6.5 Trapezoids and Kites

What *you should learn*

GOAL 1 Use properties of trapezoids.

GOAL 2 Use properties of kites.

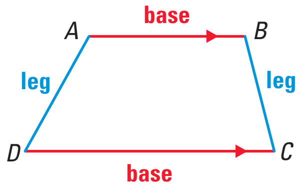
Why *you should learn it*

▼ To solve **real-life** problems, such as planning the layers of a layer cake in **Example 3**.

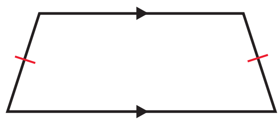
REAL LIFE

GOAL 1 USING PROPERTIES OF TRAPEZOIDS

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**. A trapezoid has two pairs of **base angles**. For instance, in trapezoid *ABCD*, $\angle D$ and $\angle C$ are one pair of base angles. The other pair is $\angle A$ and $\angle B$. The nonparallel sides are the **legs** of the trapezoid.



If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.



isosceles trapezoid

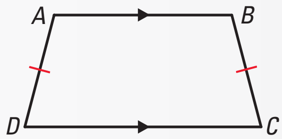
You are asked to prove the following theorems in the exercises.

THEOREMS

THEOREM 6.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

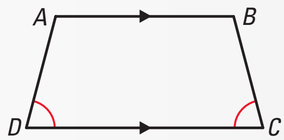
$\angle A \cong \angle B$, $\angle C \cong \angle D$



THEOREM 6.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

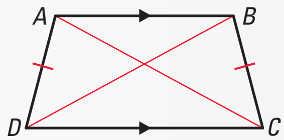
ABCD is an isosceles trapezoid.



THEOREM 6.16

A trapezoid is isosceles if and only if its diagonals are congruent.

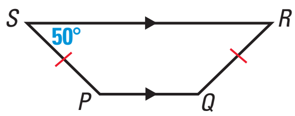
ABCD is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



$AD \cong BC$

EXAMPLE 1 *Using Properties of Isosceles Trapezoids*

PQRS is an isosceles trapezoid. Find $m\angle P$, $m\angle Q$, and $m\angle R$.



SOLUTION *PQRS* is an isosceles trapezoid, so $m\angle R = m\angle S = 50°$. Because $\angle S$ and $\angle P$ are consecutive interior angles formed by parallel lines, they are supplementary. So, $m\angle P = 180° - 50° = 130°$, and $m\angle Q = m\angle P = 130°$.

STUDENT HELP

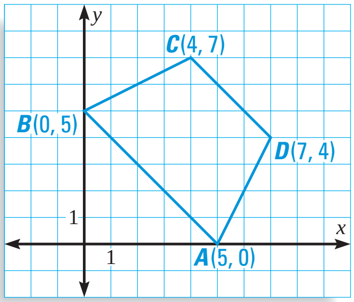
INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 2 Using Properties of Trapezoids

Show that $ABCD$ is a trapezoid.

SOLUTION

Compare the slopes of opposite sides.



The slope of $\overline{AB} = \frac{5-0}{0-5} = \frac{5}{-5} = -1$.

The slope of $\overline{CD} = \frac{4-7}{7-4} = \frac{-3}{3} = -1$.

The slopes of $\overline{AB}$ and $\overline{CD}$ are equal, so $\overline{AB} \parallel \overline{CD}$.

The slope of $\overline{BC} = \frac{7-5}{4-0} = \frac{2}{4} = \frac{1}{2}$.

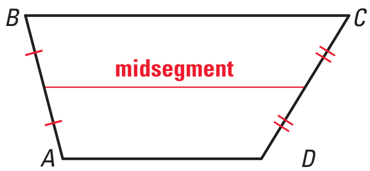
The slope of $\overline{AD} = \frac{4-0}{7-5} = \frac{4}{2} = 2$.

The slopes of $\overline{BC}$ and $\overline{AD}$ are not equal, so $\overline{BC}$ is not parallel to $\overline{AD}$.

▶ So, because $\overline{AB} \parallel \overline{CD}$ and $\overline{BC}$ is not parallel to $\overline{AD}$, $ABCD$ is a trapezoid.

. . . .

The **midsegment** of a trapezoid is the segment that connects the midpoints of its legs. Theorem 6.17 is similar to the Midsegment Theorem for triangles. You will justify part of this theorem in Exercise 42. A proof appears on page 839.

