6.6 Special Quadrilaterals

What you should learn

GOAL 1 Identify special quadrilaterals based on limited information.

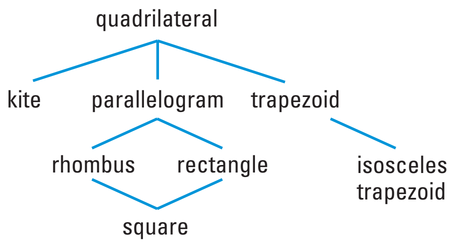
GOAL 2 Prove that a quadrilateral is a special type of quadrilateral, such as a rhombus or a trapezoid.

Why you should learn it

▼ To understand and describe **real-world** shapes such as gem facets in **Exs. 42 and 43**. REAL LIFE

GOAL 1 SUMMARIZING PROPERTIES OF QUADRILATERALS

In this chapter, you have studied the seven special types of quadrilaterals at the right. Notice that each shape has all the properties of the shapes linked above it. For instance, squares have the properties of rhombuses, rectangles, parallelograms, and quadrilaterals.

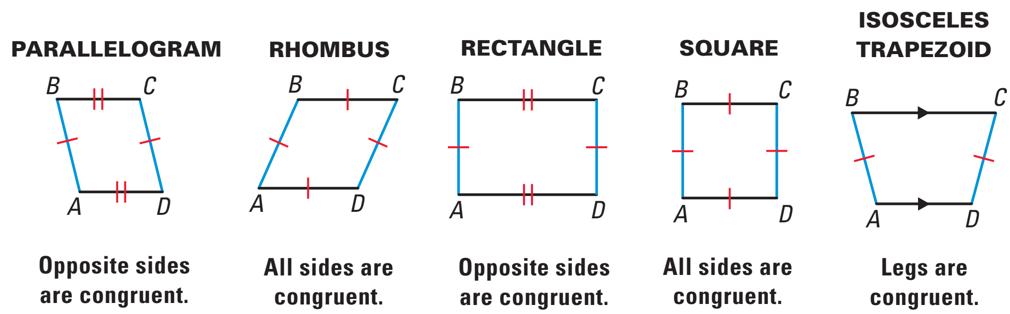


EXAMPLE 1 *Identifying Quadrilaterals*

Quadrilateral *ABCD* has at least one pair of opposite sides congruent. What kinds of quadrilaterals meet this condition?

SOLUTION

There are many possibilities.

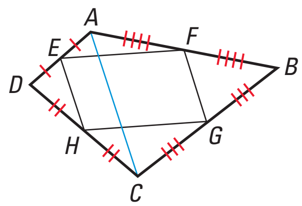


EXAMPLE 2 *Connecting Midpoints of Sides*

When you join the midpoints of the sides of any quadrilateral, what special quadrilateral is formed? Why?

SOLUTION

Let *E*, *F*, *G*, and *H* be the midpoints of the sides of any quadrilateral, *ABCD*, as shown.



If you draw $\overline{AC}$, the Midsegment Theorem for triangles says $\overline{FG} \parallel \overline{AC}$ and $\overline{EH} \parallel \overline{AC}$, so $\overline{FG} \parallel \overline{EH}$. Similar reasoning shows that $\overline{EF} \parallel \overline{HG}$.

▶ So, by definition, *EFGH* is a parallelogram.

GOAL 2 PROOF WITH SPECIAL QUADRILATERALS

When you want to prove that a quadrilateral has a specific shape, you can use either the definition of the shape as in Example 2, *or* you can use a theorem.

CONCEPT SUMMARY — PROVING QUADRILATERALS ARE RHOMBUSES

You have learned three ways to prove that a quadrilateral is a rhombus.

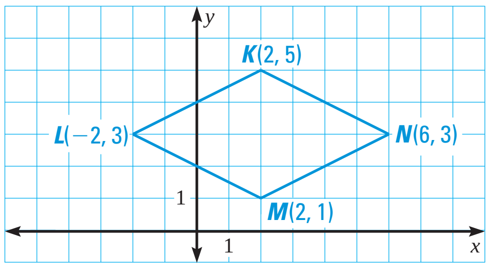
1. You can use the definition and show that the quadrilateral is a *parallelogram* that has four congruent sides. It is easier, however, to use the Rhombus Corollary and simply show that all four sides of the quadrilateral are congruent.
2. Show that the quadrilateral is a parallelogram *and* that the diagonals are perpendicular. (*Theorem 6.11*)
3. Show that the quadrilateral is a parallelogram *and* that each diagonal bisects a pair of opposite angles. (*Theorem 6.12*)

STUDENT HELP

Look Back
For help with proving a quadrilateral is a parallelogram, see pp. 338–341.

