7.2 Reflections

What you should learn

GOAL 1 Identify and use reflections in a plane.

GOAL 2 Identify relationships between reflections and line symmetry.

Why you should learn it

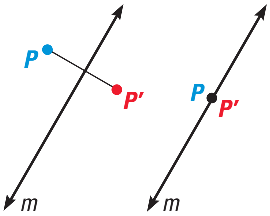
▼ Reflections and line symmetry can help you understand how mirrors in a kaleidoscope create interesting patterns, as in **Example 5**.

REAL LIFE

GOAL 1 USING REFLECTIONS IN A PLANE

One type of transformation uses a line that acts like a mirror, with an image reflected in the line. This transformation is a **reflection** and the mirror line is the **line of reflection**.

A reflection in a line m is a transformation that maps every point P in the plane to a point P', so that the following properties are true:



1. If P is not on m, then m is the perpendicular bisector of $\overline{PP'}$.

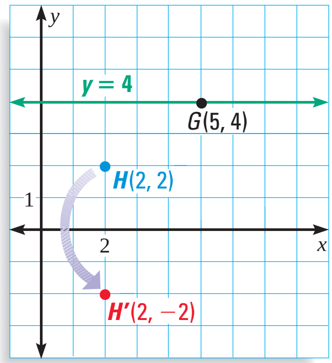
2. If P is on m, then $P = P'$.

EXAMPLE 1 *Reflections in a Coordinate Plane*

Graph the given reflection.

a. $H(2, 2)$ in the x-axis

b. $G(5, 4)$ in the line $y = 4$



SOLUTION

a. Since H is two units above the x-axis, its reflection, H', is two units below the x-axis.

b. Start by graphing $y = 4$ and G. From the graph, you can see that G is on the line. This implies that $G = G'$.

Reflections in the coordinate axes have the following properties:

1. If (x, y) is reflected in the x-axis, its image is the point $(x, -y)$.

2. If (x, y) is reflected in the y-axis, its image is the point $(-x, y)$.

In Lesson 7.1, you learned that an isometry preserves lengths. Theorem 7.1 relates isometries and reflections.

THEOREM

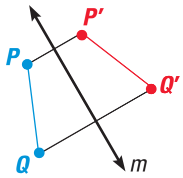
THEOREM 7.1 ***Reflection Theorem***

A reflection is an isometry.

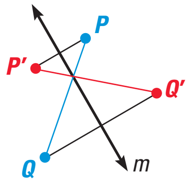
STUDENT HELP

Study Tip
Some theorems have more than one case, such as the Reflection Theorem. To fully prove this type of theorem, all of the cases must be proven.

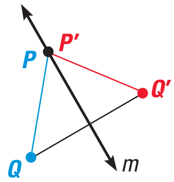
To prove the Reflection Theorem, you need to show that a reflection preserves the length of a segment. Consider a segment $\overline{PQ}$ that is reflected in a line m to produce $\overline{P'Q'}$. The four cases to consider are shown below.



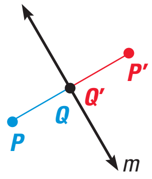
Case 1
P and Q are on the same side of m.



Case 2
P and Q are on opposite sides of m.



Case 3
One point lies on m and $\overline{PQ}$ is not perpendicular to m.



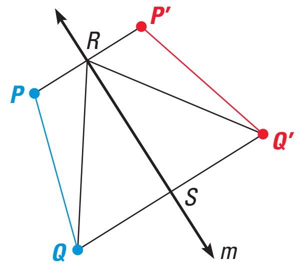
Case 4
Q lies on m and $\overline{PQ} \perp m$.

EXAMPLE 2 *Proof of Case 1 of Theorem 7.1*

Proof

GIVEN ▶ A reflection in m maps P onto P' and Q onto Q'.

PROVE ▶ $PQ = P'Q'$



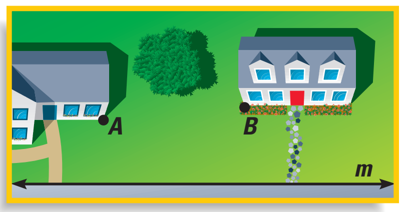
Paragraph Proof For this case, P and Q are on the same side of line m. Draw $\overline{PP'}$ and $\overline{QQ'}$, intersecting line m at R and S. Draw $\overline{RQ}$ and $\overline{RQ'}$.