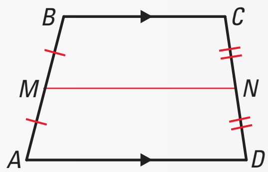


THEOREM

THEOREM 6.17 *Midsegment Theorem for Trapezoids*

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

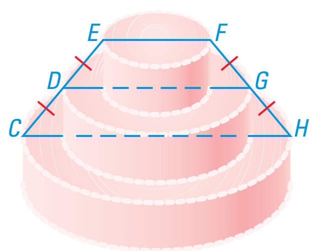
$$\overline{MN} \parallel \overline{AD},\ \overline{MN} \parallel \overline{BC},\ MN = \frac{1}{2}(AD + BC)$$



EXAMPLE 3 Finding Midsegment Lengths of Trapezoids

REAL LIFE

LAYER CAKE A baker is making a cake like the one at the right. The top layer has a diameter of 8 inches and the bottom layer has a diameter of 20 inches. How big should the middle layer be?



SOLUTION

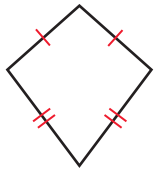
Use the Midsegment Theorem for Trapezoids.

$$DG = \frac{1}{2}(EF + CH) = \frac{1}{2}(8 + 20) = 14 \text{ inches}$$

The simplest of flying kites often use the geometric kite shape.

GOAL 2 USING PROPERTIES OF KITES

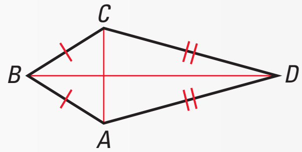
A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent. You are asked to prove Theorem 6.18 and Theorem 6.19 in Exercises 46 and 47.



THEOREMS ABOUT KITES

THEOREM 6.18

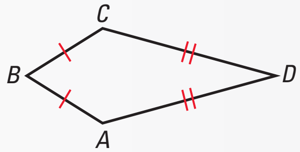
If a quadrilateral is a kite, then its diagonals are perpendicular.



$\overline{AC} \perp \overline{BD}$

THEOREM 6.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

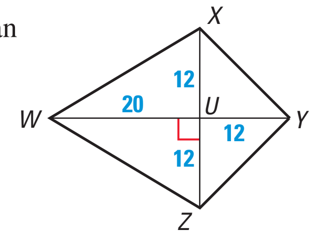


$\angle A \cong \angle C$, $\angle B \not\cong \angle D$

Using Algebra

EXAMPLE 4 *Using the Diagonals of a Kite*

WXYZ is a kite so the diagonals are perpendicular. You can use the Pythagorean Theorem to find the side lengths.



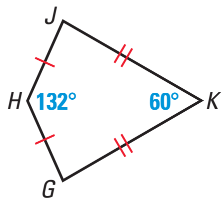
$$WX = \sqrt{20^2 + 12^2} \approx 23.32$$

$$XY = \sqrt{12^2 + 12^2} \approx 16.97$$

Because *WXYZ* is a kite, $WZ = WX \approx 23.32$ and $ZY = XY \approx 16.97$.

EXAMPLE 5 *Angles of a Kite*

Find $m\angle G$ and $m\angle J$ in the diagram at the right.



SOLUTION

GHJK is a kite, so $\angle G \cong \angle J$ and $m\angle G = m\angle J$.

$2(m\angle G) + 132° + 60° = 360°$ — Sum of measures of int. ∠s of a quad. is 360°.

$2(m\angle G) = 168°$ — Simplify.

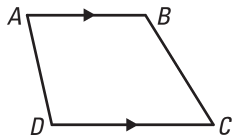
$m\angle G = 84°$ — Divide each side by 2.

▶ So, $m\angle J = m\angle G = 84°$.

GUIDED PRACTICE

Vocabulary Check ✓

1. Name the bases of trapezoid *ABCD*.



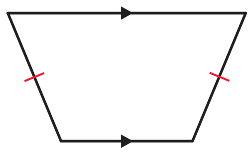
Concept Check ✓

2. Explain why a rhombus is not a kite. Use the definition of a kite.

Skill Check ✓

Decide whether the quadrilateral is a *trapezoid*, an *isosceles trapezoid*, a *kite*, or *none of these*.