EXAMPLE 3 *Proving a Quadrilateral is a Rhombus*

Show that *KLMN* is a rhombus.



SOLUTION You can use any of the three ways described in the concept summary above. For instance, you could show that opposite sides have the same slope and that the diagonals are perpendicular. Another way, shown below, is to prove that all four sides have the same length.

$$LM = \sqrt{[2-(-2)]^2 + (1-3)^2} = \sqrt{4^2 + (-2)^2} = \sqrt{20}$$

$$NK = \sqrt{(2-6)^2 + (5-3)^2} = \sqrt{(-4)^2 + 2^2} = \sqrt{20}$$

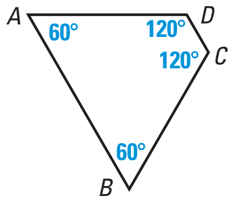
$$MN = \sqrt{(6-2)^2 + (3-1)^2} = \sqrt{4^2 + 2^2} = \sqrt{20}$$

$$KL = \sqrt{(-2-2)^2 + (3-5)^2} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{20}$$

▶ So, because $LM = NK = MN = KL$, *KLMN* is a rhombus.

EXAMPLE 4 Identifying a Quadrilateral

What type of quadrilateral is $ABCD$?
Explain your reasoning.



SOLUTION

$\angle A$ and $\angle D$ are supplementary, but $\angle A$ and $\angle B$ are not. So, $\overline{AB} \parallel \overline{DC}$ but $\overline{AD}$ is not parallel to $\overline{BC}$. By definition, $ABCD$ is a trapezoid. Because base angles are congruent, $ABCD$ is an isosceles trapezoid.

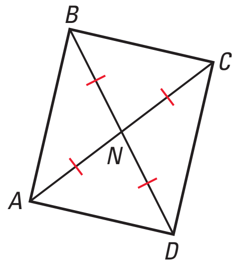
EXAMPLE 5 Identifying a Quadrilateral

The diagonals of quadrilateral $ABCD$ intersect at point N to produce four congruent segments: $\overline{AN} \cong \overline{BN} \cong \overline{CN} \cong \overline{DN}$. What type of quadrilateral is $ABCD$? Prove that your answer is correct.

SOLUTION

Draw a diagram:

Draw the diagonals as described. Then connect the endpoints to draw quadrilateral $ABCD$.



Make a conjecture:

Quadrilateral $ABCD$ looks like a rectangle.

Proof

Prove your conjecture:

GIVEN ▶ $\overline{AN} \cong \overline{BN} \cong \overline{CN} \cong \overline{DN}$

PROVE ▶ $ABCD$ is a rectangle.

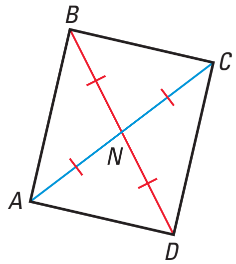
Paragraph Proof Because you are given information about the diagonals, show that $ABCD$ is a parallelogram with congruent diagonals.

First prove that $ABCD$ is a parallelogram.

Because $\overline{BN} \cong \overline{DN}$ and $\overline{AN} \cong \overline{CN}$, $\overline{BD}$ and $\overline{AC}$ bisect each other. Because the diagonals of $ABCD$ bisect each other, $ABCD$ is a parallelogram.

Then prove that the diagonals of $ABCD$ are congruent.

From the given you can write $BN = AN$ and $DN = CN$ so, by the Addition Property of Equality, $BN + DN = AN + CN$. By the Segment Addition Postulate, $BD = BN + DN$ and $AC = AN + CN$ so, by substitution, $BD = AC$.



So, $\overline{BD} \cong \overline{AC}$.

▶ $ABCD$ is a parallelogram with congruent diagonals, so $ABCD$ is a rectangle.

GUIDED PRACTICE

Concept Check ✓

1. In Example 2, explain how to prove that $\overline{EF} \parallel \overline{HG}$.

Skill Check ✓

Copy the chart. Put an X in the box if the shape *always* has the given property.

