6.2

Properties of Parallelograms

What *you should learn*

GOAL 1 Use some properties of parallelograms.

GOAL 2 Use properties of parallelograms in **real-life** situations, such as the drafting table shown in **Example 6**.

Why *you should learn it*

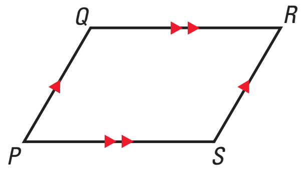
▼ You can use properties of parallelograms to understand how a scissors lift works in **Exs. 51–54**.

REAL LIFE

GOAL 1 PROPERTIES OF PARALLELOGRAMS

In this lesson and in the rest of the chapter you will study special quadrilaterals. A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

When you mark diagrams of quadrilaterals, use matching arrowheads to indicate which sides are parallel. For example, in the diagram at the right, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{SP}$. The symbol $\square PQRS$ is read "parallelogram $PQRS$."

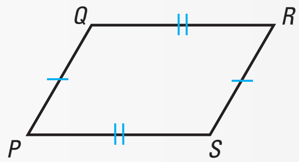


THEOREMS ABOUT PARALLELOGRAMS

THEOREM 6.2

If a quadrilateral is a parallelogram, then its **opposite sides** are congruent.

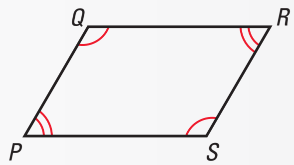
$\overline{PQ} \cong \overline{RS}$ and $\overline{SP} \cong \overline{QR}$



THEOREM 6.3

If a quadrilateral is a parallelogram, then its **opposite angles** are congruent.

$\angle P \cong \angle R$ and $\angle Q \cong \angle S$

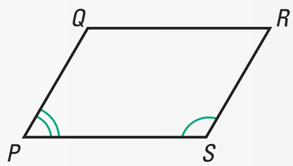


THEOREM 6.4

If a quadrilateral is a parallelogram, then its **consecutive angles** are supplementary.

$m\angle P + m\angle Q = 180°$, $m\angle Q + m\angle R = 180°$,

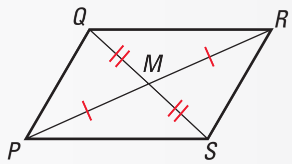
$m\angle R + m\angle S = 180°$, $m\angle S + m\angle P = 180°$



THEOREM 6.5

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$



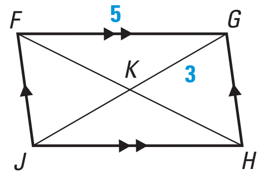
Theorem 6.2 is proved in Example 5. You are asked to prove Theorem 6.3, Theorem 6.4, and Theorem 6.5 in Exercises 38–44.

EXAMPLE 1 Using Properties of Parallelograms

FGHJ is a parallelogram. Find the unknown length. Explain your reasoning.

a. *JH*

b. *JK*



SOLUTION

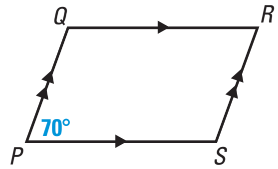
a. $JH = FG$	Opposite sides of a ▱ are ≅.
$JH = 5$	Substitute 5 for *FG*.
b. $JK = GK$	Diagonals of a ▱ bisect each other.
$JK = 3$	Substitute 3 for *GK*.

EXAMPLE 2 Using Properties of Parallelograms

PQRS is a parallelogram. Find the angle measure.

a. $m\angle R$

b. $m\angle Q$

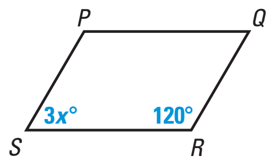


SOLUTION

a. $m\angle R = m\angle P$	Opposite angles of a ▱ are ≅.
$m\angle R = 70°$	Substitute 70° for $m\angle P$.
b. $m\angle Q + m\angle P = 180°$	Consecutive $\angle$s of a ▱ are supplementary.
$m\angle Q + 70° = 180°$	Substitute 70° for $m\angle P$.
$m\angle Q = 110°$	Subtract 70° from each side.

EXAMPLE 3 Using Algebra with Parallelograms

PQRS is a parallelogram. Find the value of *x*.



SOLUTION

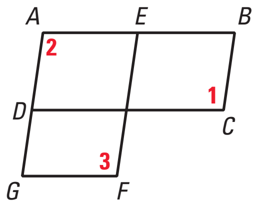
$m\angle S + m\angle R = 180°$	Consecutive angles of a ▱ are supplementary.
$3x + 120 = 180$	Substitute $3x$ for $m\angle S$ and 120 for $m\angle R$.
$3x = 60$	Subtract 120 from each side.
$x = 20$	Divide each side by 3.