By the definition of a reflection, $m \perp \overline{QQ'}$ and $\overline{QS} \cong \overline{Q'S}$. It follows that $\triangle RSQ \cong \triangle RSQ'$ using the SAS Congruence Postulate. This implies $\overline{RQ} \cong \overline{RQ'}$ and $\angle QRS \cong \angle Q'RS$. Because $\overleftrightarrow{RS}$ is a perpendicular bisector of $\overline{PP'}$, you have enough information to apply SAS to conclude that $\triangle RQP \cong \triangle RQ'P'$. Because corresponding parts of congruent triangles are congruent, $PQ = P'Q'$.

EXAMPLE 3 *Finding a Minimum Distance*

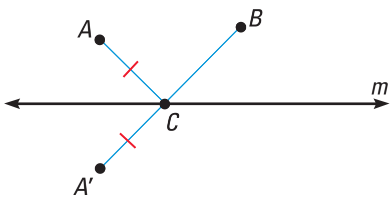
REAL LIFE

SURVEYING Two houses are located on a rural road m, as shown at the right. You want to place a telephone pole on the road at point C so that the length of the telephone cable, $AC + BC$, is a minimum. Where should you locate C?



SOLUTION

Reflect A in line m to obtain A'. Then, draw $\overline{A'B}$. Label the point at which this segment intersects m as C. Because $\overline{A'B}$ represents the shortest distance between A' and B, and $AC = A'C$, you can conclude that at point C a minimum length of telephone cable is used.

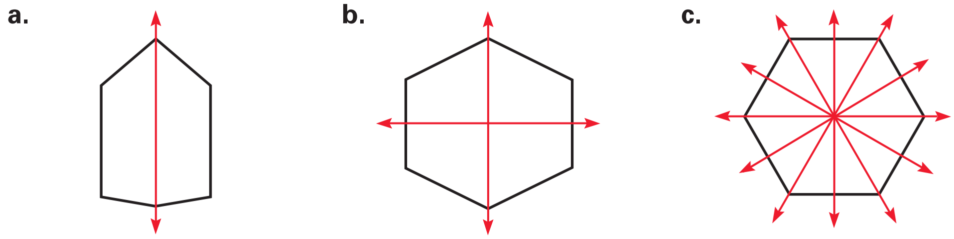


GOAL 2 REFLECTIONS AND LINE SYMMETRY

A figure in the plane has a **line of symmetry** if the figure can be mapped onto itself by a reflection in the line.

EXAMPLE 4 *Finding Lines of Symmetry*

Hexagons can have different lines of symmetry depending on their shape.



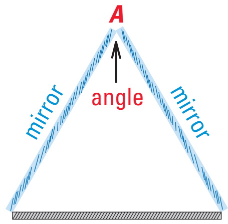
a. This hexagon has only one line of symmetry.

b. This hexagon has two lines of symmetry.

c. This hexagon has six lines of symmetry.

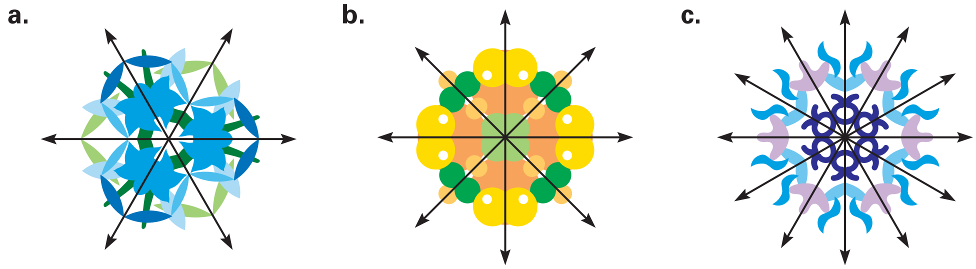
EXAMPLE 5 *Identifying Reflections*

KALEIDOSCOPES Inside a kaleidoscope, two mirrors are placed next to each other to form a V, as shown at the right. The angle between the mirrors determines the number of lines of symmetry in the image. The formula below can be used to calculate the angle between the mirrors, A, or the number of lines of symmetry in the image, n.



$$n(m\angle A) = 180°$$

Use the formula to find the angle that the mirrors must be placed for the image of a kaleidoscope to resemble the design.



a. b. c.

