7.6

Frieze Patterns

What you should learn

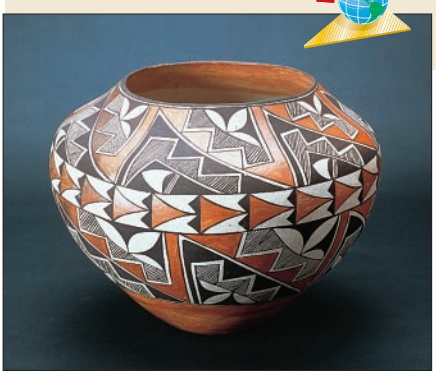
GOAL 1 Use transformations to classify frieze patterns.

GOAL 2 Use frieze patterns to design border patterns in **real life**, such as the tiling pattern in **Example 4**.

Why you should learn it

▼ You can use frieze patterns to create decorative borders for **real-life** objects, such as the pottery below and the pottery in **Exs. 35–37**.

REAL LIFE



GOAL 1 CLASSIFYING FRIEZE PATTERNS

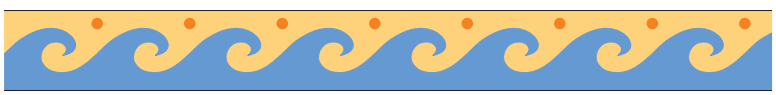
A **frieze pattern** or **border pattern** is a pattern that extends to the left and right in such a way that the pattern can be mapped onto itself by a horizontal translation. In addition to being mapped onto itself by a horizontal translation, some frieze patterns can be mapped onto themselves by other transformations.

1. Translation	T
2. 180° rotation	R
3. Reflection in a horizontal line	H
4. Reflection in a vertical line	V
5. Horizontal glide reflection	G

EXAMPLE 1 *Describing Frieze Patterns*

Describe the transformations that will map each frieze pattern onto itself.

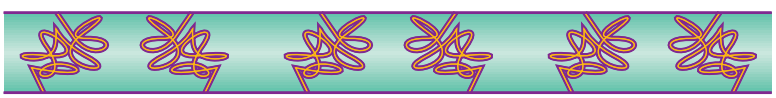
a.



b.

c.

d.



SOLUTION

a. This frieze pattern can be mapped onto itself by a horizontal translation (T).

b. This frieze pattern can be mapped onto itself by a horizontal translation (T) or by a 180° rotation (R).

c. This frieze pattern can be mapped onto itself by a horizontal translation (T) or by a horizontal glide reflection (G).

d. This frieze pattern can be mapped onto itself by a horizontal translation (T) or by a reflection in a vertical line (V).

CONCEPT SUMMARY — **CLASSIFICATIONS OF FRIEZE PATTERNS**

T	Translation	
TR	Translation and 180° rotation	
TG	Translation and horizontal glide reflection	
TV	Translation and vertical line reflection	
THG	Translation, horizontal line reflection, and horizontal glide reflection	
TRVG	Translation, 180° rotation, vertical line reflection, and horizontal glide reflection	
TRHVG	Translation, 180° rotation, horizontal line reflection, vertical line reflection, and horizontal glide reflection	

STUDENT HELP

Study Tip

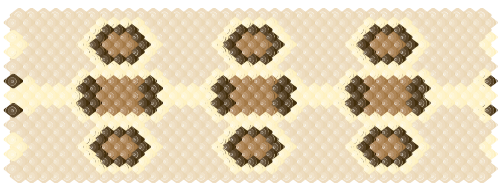
To help classify a frieze pattern, you can use a process of elimination. This process is described at the right and in the tree diagram in **Ex. 53**.

To classify a frieze pattern into one of the seven categories, you first decide whether the pattern has 180° rotation. If it does, then there are three possible classifications: TR, TRVG, and TRHVG.

If the frieze pattern does not have 180° rotation, then there are four possible classifications: T, TV, TG, and THG. Decide whether the pattern has a line of reflection. By a process of elimination, you will reach the correct classification.

EXAMPLE 2 *Classifying a Frieze Pattern*

SNAKES Categorize the snakeskin pattern of the mountain adder.



