7.1 Rigid Motion in a Plane

What you should learn

GOAL 1 Identify the three basic rigid transformations.

GOAL 2 Use transformations in **real-life** situations, such as building a kayak in **Example 5**.

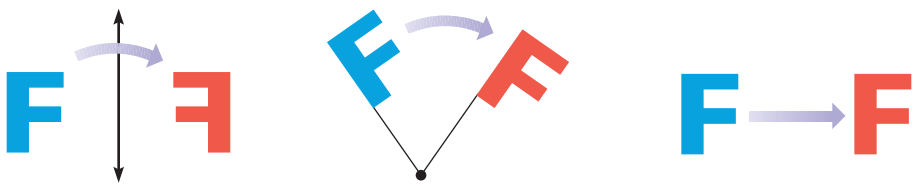
Why you should learn it

▼ Transformations help you when planning a stenciled design, such as on the wall below and the stencil in **Ex. 41**.

GOAL 1 IDENTIFYING TRANSFORMATIONS

Figures in a plane can be reflected, rotated, or translated to produce new figures. The new figure is called the **image**, and the original figure is called the **preimage**. The operation that *maps*, or moves, the preimage onto the image is called a **transformation**.

In this chapter, you will learn about three basic transformations—*reflections*, *rotations*, and *translations*—and combinations of these. For each of the three transformations below, the blue figure is the preimage and the red figure is the image. This color convention will be used throughout this book.



Reflection in a line | **Rotation about a point** | **Translation**

Some transformations involve labels. When you name an image, take the corresponding point of the preimage and add a prime symbol. For instance, if the preimage is A, then the image is A', read as "*A prime*."

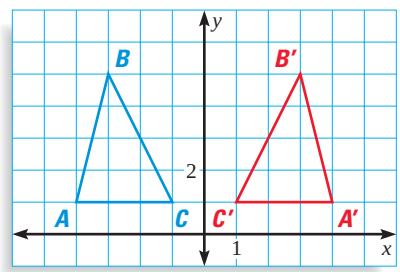
EXAMPLE 1 *Naming Transformations*

Use the graph of the transformation at the right.

a. Name and describe the transformation.

b. Name the coordinates of the vertices of the image.

c. Is $\triangle ABC$ congruent to its image?



SOLUTION

a. The transformation is a reflection in the y-axis. You can imagine that the image was obtained by flipping $\triangle ABC$ over the y-axis.

b. The coordinates of the vertices of the image, $\triangle A'B'C'$, are $A'(4, 1)$, $B'(3, 5)$, and $C'(1, 1)$.

c. Yes, $\triangle ABC$ is congruent to its image $\triangle A'B'C'$. One way to show this would be to use the Distance Formula to find the lengths of the sides of both triangles. Then use the SSS Congruence Postulate.

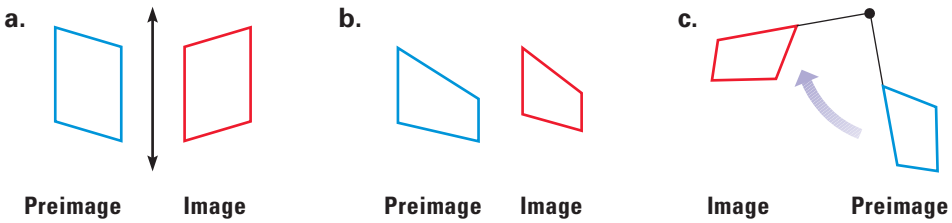
STUDENT HELP

Study Tip
The term **isometry** comes from the Greek phrase *isos metrom*, meaning *equal measure*.

An **isometry** is a transformation that preserves lengths. Isometries also preserve angle measures, parallel lines, and distances between points. Transformations that are isometries are called *rigid transformations*.

EXAMPLE 2 Identifying Isometries

Which of the following transformations appear to be isometries?



SOLUTION

a. This transformation appears to be an isometry. The blue parallelogram is reflected in a line to produce a congruent red parallelogram.

b. This transformation is not an isometry. The image is not congruent to the preimage.