	Property	▱	Rectangle	Rhombus	Square	Kite	Trapezoid
2.	Both pairs of opp. sides are $\parallel$.	?	?	?	?	?	?
3.	Exactly 1 pair of opp. sides are $\parallel$.	?	?	?	?	?	?
4.	Diagonals are $\perp$.	?	?	?	?	?	?
5.	Diagonals are $\cong$.	?	?	?	?	?	?
6.	Diagonals bisect each other.	?	?	?	?	?	?

7. Which quadrilaterals can you form with four sticks of the same length? You must attach the sticks at their ends and cannot bend or break any of them.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 814.

PROPERTIES OF QUADRILATERALS **Copy the chart. Put an X in the box if the shape *always* has the given property.**

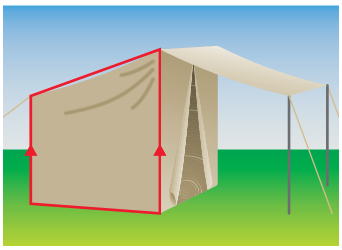
	Property	▱	Rectangle	Rhombus	Square	Kite	Trapezoid
8.	Both pairs of opp. sides are $\cong$.	?	?	?	?	?	?
9.	Exactly 1 pair of opp. sides are $\cong$.	?	?	?	?	?	?
10.	All sides are $\cong$.	?	?	?	?	?	?
11.	Both pairs of opp. $\angle$s are $\cong$.	?	?	?	?	?	?
12.	Exactly 1 pair of opp. $\angle$s are $\cong$.	?	?	?	?	?	?
13.	All $\angle$s are $\cong$.	?	?	?	?	?	?

FOCUS ON APPLICATIONS

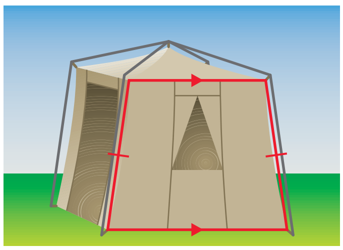
REAL LIFE Tents are designed differently for different climates. For example, winter tents are designed to shed snow. Desert tents can have flat roofs because they don't need to shed rain.

TENT SHAPES **What kind of special quadrilateral is the red shape?**

14.



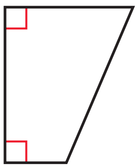
15.

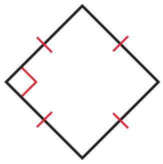


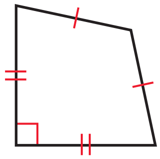
STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 8–24, 30–35
Example 2: Exs. 14–18, 42–44
Example 3: Exs. 25–29, 36–41
Example 4: Exs. 14–18, 42, 43
Example 5: Exs. 45–47

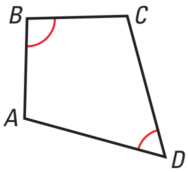
IDENTIFYING QUADRILATERALS **Identify the special quadrilateral. Use the most specific name.**

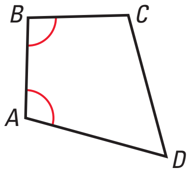
16.


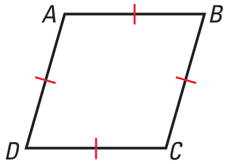
17.


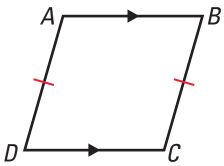
18.


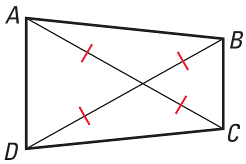
IDENTIFYING QUADRILATERALS **What kinds of quadrilaterals meet the conditions shown? *ABCD* is not drawn to scale.**

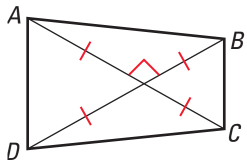
19.


20.


21.


22.


23.


24.


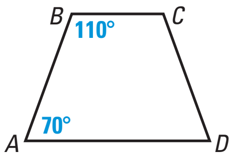
STUDENT HELP

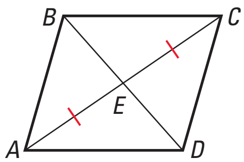
Study Tip
See the summaries for parallelograms and rhombuses on pp. 340 and 365, and the list of postulates and theorems on pp. 828–837. You can refer to your summaries as you do the rest of the exercises.