SOLUTION

This pattern is a TRHVG. The pattern can be mapped onto itself by a translation, a 180° rotation, a reflection in a horizontal line, a reflection in a vertical line, and a horizontal glide reflection.

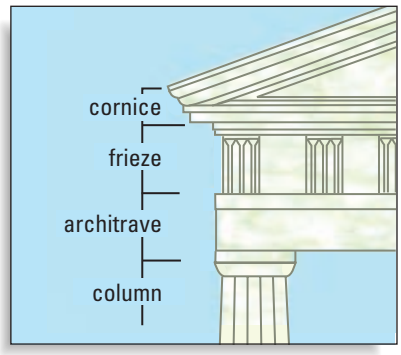
FOCUS ON APPLICATIONS

ARCHITECTURE Features of classical architecture from Greece and Rome are seen in "neo-classical" buildings today, such as the Supreme Court building shown.

GOAL 2 USING FRIEZE PATTERNS IN REAL LIFE

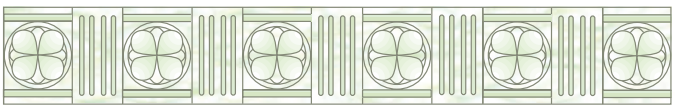
EXAMPLE 3 *Identifying Frieze Patterns*

ARCHITECTURE The frieze patterns of ancient Doric buildings are located between the cornice and the architrave, as shown at the right. The frieze patterns consist of alternating sections. Some sections contain a person or a symmetric design. Other sections have simple patterns of three or four vertical lines.



Portions of two frieze patterns are shown below. Classify the patterns.

a.



b.

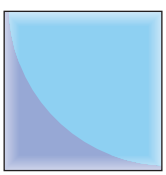
SOLUTION

a. Following the diagrams on the previous page, you can see that this frieze pattern has rotational symmetry, line symmetry about a horizontal line and a vertical line, and that the pattern can be mapped onto itself by a glide reflection. So, the pattern can be classified as TRHVG.

b. The only transformation that maps this pattern onto itself is a translation. So, the pattern can be classified as T.

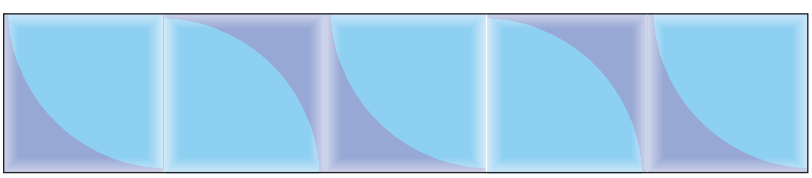
EXAMPLE 4 *Drawing a Frieze Pattern*

TILING A border on a bathroom wall is created using the decorative tile at the right. The border pattern is classified as TR. Draw one such pattern.



SOLUTION

Begin by rotating the given tile 180°. Use this tile and the original tile to create a pattern that has rotational symmetry. Then translate the pattern several times to create the frieze pattern.



GUIDED PRACTICE

Vocabulary Check ✓

1. Describe the term *frieze pattern* in your own words.

Concept Check ✓

2. **ERROR ANALYSIS** Describe Lucy's error below.

Skill Check ✓

In Exercises 3–6, describe the transformations that map the frieze pattern onto itself.

3. 

4. 

5. 

6. 

7. List the five possible transformations, along with their letter abbreviations, that can be found in a frieze pattern.

PRACTICE AND APPLICATIONS

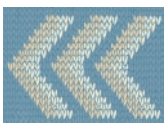
STUDENT HELP

Extra Practice to help you master skills is on p. 816.

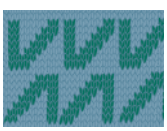
SWEATER PATTERN **Each row of the sweater is a frieze pattern. Match the row with its classification.**

A. TRHVG **B.** TR **C.** TRVG **D.** THG

8. 

9. 

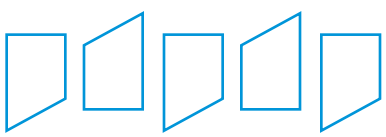
10. 