SOLUTION

a. There are 3 lines of symmetry. So, you can write $3(m\angle A) = 180°$. The solution is $m\angle A = 60°$.

b. There are 4 lines of symmetry. So, you can write $4(m\angle A) = 180°$. The solution is $m\angle A = 45°$.

c. There are 6 lines of symmetry. So, you can write $6(m\angle A) = 180°$. The solution is $m\angle A = 30°$.

FOCUS ON PEOPLE

KALEIDOSCOPES Sue and Bob Rioux design and make kaleidoscopes. The kaleidoscope in front of Sue is called *Sea Angel*.

INTERNET APPLICATION LINK www.mcdougallittell.com

GUIDED PRACTICE

Vocabulary Check ✓

1. Describe what a *line of symmetry* is.

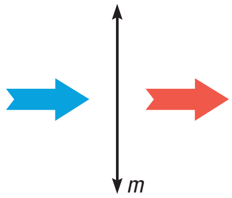
Concept Check ✓

2. When a point is reflected in the *x*-axis, how are the coordinates of the image related to the coordinates of the preimage?

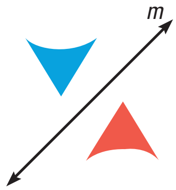
Skill Check ✓

Determine whether the blue figure maps onto the red figure by a reflection in line *m*.

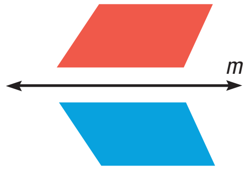
3.



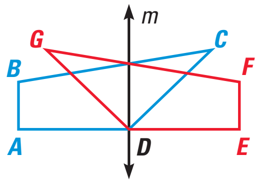
4.



5.



Use the diagram at the right to complete the statement.



6. $\overline{AB} \rightarrow$ __?__

7. __?__ $\rightarrow \angle DEF$

8. $C \rightarrow$ __?__

9. $D \rightarrow$ __?__

10. __?__ $\rightarrow \angle GFE$

11. __?__ $\rightarrow \overline{DG}$

FLOWERS **Determine the number of lines of symmetry in the flower.**

12.

13.

14.

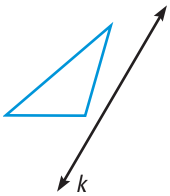
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master skills is on pp. 815 and 816.

DRAWING REFLECTIONS **Trace the figure and draw its reflection in line *k*.**

15.



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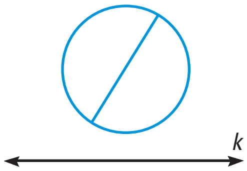

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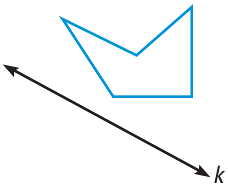


STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 15–30
Example 2: Exs. 33–35
Example 3: Exs. 36–40
Example 4: Exs. 31, 32
Example 5: Exs. 44–46

ANALYZING STATEMENTS **Decide whether the conclusion is *true* or *false*. Explain your reasoning.**

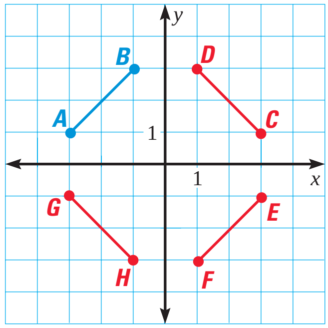
18. If $N(2, 4)$ is reflected in the line $y = 2$, then N' is $(2, 0)$.

19. If $M(6, -2)$ is reflected in the line $x = 3$, then M' is $(0, -2)$.

20. If $W(-6, -3)$ is reflected in the line $y = -2$, then W' is $(-6, 1)$.

21. If $U(5, 3)$ is reflected in the line $x = 1$, then U' is $(-3, 3)$.

REFLECTIONS IN A COORDINATE PLANE **Use the diagram at the right to name the image of $\overline{AB}$ after the reflection.**



22. Reflection in the x-axis

23. Reflection in the y-axis

24. Reflection in the line $y = x$

25. Reflection in the y-axis, followed by a reflection in the x-axis.

REFLECTIONS **In Exercises 26–29, find the coordinates of the reflection without using a coordinate plane. Then check your answer by plotting the image and preimage on a coordinate plane.**

26. $S(0, 2)$ reflected in the x-axis

27. $T(3, 8)$ reflected in the x-axis

28. $Q(-3, -3)$ reflected in the y-axis

29. $R(7, -2)$ reflected in the y-axis

30. CRITICAL THINKING Draw a triangle on the coordinate plane and label its vertices. Then reflect the triangle in the line $y = x$. What do you notice about the coordinates of the vertices of the preimage and the image?