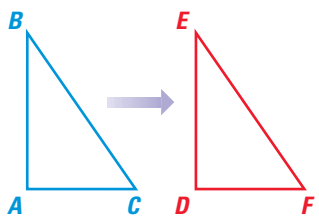
c. This transformation appears to be an isometry. The blue parallelogram is rotated about a point to produce a congruent red parallelogram.

. . . . . .

MAPPINGS You can describe the transformation in the diagram by writing "$\triangle ABC$ is *mapped onto* $\triangle DEF$." You can also use arrow notation as follows:

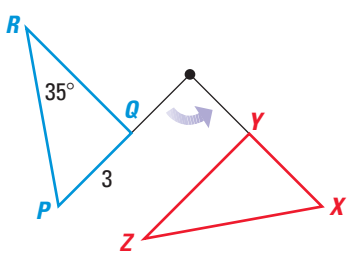
$$\triangle ABC \rightarrow \triangle DEF$$

The order in which the vertices are listed specifies the correspondence. Either of the descriptions implies that $A \rightarrow D$, $B \rightarrow E$, and $C \rightarrow F$.



EXAMPLE 3 Preserving Length and Angle Measure

In the diagram, $\triangle PQR$ is mapped onto $\triangle XYZ$. The mapping is a rotation. Given that $\triangle PQR \rightarrow \triangle XYZ$ is an isometry, find the length of $\overline{XY}$ and the measure of $\angle Z$.



SOLUTION

The statement "$\triangle PQR$ is mapped onto $\triangle XYZ$" implies that $P \rightarrow X$, $Q \rightarrow Y$, and $R \rightarrow Z$. Because the transformation is an isometry, the two triangles are congruent.

▶ So, $XY = PQ = 3$ and $m\angle Z = m\angle R = 35°$.

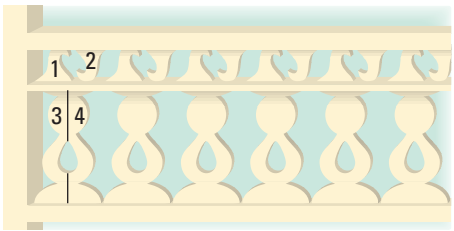
FOCUS ON APPLICATIONS

REAL LIFE **CARPENTER GOTHIC** The woodwork of carpenter gothic houses contains decorative patterns. Notice the translations in the patterns of the carpenter gothic house above.

GOAL 2 USING TRANSFORMATIONS IN REAL LIFE

EXAMPLE 4 *Identifying Transformations*

CARPENTRY You are assembling pieces of wood to complete a railing for your porch. The finished railing should resemble the one below.



a. How are pieces 1 and 2 related? pieces 3 and 4?

b. In order to assemble the rail as shown, explain why you need to know how the pieces are related.

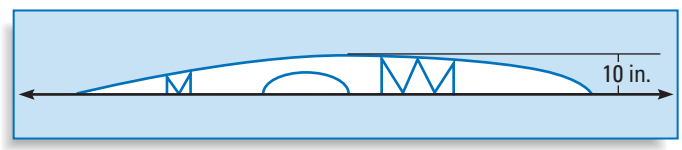
SOLUTION

a. Pieces 1 and 2 are related by a rotation. Pieces 3 and 4 are related by a reflection.

b. Knowing how the pieces are related helps you manipulate the pieces to create the desired pattern.

EXAMPLE 5 *Using Transformations*

REAL LIFE **BUILDING A KAYAK** Many building plans for kayaks show the layout and dimensions for only half of the kayak. A plan of the top view of a kayak is shown below.



a. What type of transformation can a builder use to visualize plans for the entire body of the kayak?

b. Using the plan above, what is the maximum width of the entire kayak?

SOLUTION

a. The builder can use a reflection to visualize the entire kayak. For instance, when one half of the kayak is reflected in a line through its center, you obtain the other half of the kayak.

b. The two halves of the finished kayak are congruent, so the width of the entire kayak will be 2(10), or 20 inches.