11. 

CLASSIFYING PATTERNS **Name the isometries that map the frieze pattern onto itself.**

12. 

13. 

14. 

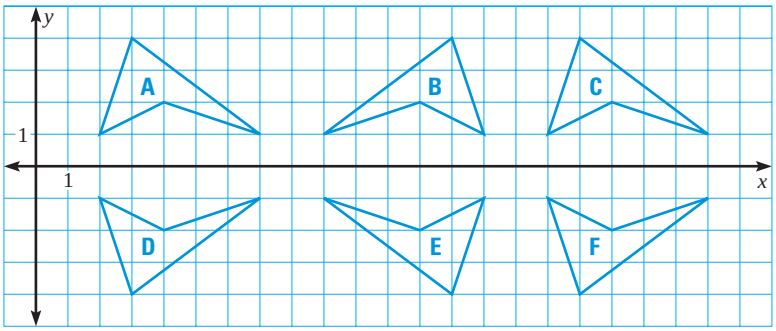
15. 

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 8–15
Example 2: Exs. 16–23
Example 3: Exs. 32–39
Example 4: Exs. 40–43

DESCRIBING TRANSFORMATIONS **Use the diagram of the frieze pattern.**



16. Is there a reflection in a vertical line? If so, describe the reflection(s).

17. Is there a reflection in a horizontal line? If so, describe the reflection(s).

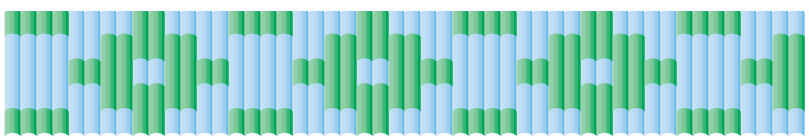
18. Name and describe the transformation that maps A onto F.

19. Name and describe the transformation that maps D onto B.

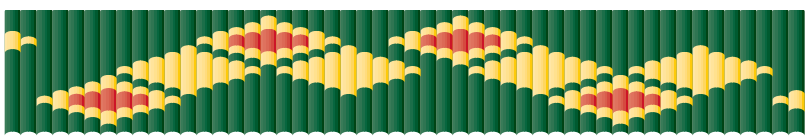
20. Classify the frieze pattern.

PET COLLARS **In Exercises 21–23, use the chart on page 438 to classify the frieze pattern on the pet collars.**

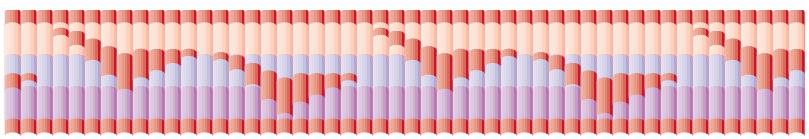
21.



22.



23.



24. **TECHNOLOGY** Pick one of the seven classifications of patterns and use geometry software to create a frieze pattern of that classification. Print and color your frieze pattern.

25. **DATA COLLECTION** Use a library, magazines, or some other reference source to find examples of frieze patterns. How many of the seven classifications of patterns can you find?

CREATING A FRIEZE PATTERN **Use the design below to create a frieze pattern with the given classification.**

26. TR

27. TV

28. TG

29. THG

30. TRVG

31. TRHVG

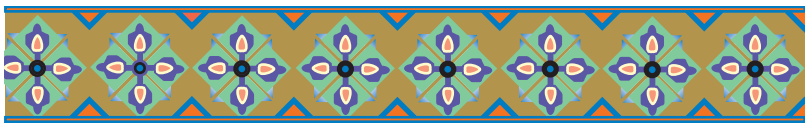
FOCUS ON APPLICATIONS

NIKKO MEMORIAL
The building shown is a memorial to Tokugawa Ieyasu (1543–1616), the founder of the Tokugawa Shogunate.