3.



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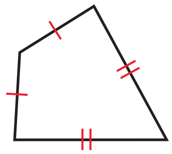

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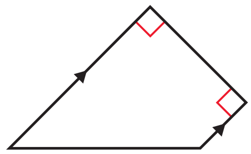
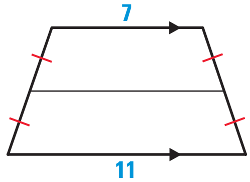


6. How can you prove that trapezoid *ABCD* in Example 2 is isosceles?

Find the length of the midsegment.

7.



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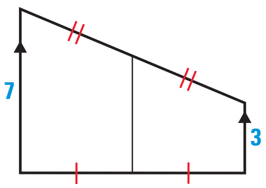

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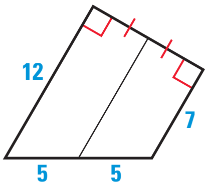


PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 814.

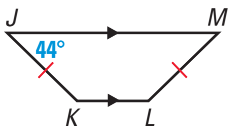
STUDYING A TRAPEZOID **Draw a trapezoid *PQRS* with $\overline{QR} \parallel \overline{PS}$. Identify the segments or angles of *PQRS* as *bases, consecutive sides, legs, diagonals, base angles,* or *opposite angles*.**

10. $\overline{QR}$ and $\overline{PS}$ **11.** $\overline{PQ}$ and $\overline{RS}$ **12.** $\overline{PQ}$ and $\overline{QR}$

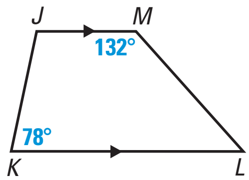
13. $\overline{QS}$ and $\overline{PR}$ **14.** $\angle Q$ and $\angle S$ **15.** $\angle S$ and $\angle P$

FINDING ANGLE MEASURES **Find the angle measures of *JKLM*.**

16.



17.



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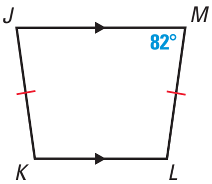


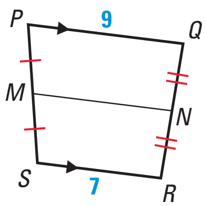
STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 16–18
Example 2: Exs. 34, 37, 38, 48–50
Example 3: Exs. 19–24, 35, 39
Example 4: Exs. 28–30
Example 5: Exs. 31–33

FINDING MIDSEGMENTS **Find the length of the midsegment $\overline{MN}$.**

19.



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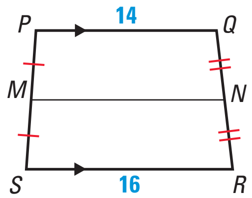

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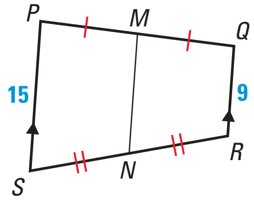
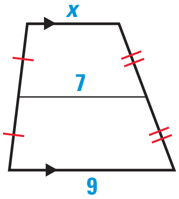


USING ALGEBRA **Find the value of x.**

22.



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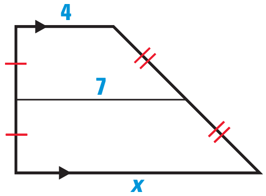

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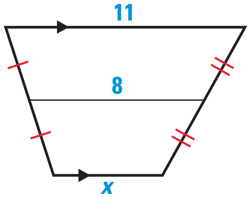
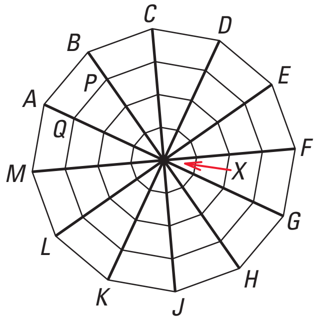


CONCENTRIC POLYGONS **In the diagram, *ABCDEFGHJKLM* is a regular dodecagon, $\overline{AB} \parallel \overline{PQ}$, and X is equidistant from the vertices of the dodecagon.**



25. Are you given enough information to prove that *ABPQ* is isosceles? Explain your reasoning.

26. What is the measure of $\angle AXB$?

27. What is the measure of each interior angle of *ABPQ*?

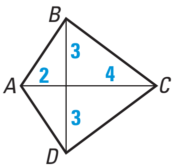
FOCUS ON APPLICATIONS

REAL LIFE

WEBS The spider web above is called an orb web. Although it looks like concentric polygons, the spider actually followed a spiral path to spin the web.