GUIDED PRACTICE

Vocabulary Check ✓

1. An operation that maps a preimage onto an image is called a __?__.

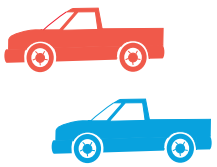
Concept Check ✓

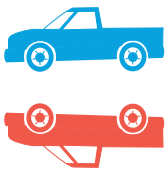
Complete the statement with *always, sometimes,* or *never*.

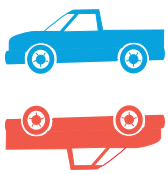
2. The preimage and the image of a transformation are __?__ congruent.
3. A transformation that is an isometry __?__ preserves length.
4. An isometry __?__ maps an acute triangle onto an obtuse triangle.

Skill Check ✓

Name the transformation that maps the blue pickup truck (preimage) onto the red pickup (image).

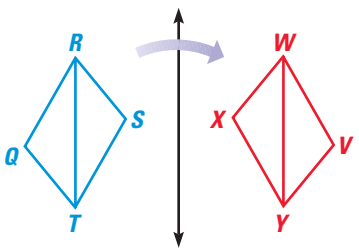
5. 

6. 

7. 

Use the figure shown, where figure *QRST* is mapped onto figure *VWXY*.

8. Name the preimage of $\overline{XY}$.
9. Name the image of $\overline{QR}$.
10. Name two angles that have the same measure.
11. Name a triangle that appears to be congruent to $\triangle RST$.



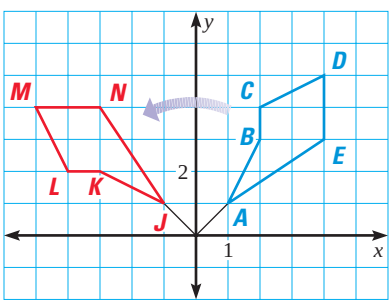
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 815.

NAMING TRANSFORMATIONS **Use the graph of the transformation below.**

12. Figure $ABCDE \rightarrow$ Figure __?__
13. Name and describe the transformation.
14. Name two sides with the same length.
15. Name two angles with the same measure.
16. Name the coordinates of the preimage of point L.
17. Show two corresponding sides have the same length, using the Distance Formula.



STUDENT HELP

HOMEWORK HELP

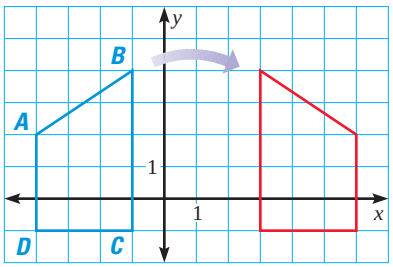
Example 1: Exs. 12–22
Example 2: Exs. 23–25
Example 3: Exs. 26–31
Example 4: Exs. 36–39
Example 5: Ex. 41

ANALYZING STATEMENTS **Is the statement *true* or *false*?**

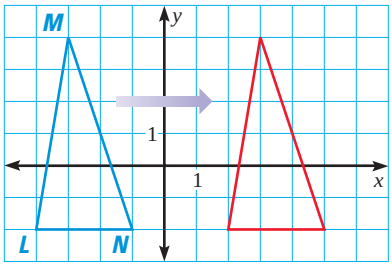
18. Isometries preserve angle measures and parallel lines.
19. Transformations that are *not* isometries are called rigid transformations.
20. A reflection in a line is a type of transformation.

DESCRIBING TRANSFORMATIONS Name and describe the transformation. Then name the coordinates of the vertices of the image.

21.

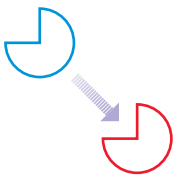


22.

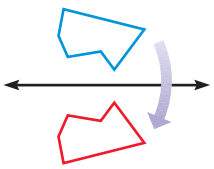


ISOMETRIES Does the transformation appear to be an isometry? Explain.

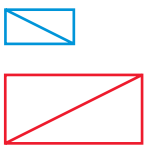
23.