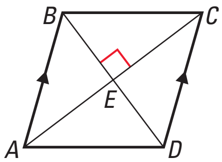
DESCRIBING METHODS OF PROOF **Summarize the ways you have learned to prove that a quadrilateral is the given special type of quadrilateral.**

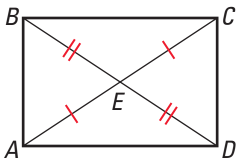
25. kite
26. square
27. rectangle
28. trapezoid
29. isosceles trapezoid

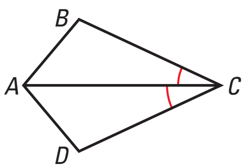
DEVELOPING PROOF **Which two segments or angles must be congruent to enable you to prove *ABCD* is the given quadrilateral? Explain your reasoning. There may be more than one right answer.**

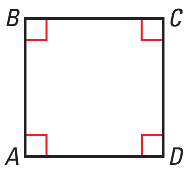
30. isosceles trapezoid


31. parallelogram


32. rhombus


33. rectangle


34. kite


35. square


QUADRILATERALS **What kind of quadrilateral is *PQRS*? Justify your answer.**

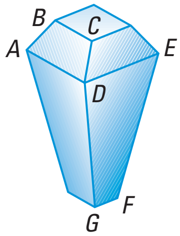
36. $P(0, 0)$, $Q(0, 2)$, $R(5, 5)$, $S(2, 0)$
37. $P(1, 1)$, $Q(5, 1)$, $R(4, 8)$, $S(2, 8)$
38. $P(2, 1)$, $Q(7, 1)$, $R(7, 7)$, $S(2, 5)$
39. $P(0, 7)$, $Q(4, 8)$, $R(5, 2)$, $S(1, 1)$
40. $P(1, 7)$, $Q(5, 9)$, $R(8, 3)$, $S(4, 1)$
41. $P(5, 1)$, $Q(9, 6)$, $R(5, 11)$, $S(1, 6)$

FOCUS ON CAREERS

GEMOLOGISTS analyze the cut of a gem when determining its value.

CAREER LINK
www.mcdougallittell.com

GEM CUTTING **In Exercises 42 and 43, use the following information.**
There are different ways of cutting gems to enhance the beauty of the jewel. One of the earliest shapes used for diamonds is called the *table cut*, as shown at the right. Each face of a cut gem is called a *facet*.



42. $\overline{BC} \parallel \overline{AD}$, $\overline{AB}$ and $\overline{DC}$ are not parallel. What shape is the facet labeled $ABCD$?

43. $\overline{DE} \parallel \overline{GF}$, $\overline{DG}$ and $\overline{EF}$ are congruent, but not parallel. What shape is the facet labeled $DEFG$?

44. JUSTIFYING A CONSTRUCTION Look back at the *Perpendicular to a Line* construction on page 130. Explain why this construction works.

DRAWING QUADRILATERALS **Draw $\overline{AC}$ and $\overline{BD}$ as described. What special type of quadrilateral is *ABCD*? Prove that your answer is correct.**

45. $\overline{AC}$ and $\overline{BD}$ bisect each other, but they are not perpendicular or congruent.

46. $\overline{AC}$ and $\overline{BD}$ bisect each other. $\overline{AC} \perp \overline{BD}$, $\overline{AC} \not\cong \overline{BD}$

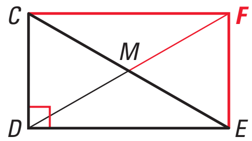
47. $\overline{AC} \perp \overline{BD}$, and $\overline{AC}$ bisects $\overline{BD}$. $\overline{BD}$ does not bisect $\overline{AC}$.

48. LOGICAL REASONING $EFGH$, $GHJK$, and $JKLM$ are all parallelograms. If $\overline{EF}$ and $\overline{LM}$ are not collinear, what kind of quadrilateral is $EFLM$? Prove that your answer is correct.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 45–47.