USING ALGEBRA **What are the lengths of the sides of the kite? Give your answer to the nearest hundredth.**

28.



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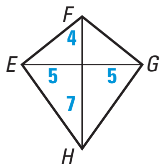

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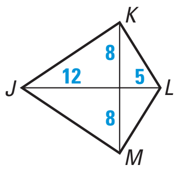
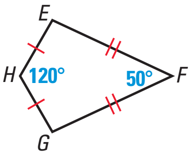


ANGLES OF KITES ***EFGH* is a kite. What is $m\angle G$?**

31.



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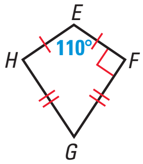
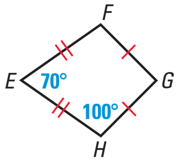
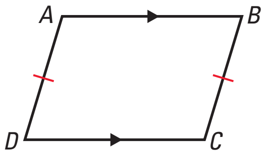


33.



34. ERROR ANALYSIS A student says that parallelogram *ABCD* is an isosceles trapezoid because $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \cong \overline{BC}$. Explain what is wrong with this reasoning.



35. CRITICAL THINKING The midsegment of a trapezoid is 5 inches long. What are possible lengths of the bases?

36. COORDINATE GEOMETRY Determine whether the points $A(4, 5)$, $B(-3, 3)$, $C(-6, -13)$, and $D(6, -2)$ are the vertices of a kite. Explain your answer.

TRAPEZOIDS **Determine whether the given points represent the vertices of a trapezoid. If so, is the trapezoid isosceles? Explain your reasoning.**

37. $A(-2, 0)$, $B(0, 4)$, $C(5, 4)$, $D(8, 0)$

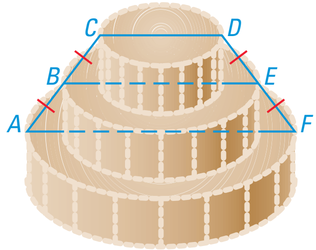
38. $E(1, 9)$, $F(4, 2)$, $G(5, 2)$, $H(8, 9)$

FOCUS ON CAREERS

REAL LIFE **CAKE DESIGNERS** design cakes for many occasions, including weddings, birthdays, anniversaries, and graduations.

INTERNET **CAREER LINK**
www.mcdougallittell.com

39. **LAYER CAKE** The top layer of the cake has a diameter of 10 inches. The bottom layer has a diameter of 22 inches. What is the diameter of the middle layer?

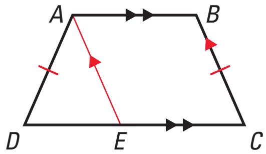


40. **PROVING THEOREM 6.14** Write a proof of Theorem 6.14.

GIVEN ▶ $ABCD$ is an isosceles trapezoid.

$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \cong \overline{BC}$

PROVE ▶ $\angle D \cong \angle C$, $\angle DAB \cong \angle B$



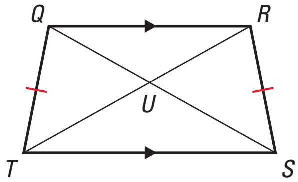
Plan for Proof To show $\angle D \cong \angle C$, first draw $\overline{AE}$ so $ABCE$ is a parallelogram. Then show $\overline{BC} \cong \overline{AE}$, so $\overline{AE} \cong \overline{AD}$ and $\angle D \cong \angle AED$. Finally, show $\angle D \cong \angle C$. To show $\angle DAB \cong \angle B$, use the consecutive interior angles theorem and substitution.

41. **PROVING THEOREM 6.16** Write a proof of one conditional statement of Theorem 6.16.

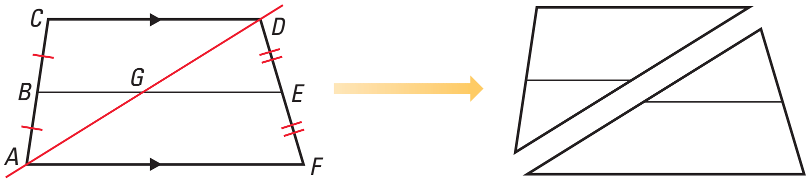
GIVEN ▶ $TQRS$ is an isosceles trapezoid.