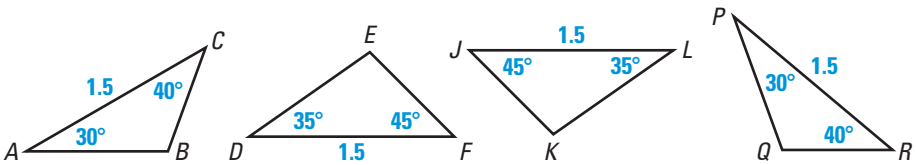
24.



25.



COMPLETING STATEMENTS Use the diagrams to complete the statement.



26. $\triangle ABC \rightarrow \triangle \_?\_$

27. $\triangle DEF \rightarrow \triangle \_?\_$

28. $\triangle \_?\_ \rightarrow \triangle EFD$

29. $\triangle \_?\_ \rightarrow \triangle ACB$

30. $\triangle LJK \rightarrow \triangle \_?\_$

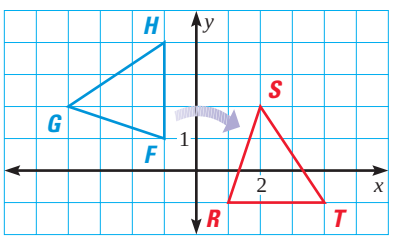
31. $\triangle \_?\_ \rightarrow \triangle CBA$

STUDENT HELP

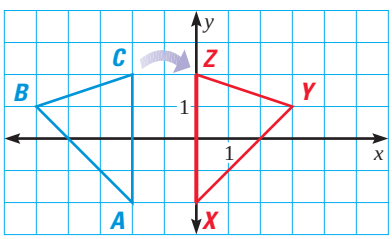
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 32 and 33.

SHOWING AN ISOMETRY Show that the transformation is an isometry by using the Distance Formula to compare the side lengths of the triangles.

32. $\triangle FGH \rightarrow \triangle RST$

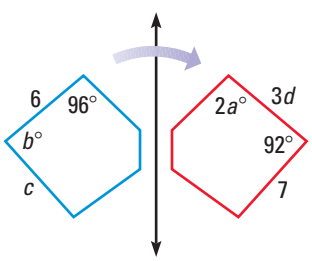


33. $\triangle ABC \rightarrow \triangle XYZ$

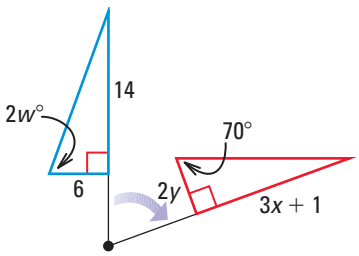


USING ALGEBRA Find the value of each variable, given that the transformation is an isometry.

34.



35.



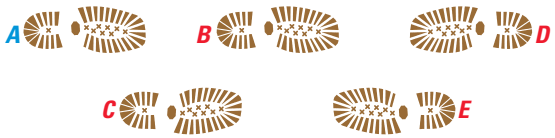
FOOTPRINTS **In Exercises 36–39, name the transformation that will map footprint *A* onto the indicated footprint.**

36. Footprint *B*

37. Footprint *C*

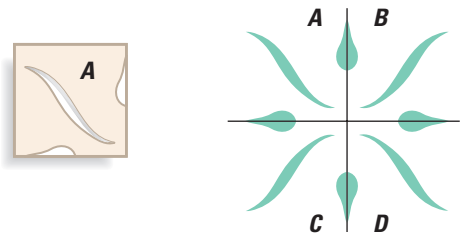
38. Footprint *D*

39. Footprint *E*



40. *Writing* Can a point or a line segment be its own preimage? Explain and illustrate your answer.

41. **STENCILING** You are stenciling the living room of your home. You want to use the stencil pattern below on the left to create the design shown. What type of transformation will you use to manipulate the stencil from *A* to *B*? from *A* to *C*? from *A* to *D*?