LINES OF SYMMETRY **Sketch the figure, if possible.**

31. An octagon with exactly two lines of symmetry

32. A quadrilateral with exactly four lines of symmetry

PARAGRAPH PROOF **In Exercises 33–35, write a paragraph proof for each case of Theorem 7.1. (Refer to the diagrams on page 405.)**

33. In Case 2, it is given that a reflection in m maps P onto P' and Q onto Q'. Also, $\overline{PQ}$ intersects m at point R.

PROVE ▶ $PQ = P'Q'$

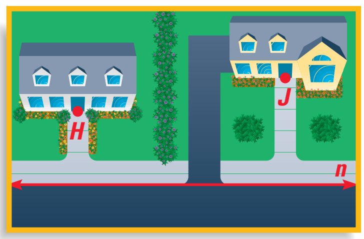
34. In Case 3, it is given that a reflection in m maps P onto P' and Q onto Q'. Also, P lies on line m and $\overline{PQ}$ is not perpendicular to m.

PROVE ▶ $PQ = P'Q'$

35. In Case 4, it is given that a reflection in m maps P onto P' and Q onto Q'. Also, Q lies on line m and $\overline{PQ}$ is perpendicular to line m.

PROVE ▶ $PQ = P'Q'$

36. DELIVERING PIZZA You park your car at some point K on line n. You deliver a pizza to house H, go back to your car, and deliver a pizza to house J. Assuming that you cut across both lawns, explain how to estimate K so the distance that you travel is as small as possible.



STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with Exs. 37–40.

MINIMUM DISTANCE **Find point C on the x-axis so $AC + BC$ is a minimum.**

37. $A(1, 5)$, $B(7, 1)$

38. $A(2, -2)$, $B(11, -4)$

39. $A(-1, 4)$, $B(6, 3)$

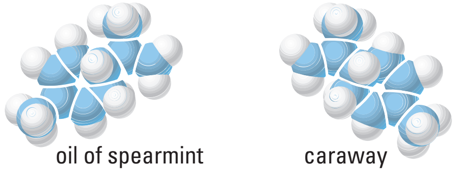
40. $A(-4, 6)$, $B(3.5, 9)$

FOCUS ON CAREERS

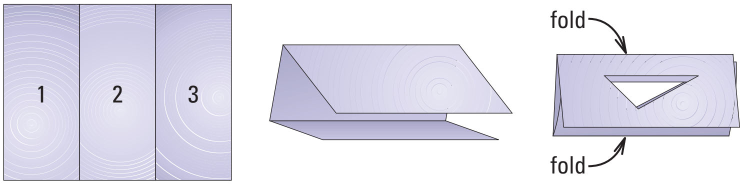
REAL LIFE **CHEMIST** Some chemists study the molecular structure of living things. The research done by these chemists has led to important discoveries in the field of medicine.

INTERNET CAREER LINK www.mcdougallittell.com

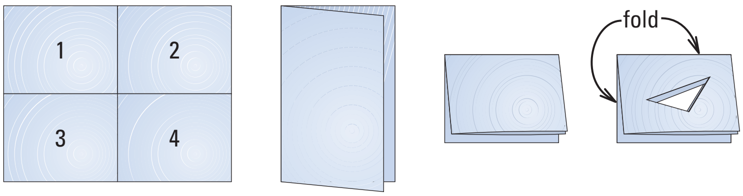
41. **CHEMISTRY CONNECTION** The figures at the right show two versions of the carvone molecule. One version is oil of spearmint and the other is caraway. How are the structures of these two molecules related?



42. **PAPER FOLDING** Fold a piece of paper and label it as shown. Cut a scalene triangle out of the folded paper and unfold the paper. How are triangle 2 and triangle 3 related to triangle 1?

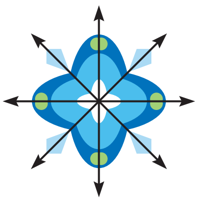


43. **PAPER FOLDING** Fold a piece of paper and label it as shown. Cut a scalene triangle out of the folded paper and unfold the paper. How are triangles 2, 3, and 4 related to triangle 1?