$\overline{QR} \parallel \overline{TS}$ and $\overline{QT} \cong \overline{RS}$

PROVE ▶ $\overline{TR} \cong \overline{SQ}$



42. **JUSTIFYING THEOREM 6.17** In the diagram below, $\overline{BG}$ is the midsegment of $\triangle ACD$ and $\overline{GE}$ is the midsegment of $\triangle ADF$. Explain why the midsegment of trapezoid $ACDF$ is parallel to each base and why its length is one half the sum of the lengths of the bases.

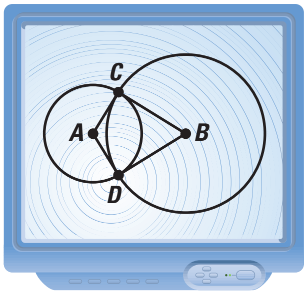


STUDENT HELP

INTERNET **SOFTWARE HELP**
Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

USING TECHNOLOGY **In Exercises 43–45, use geometry software.**

Draw points A, B, C and segments $\overline{AC}$ and $\overline{BC}$. Construct a circle with center A and radius AC. Construct a circle with center B and radius BC. Label the other intersection of the circles D. Draw $\overline{BD}$ and $\overline{AD}$.

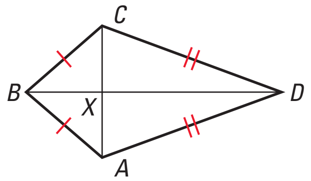


43. What kind of shape is $ACBD$? How do you know? What happens to the shape as you drag A? drag B? drag C?

44. Measure $\angle ACB$ and $\angle ADB$. What happens to the angle measures as you drag A, B, or C?

45. Which theorem does this construction illustrate?

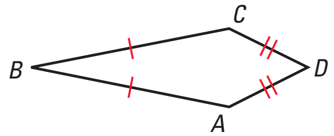
46. **PROVING THEOREM 6.18** Write a two-column proof of Theorem 6.18.



GIVEN ▶ $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

PROVE ▶ $\overline{AC} \perp \overline{BD}$

47. **PROVING THEOREM 6.19** Write a paragraph proof of Theorem 6.19.

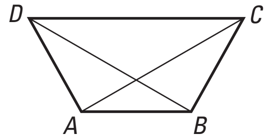


GIVEN ▶ $ABCD$ is a kite with $\overline{AB} \cong \overline{CB}$ and $\overline{AD} \cong \overline{CD}$.

PROVE ▶ $\angle A \cong \angle C$, $\angle B \not\cong \angle D$

Plan for Proof First show that $\angle A \cong \angle C$. Then use an indirect argument to show $\angle B \not\cong \angle D$: If $\angle B \cong \angle D$, then $ABCD$ is a parallelogram. But opposite sides of a parallelogram are congruent. This contradicts the definition of a kite.

TRAPEZOIDS **Decide whether you are given enough information to conclude that *ABCD* is an isosceles trapezoid. Explain your reasoning.**



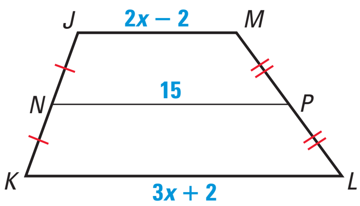
48. $\overline{AB} \parallel \overline{DC}$
$\overline{AD} \cong \overline{BC}$
$\overline{AD} \cong \overline{AB}$

49. $\overline{AB} \parallel \overline{DC}$
$\overline{AC} \cong \overline{BD}$
$\angle A \not\cong \angle C$

50. $\angle A \cong \angle B$
$\angle D \cong \angle C$
$\angle A \not\cong \angle C$

Test Preparation

51. **MULTIPLE CHOICE** In the trapezoid at the right, $NP = 15$. What is the value of x?



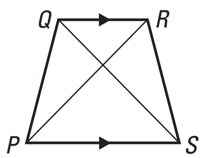
Ⓐ 2 Ⓑ 3 Ⓒ 4
Ⓓ 5 Ⓔ 6

52. **MULTIPLE CHOICE** Which one of the following can a trapezoid have?