49. PROOF Prove that the median of a right triangle is one half the length of the hypotenuse.



GIVEN ▶ $\angle CDE$ is a right angle. $\overline{CM} \cong \overline{EM}$

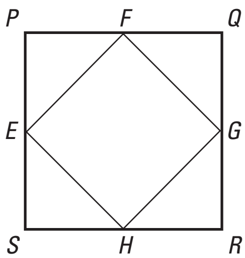
PROVE ▶ $\overline{DM} \cong \overline{CM}$

Plan for Proof First draw $\overline{CF}$ and $\overline{EF}$ so $CDEF$ is a rectangle. (How?)

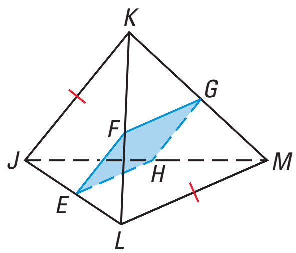
50. PROOF Use facts about angles to prove that the quadrilateral in Example 5 is a rectangle. (*Hint:* Let $x°$ be the measure of $\angle ABN$. Find the measures of the other angles in terms of x.)

PROOF **What special type of quadrilateral is *EFGH*? Prove that your answer is correct.**

51. GIVEN ▶ $PQRS$ is a square. E, F, G, and H are midpoints of the sides of the square.

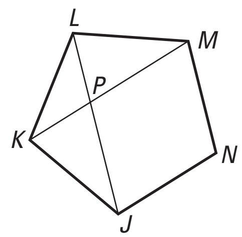


52. GIVEN ▶ $\overline{JK} \cong \overline{LM}$, E, F, G, and H are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$.



Test Preparation

53. MULTI-STEP PROBLEM Copy the diagram. $JKLMN$ is a regular pentagon. You will identify $JPMN$.



a. What kind of triangle is $\triangle JKL$? Use $\triangle JKL$ to prove that $\angle LJN \cong \angle JLM$.

b. List everything you know about the interior angles of $JLMN$. Use these facts to prove that $\overline{JL} \parallel \overline{NM}$.

c. Reasoning similar to parts (a) and (b) shows that $\overline{KM} \parallel \overline{JN}$. Based on this and the result from part (b), what kind of shape is $JPMN$?

d. *Writing* Is $JPMN$ a rhombus? Justify your answer.

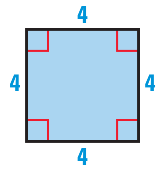
★ **Challenge**

54. PROOF $\overline{AC}$ and $\overline{BD}$ intersect each other at N. $\overline{AN} \cong \overline{BN}$ and $\overline{CN} \cong \overline{DN}$, but $\overline{AC}$ and $\overline{BD}$ do not bisect each other. Draw $\overline{AC}$ and $\overline{BD}$, and $ABCD$. What special type of quadrilateral is $ABCD$? Write a plan for a proof of your answer.

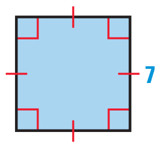
MIXED REVIEW

FINDING AREA **Find the area of the figure.** **(Review 1.7 for 6.7)**

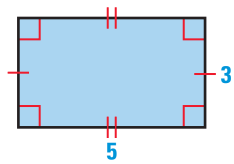
55.



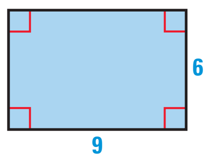
56.



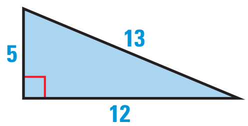
57.



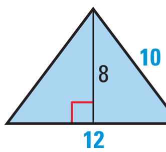
58.



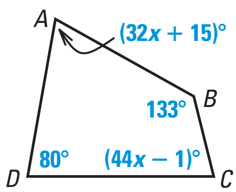
59.



60.



USING ALGEBRA **In Exercises 61 and 62, use the diagram at the right.** **(Review 6.1)**

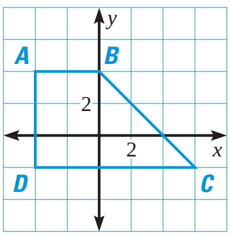


61. What is the value of x?

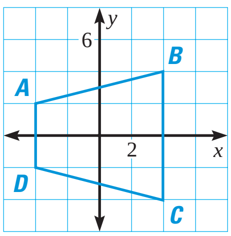
62. What is $m\angle A$? Use your result from Exercise 61.

FINDING THE MIDSEGMENT **Find the length of the midsegment of the trapezoid.** **(Review 6.5 for 6.7)**

63.



64.



65.