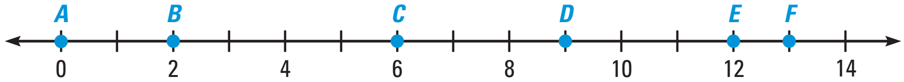
17. $\frac{3 \text{ ft}}{12 \text{ in.}}$ **18.** $\frac{60 \text{ cm}}{1 \text{ m}}$ **19.** $\frac{350 \text{ g}}{1 \text{ kg}}$ **20.** $\frac{2 \text{ mi}}{3000 \text{ ft}}$

21. $\frac{6 \text{ yd}}{10 \text{ ft}}$ **22.** $\frac{2 \text{ lb}}{20 \text{ oz}}$ **23.** $\frac{400 \text{ m}}{0.5 \text{ km}}$ **24.** $\frac{20 \text{ oz}}{4 \text{ lb}}$

STUDENT HELP

HOMEWORK HELP
continued from p. 461
Example 5: Exs. 33–44
Example 6: Exs. 48–53, 59–61
Example 7: Exs. 48–53, 59–61

FINDING RATIOS **Use the number line to find the ratio of the distances.**



25. $\frac{AB}{CD} = \underline{\ ?\ }$ **26.** $\frac{BD}{CF} = \underline{\ ?\ }$ **27.** $\frac{BF}{AD} = \underline{\ ?\ }$ **28.** $\frac{CF}{AB} = \underline{\ ?\ }$

29. The perimeter of a rectangle is 84 feet. The ratio of the width to the length is 2:5. Find the length and the width.

30. The area of a rectangle is 108 cm^2. The ratio of the width to the length is 3:4. Find the length and the width.

31. The measures of the angles in a triangle are in the extended ratio of 1:4:7. Find the measures of the angles.

32. The measures of the angles in a triangle are in the extended ratio of 2:15:19. Find the measures of the angles.

SOLVING PROPORTIONS **Solve the proportion.**

33. $\frac{x}{4} = \frac{5}{7}$ **34.** $\frac{y}{8} = \frac{9}{10}$ **35.** $\frac{7}{z} = \frac{10}{25}$

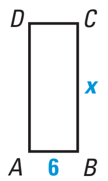
36. $\frac{4}{b} = \frac{10}{3}$ **37.** $\frac{30}{5} = \frac{14}{c}$ **38.** $\frac{16}{3} = \frac{d}{6}$

39. $\frac{5}{x+3} = \frac{4}{x}$ **40.** $\frac{4}{y-3} = \frac{8}{y}$ **41.** $\frac{7}{2z+5} = \frac{3}{z}$

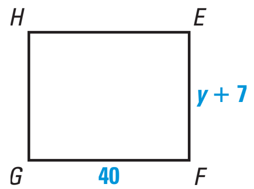
42. $\frac{3x-8}{6} = \frac{2x}{10}$ **43.** $\frac{5y-8}{7} = \frac{5y}{6}$ **44.** $\frac{4}{2z+6} = \frac{10}{7z-2}$

USING PROPORTIONS **In Exercises 45–47, the ratio of the width to the length for each rectangle is given. Solve for the variable.**

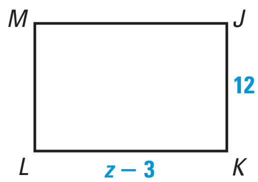
45. $AB:BC$ is 3:8.



46. $EF:FG$ is 4:5.



47. $JK:KL$ is 2:3.



SCIENCE CONNECTION **Use the following information.**
The table gives the ratios of the gravity of four different planets to the gravity of Earth. Round your answers to the nearest whole number.

Planet	Venus	Mars	Jupiter	Pluto
Ratio of gravity	$\frac{9}{10}$	$\frac{38}{100}$	$\frac{236}{100}$	$\frac{7}{100}$

48. Which of the planets listed above has a gravity closest to the gravity of Earth?

49. Estimate how much a person who weighs 140 pounds on Earth would weigh on Venus, Mars, Jupiter, and Pluto.

50. If a person weighed 46 pounds on Mars, estimate how much he or she would weigh on Earth.

FOCUS ON APPLICATIONS

REAL LIFE
MOON'S GRAVITY Neil Armstrong's space suit weighed about 185 pounds on Earth and just over 30 pounds on the moon, due to the weaker force of gravity.

INTERNET
APPLICATION LINK
www.mcdougallittell.com

BASEBALL BAT SCULPTURE **A huge, free-standing baseball bat sculpture stands outside a sports museum in Louisville, Kentucky. It was patterned after Babe Ruth's 35 inch bat. The sculpture is 120 feet long. Round your answers to the nearest tenth of an inch.**

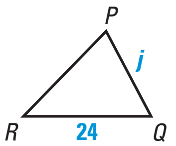
51. How long is the sculpture in inches?

52. The diameter of the sculpture near the base is 9 feet. Estimate the corresponding diameter of Babe Ruth's bat.

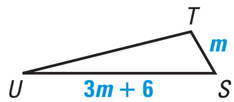
53. The diameter of the handle of the sculpture is 3.5 feet. Estimate the diameter of the handle of Babe Ruth's bat.