Ⓐ congruent bases
Ⓑ diagonals that bisect each other
Ⓒ exactly two congruent sides
Ⓓ a pair of congruent opposite angles
Ⓔ exactly three congruent angles

★ **Challenge**

53. **PROOF** Prove one direction of Theorem 6.16: If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles.



GIVEN ▶ $PQRS$ is a trapezoid.
$\overline{QR} \parallel \overline{PS}$, $\overline{PR} \cong \overline{SQ}$

PROVE ▶ $\overline{QP} \cong \overline{RS}$

Plan for Proof Draw a perpendicular segment from Q to $\overline{PS}$ and label the intersection M. Draw a perpendicular segment from R to $\overline{PS}$ and label the intersection N. Prove that $\triangle QMS \cong \triangle RNP$. Then prove that $\triangle QPS \cong \triangle RSP$.

EXTRA CHALLENGE
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MIXED REVIEW

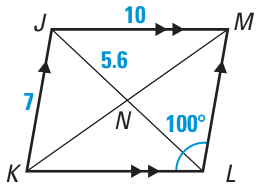
CONDITIONAL STATEMENTS **Rewrite the statement in if-then form.** (Review 2.1)

54. A scalene triangle has no congruent sides.

55. A kite has perpendicular diagonals.

56. A polygon is a pentagon if it has five sides.

FINDING MEASUREMENTS **Use the diagram to find the side length or angle measure.** (Review 6.2 for 6.6)



57. LN **58.** KL **59.** ML

60. JL **61.** $m\angle JML$ **62.** $m\angle MJK$

PARALLELOGRAMS **Determine whether the given points represent the vertices of a parallelogram. Explain your answer.** (Review 6.3 for 6.6)

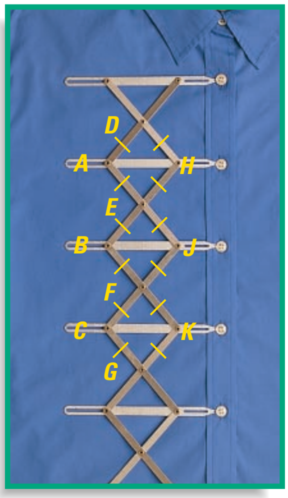
63. $A(-2, 8)$, $B(5, 8)$, $C(2, 0)$, $D(-5, 0)$

64. $P(4, -3)$, $Q(9, -1)$, $R(8, -6)$, $S(3, -8)$

QUIZ 2

Self-Test for Lessons 6.4 and 6.5

1. **POSITIONING BUTTONS** The tool at the right is used to decide where to put buttons on a shirt. The tool is stretched to fit the length of the shirt, and the pointers show where to put the buttons. Why are the pointers always evenly spaced? (*Hint:* You can prove that $\overline{HJ} \cong \overline{JK}$ if you know that $\triangle JFK \cong \triangle HEJ$.) (Lesson 6.4)



Determine whether the given points represent the vertices of a *rectangle*, a *rhombus*, a *square*, a *trapezoid*, or a *kite*. (Lessons 6.4, 6.5)

2. $P(2, 5)$, $Q(-4, 5)$, $R(2, -7)$, $S(-4, -7)$

3. $A(-3, 6)$, $B(0, 9)$, $C(3, 6)$, $D(0, -10)$

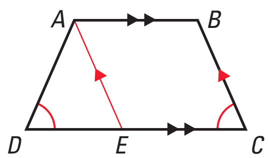
4. $J(-5, 6)$, $K(-4, -2)$, $L(4, -1)$, $M(3, 7)$

5. $P(-5, -3)$, $Q(1, -2)$, $R(6, 3)$, $S(7, 9)$

6. **PROVING THEOREM 6.15** Write a proof of Theorem 6.15.

GIVEN ▶ $ABCD$ is a trapezoid with $\overline{AB} \parallel \overline{DC}$.
$\angle D \cong \angle C$

PROVE ▶ $\overline{AD} \cong \overline{BC}$



Plan for Proof Draw $\overline{AE}$ so $ABCE$ is a parallelogram. Use the Transitive Property of Congruence to show $\angle AED \cong \angle D$. Then $\overline{AD} \cong \overline{AE}$, so $\overline{AD} \cong \overline{BC}$. (Lesson 6.5)