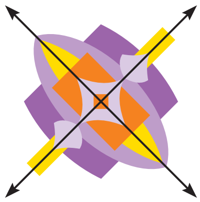


KALEIDOSCOPES **In Exercises 44–46, calculate the angle at which the mirrors must be placed for the image of a kaleidoscope to resemble the given design. (Use the formula in Example 5 on page 406.)**

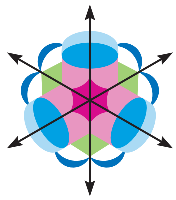
44.



45.



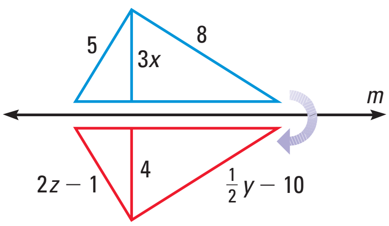
46.



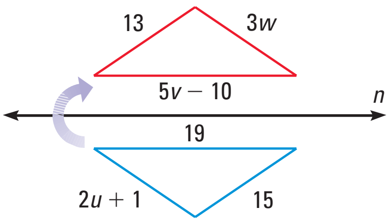
47. **TECHNOLOGY** Use geometry software to draw a polygon reflected in line m. Connect the corresponding vertices of the preimage and image. Measure the distance between each vertex and line m. What do you notice about these measures?

USING ALGEBRA **Find the value of each variable, given that the diagram shows a reflection in a line.**

48.

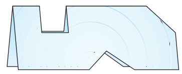


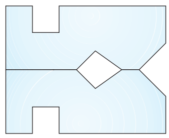
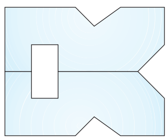
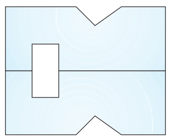
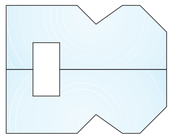
49.



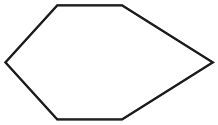
Test Preparation

50. MULTIPLE CHOICE A piece of paper is folded in half and some cuts are made, as shown. Which figure represents the piece of paper unfolded?



Ⓐ  Ⓑ  Ⓒ  Ⓓ 

51. MULTIPLE CHOICE How many lines of symmetry does the figure at the right have?



Ⓐ 0　Ⓑ 1　Ⓒ 2

Ⓓ 3　Ⓔ 6

★ Challenge

WRITING AN EQUATION Follow the steps to write an equation for the line of reflection.

52. Graph $R(2, 1)$ and $R'(-2, -1)$. Draw a segment connecting the two points.

53. Find the midpoint of $\overline{RR'}$ and name it Q.

54. Find the slope of $\overline{RR'}$. Then write the slope of a line perpendicular to $\overline{RR'}$.

55. Write an equation of the line that is perpendicular to $\overline{RR'}$ and passes through Q.

56. Repeat Exercises 52–55 using $R(-2, 3)$ and $R'(3, -2)$.

EXTRA CHALLENGE
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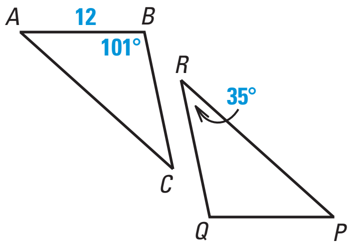
MIXED REVIEW

CONGRUENT TRIANGLES Use the diagram, in which $\triangle ABC \cong \triangle PQR$, to complete the statement. (Review 4.2 for 7.3)



57. $\angle A \cong$ __?__　**58.** $PQ =$ __?__

59. $\overline{QR} \cong$ __?__　**60.** $m\angle C =$ __?__

61. $m\angle Q =$ __?__　**62.** $\angle R \cong$ __?__

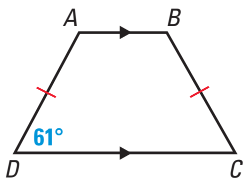
FINDING SIDE LENGTHS OF A TRIANGLE Two side lengths of a triangle are given. Describe the length of the third side, *c*, with an inequality. (Review 5.5)

63. $a = 7, b = 17$　**64.** $a = 9, b = 21$　**65.** $a = 12, b = 33$

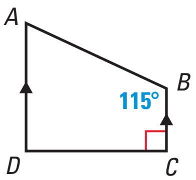
66. $a = 26, b = 6$　**67.** $a = 41.2, b = 15.5$　**68.** $a = 7.1, b = 11.9$

FINDING ANGLE MEASURES Find the angle measures of *ABCD*. (Review 6.5)

69.



70.



71.