USING PROPORTIONS **In Exercises 54–56, the ratio of two side lengths of the triangle is given. Solve for the variable.**

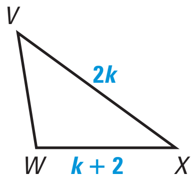
54. $PQ : QR$ is $3 : 4$.



55. $SU : ST$ is $4 : 1$.



56. $WX : XV$ is $5 : 7$.

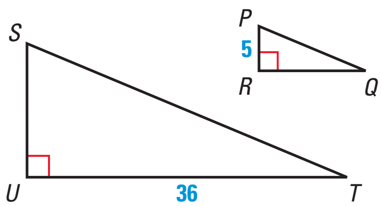


STUDENT HELP

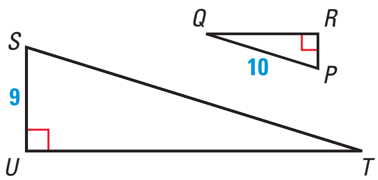
INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 57 and 58.

PYTHAGOREAN THEOREM **The ratios of the side lengths of $\triangle PQR$ to the corresponding side lengths of $\triangle STU$ are 1 : 3. Find the unknown lengths.**

57.



58.



GULLIVER'S TRAVELS **In Exercises 59–61, use the following information.** *Gulliver's Travels* was written by Jonathan Swift in 1726. In the story, Gulliver is shipwrecked and wanders ashore to the island of Lilliput. The average height of the people in Lilliput is 6 inches.

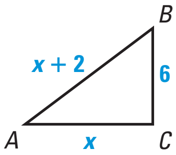
59. Gulliver is 6 feet tall. What is the ratio of his height to the average height of a Lilliputian?

60. After leaving Lilliput, Gulliver visits the island of Brobdingnag. The ratio of the average height of these natives to Gulliver's height is proportional to the ratio of Gulliver's height to the average height of a Lilliputian. What is the average height of a Brobdingnagian?

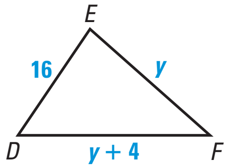
61. What is the ratio of the average height of a Brobdingnagian to the average height of a Lilliputian?

USING ALGEBRA **You are given an extended ratio that compares the lengths of the sides of the triangle. Find the lengths of all unknown sides.**

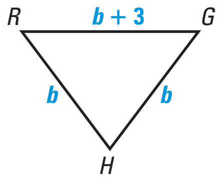
62. $BC:AC:AB$ is $3:4:5$.



63. $DE:EF:DF$ is $4:5:6$.



64. $GH:HR:GR$ is $5:5:6$.



Test Preparation

65. **MULTIPLE CHOICE** For planting roses, a gardener uses a special mixture of soil that contains sand, peat moss, and compost in the ratio $2:5:3$. How many pounds of compost does she need to add if she uses three 10 pound bags of peat moss?

(A) 12 (B) 14 (C) 15 (D) 18 (E) 20

66. **MULTIPLE CHOICE** If the measures of the angles of a triangle have the ratio $2:3:7$, the triangle is

(A) acute. (B) right. (C) isosceles.

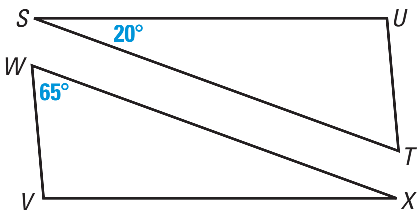
(D) obtuse. (E) equilateral.

★ **Challenge**

67. **FINDING SEGMENT LENGTHS** Suppose the points B and C lie on $\overline{AD}$. What is the length of $\overline{AC}$ if $\frac{AB}{BD} = \frac{2}{3}$, $\frac{CD}{AC} = \frac{1}{9}$, and $BD = 24$?

MIXED REVIEW

FINDING UNKNOWN MEASURES **Use the figure shown, in which $\triangle STU \cong \triangle XWV$.** (Review 4.2)



68. What is the measure of $\angle X$?

69. What is the measure of $\angle V$?

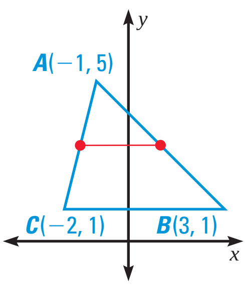
70. What is the measure of $\angle T$?

71. What is the measure of $\angle U$?

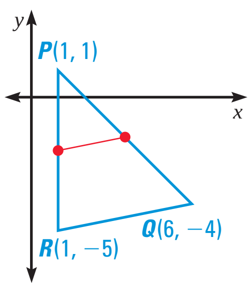
72. Which side is congruent to $\overline{TU}$?

FINDING COORDINATES **Find the coordinates of the endpoints of each midsegment shown in red.** (Review 5.4 for 8.2)

73.



74.



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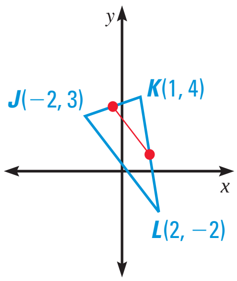

76. A line segment has endpoints $A(1, -3)$ and $B(6, -7)$. Graph $\overline{AB}$ and its image $\overline{A'B'}$ if $\overline{AB}$ is reflected in the line $x = 2$. (Review 7.2)