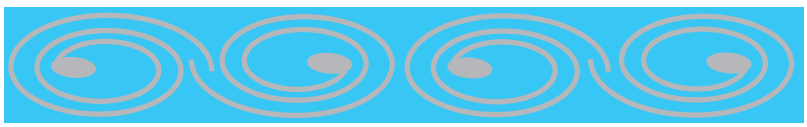
JAPANESE PATTERNS **The patterns shown were used in Japan during the Tokugawa Shogunate. Classify the frieze patterns.**

32.

33.



34.



POTTERY **In Exercises 35–37, use the pottery shown below. This pottery was created by the Acoma Indians. The Acoma pueblo is America's oldest continually inhabited city.**

35. Identify any frieze patterns on the pottery.

36. Classify the frieze pattern(s) you found in Exercise 35.

37. Create your own frieze pattern similar to the patterns shown on the pottery.

38. Look back to the southwestern pottery on page 437. Describe and classify one of the frieze patterns on the pottery.

39. **LOGICAL REASONING** You are decorating a large circular vase and decide to place a frieze pattern around its base. You want the pattern to consist of ten repetitions of a design. If the diameter of the base is about 9.5 inches, how wide should each design be?

TILING **In Exercises 40–42, use the tile to create a border pattern with the given classification. Your border should consist of one row of tiles.**

40. TR

41. TRVG

42. TRHVG

43. *Writing* Explain how the design of the tiles in Exercises 40–42 is a factor in the classification of the patterns. For instance, could the tile in Exercise 40 be used to create a single row of tiles classified as THG?

CRITICAL THINKING **Explain why the combination is not a category for frieze pattern classification.**

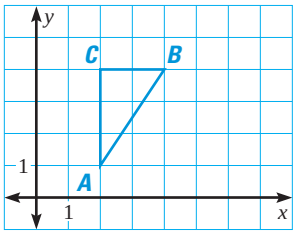
44. TVG **45.** THV **46.** TRG

STUDENT HELP

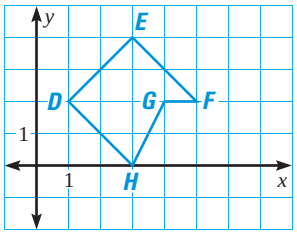
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 47 and 48.

USING THE COORDINATE PLANE **The figure shown in the coordinate plane is part of a frieze pattern with the given classification. Copy the graph and draw the figures needed to complete the pattern.**

47. TR



48. TRVG

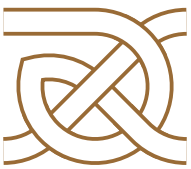


Test Preparation

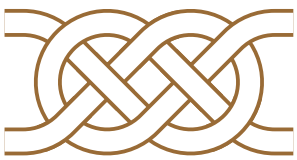
MULTI-STEP PROBLEM **In Exercises 49–52, use the following information.**

In Celtic art and design, border patterns are used quite frequently, especially in jewelry. Three different designs are shown.

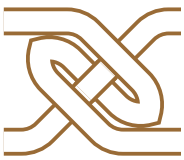
A.



B.



C.



49. Use translations to create a frieze pattern of each design.

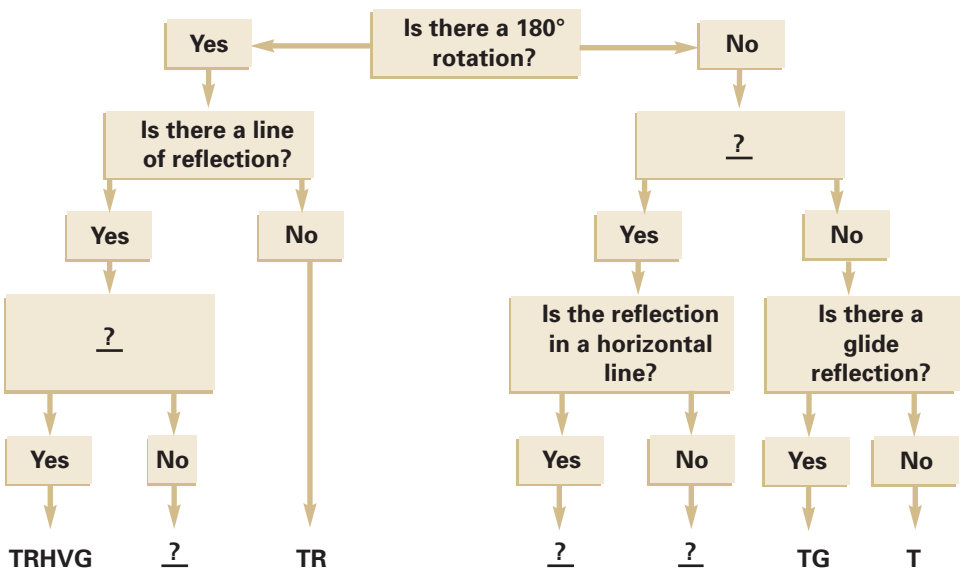
50. Classify each frieze pattern that you created.