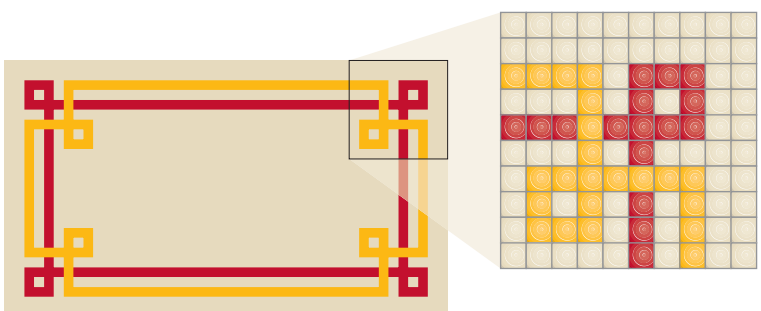
FOCUS ON APPLICATIONS

REAL LIFE **EMBROIDERY** Before machines, all stitching was done by hand. Completing samplers, such as the one above, served as practice for those learning how to stitch.

42. **MACHINE EMBROIDERY** Computerized embroidery machines are used to sew letters and designs on fabric. A computerized embroidery machine can use the same symbol to create several different letters. Which of the letters below are rigid transformations of other letters? Explain how a computerized embroidery machine can create these letters from one symbol.

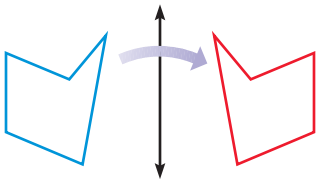
abcdefghijklm
nopqrstuvwxyz

43. **TILING A FLOOR** You are tiling a kitchen floor using the design shown below. You use a plan to lay the tile for the upper right corner of the floor design. Describe how you can use the plan to complete the other three corners of the floor.



Test Preparation

44. MULTIPLE CHOICE What type of transformation is shown?



Ⓐ slide Ⓑ reflection

Ⓒ translation Ⓓ rotation

45. MULTIPLE CHOICE Which of the following is *not* a rotation of the figure at right?

Ⓐ 

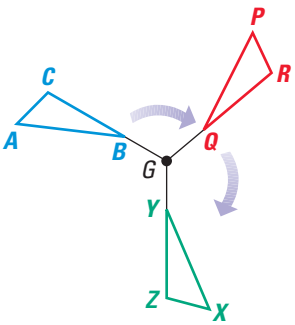
Ⓑ 

Ⓒ 

Ⓓ 

Challenge

46. TWO-COLUMN PROOF Write a two-column proof using the given information and the diagram.



GIVEN ▶ $\triangle ABC \rightarrow \triangle PQR$ and $\triangle PQR \rightarrow \triangle XYZ$ are isometries.

PROVE ▶ $\triangle ABC \rightarrow \triangle XYZ$ is an isometry.

Plan for Proof Show that $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, and $\overline{AC} \cong \overline{XZ}$.

EXTRA CHALLENGE
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MIXED REVIEW

USING THE DISTANCE FORMULA **Find the distance between the two points.** (Review 1.3 for 7.2)

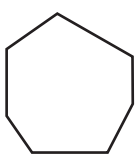
47. $A(3, 10), B(-2, -2)$

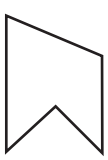
48. $C(5, -7), D(-11, 6)$

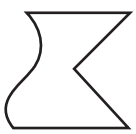
49. $E(0, 8), F(-8, 3)$

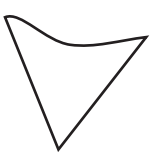
50. $G(0, -7), H(6, 3)$

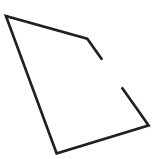
IDENTIFYING POLYGONS **Determine whether the figure is a polygon. If it is not, explain why not.** (Review 6.1 for 7.2)

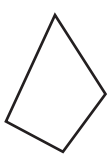
51. 

52. 

53. 

54. 

55. 

56. 

USING COORDINATE GEOMETRY **Use two different methods to show that the points represent the vertices of a parallelogram.** (Review 6.3)

57. $P(0, 4), Q(7, 6), R(8, -2), S(1, -4)$

58. $W(1, 5), X(9, 5), Y(6, -1), Z(-2, -1)$