51. Which design does not have rotational symmetry? Use rotations to create a new frieze pattern of this design.

52. *Writing* If a design has 180° rotational symmetry, it cannot be used to create a frieze pattern with classification *T*. Explain why not.

Challenge

53. **TREE DIAGRAM** The following tree diagram can help classify frieze patterns. Copy the tree diagram and fill in the missing parts.



MIXED REVIEW

RATIOS **Find the ratio of girls to boys in a class, given the number of boys and the total number of students.** (Skills Review for 8.1)

54. 12 boys, 23 students

55. 8 boys, 21 students

56. 3 boys, 13 students

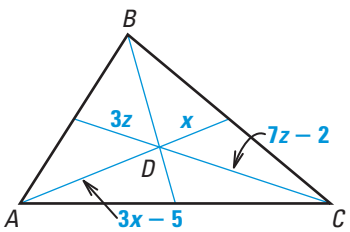
57. 19 boys, 35 students

58. 11 boys, 18 students

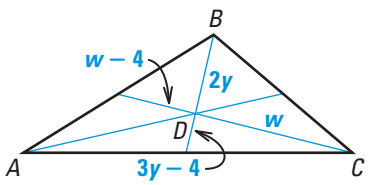
59. 10 boys, 20 students

PROPERTIES OF MEDIANS **Given that *D* is the centroid of $\triangle ABC$, find the value of each variable.** (Review 5.3)

60.

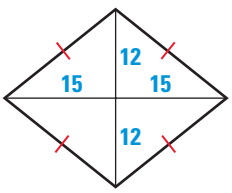


61.

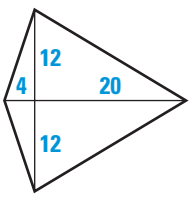


FINDING AREA **Find the area of the quadrilateral.** (Review 6.7)

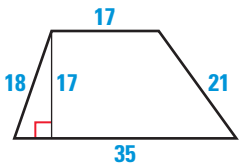
62.



63.



64.



QUIZ 2

Self-Test for Lessons 7.4–7.6

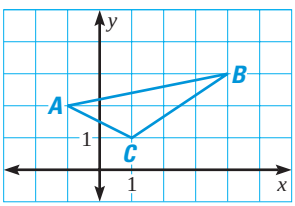
Write the coordinates of the vertices *A′*, *B′*, and *C′* after $\triangle ABC$ is translated by the given vector. (Lesson 7.4)

1. $\langle 1, 3 \rangle$

2. $\langle -3, 4 \rangle$

3. $\langle -2, -4 \rangle$

4. $\langle 5, 2 \rangle$



In Exercises 5 and 6, sketch the image of $\triangle PQR$ after a composition using the given transformations in the order they appear. (Lesson 7.5)

5. $P(5, 1)$, $Q(3, 4)$, $R(0, 1)$
Translation: $(x, y) \rightarrow (x - 2, y - 4)$
Reflection: in the y-axis

6. $P(7, 2)$, $Q(3, 1)$, $R(6, -1)$
Translation: $(x, y) \rightarrow (x - 4, y + 3)$
Rotation: 90° clockwise about origin

7. **MUSICAL NOTES** Do the notes shown form a frieze pattern? If so, classify the frieze pattern. (Lesson 7.6)