GOAL 2 REASONING ABOUT PARALLELOGRAMS

EXAMPLE 4 *Proving Facts about Parallelograms*

GIVEN ▶ *ABCD* and *AEFG* are parallelograms.

PROVE ▶ $\angle 1 \cong \angle 3$



Plan Show that both angles are congruent to $\angle 2$. Then use the Transitive Property of Congruence.

SOLUTION

Method 1 Write a two-column proof.

Statements	Reasons
1. *ABCD* is a ▱. *AEFG* is a ▱.	**1.** Given
2. $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$	**2.** Opposite angles of a ▱ are ≅.
3. $\angle 1 \cong \angle 3$	**3.** Transitive Property of Congruence

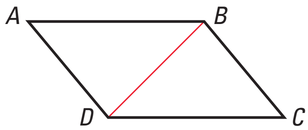
Method 2 Write a paragraph proof.

ABCD is a parallelogram, so $\angle 1 \cong \angle 2$ because opposite angles of a parallelogram are congruent. *AEFG* is a parallelogram, so $\angle 2 \cong \angle 3$. By the Transitive Property of Congruence, $\angle 1 \cong \angle 3$.

EXAMPLE 5 *Proving Theorem 6.2*

GIVEN ▶ *ABCD* is a parallelogram.

PROVE ▶ $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$



SOLUTION

Statements	Reasons
1. *ABCD* is a ▱.	**1.** Given
2. Draw $\overline{BD}$.	**2.** Through any two points there exists exactly one line.
3. $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{CB}$	**3.** Definition of parallelogram
4. $\angle ABD \cong \angle CDB$, $\angle ADB \cong \angle CBD$	**4.** Alternate Interior Angles Theorem
5. $\overline{DB} \cong \overline{DB}$	**5.** Reflexive Property of Congruence
6. $\triangle ADB \cong \triangle CBD$	**6.** ASA Congruence Postulate
7. $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$	**7.** Corresponding parts of ≅ ⚠ are ≅.

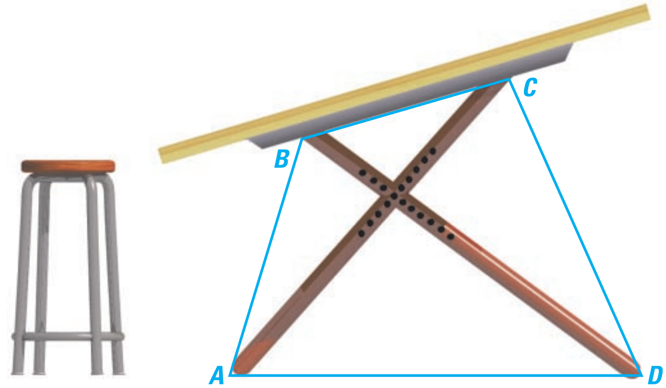
FOCUS ON CAREERS

FURNITURE DESIGN
Furniture designers use geometry, trigonometry, and other skills to create designs for furniture.

CAREER LINK
www.mcdougallittell.com

EXAMPLE 6 *Using Parallelograms in Real Life*

FURNITURE DESIGN A drafting table is made so that the legs can be joined in different ways to change the slope of the drawing surface. In the arrangement below, the legs $\overline{AC}$ and $\overline{BD}$ do *not* bisect each other. Is $ABCD$ a parallelogram?



SOLUTION

No. If $ABCD$ were a parallelogram, then by Theorem 6.5 $\overline{AC}$ would bisect $\overline{BD}$ and $\overline{BD}$ would bisect $\overline{AC}$.

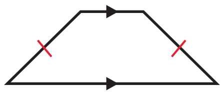
GUIDED PRACTICE

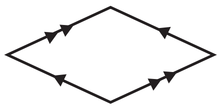
Vocabulary Check ✓

1. Write a definition of *parallelogram*.

Concept Check ✓

Decide whether the figure is a parallelogram. If it is not, explain why not.

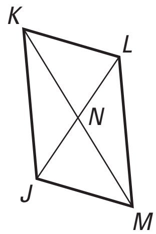
2. 

3. 

Skill Check ✓

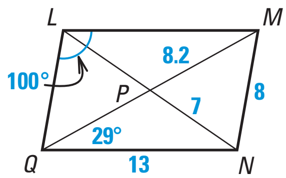
IDENTIFYING CONGRUENT PARTS Use the diagram of parallelogram *JKLM* at the right. Complete the statement, and give a reason for your answer.

4. $\overline{JK} \cong$ __?__
5. $\overline{MN} \cong$ __?__
6. $\angle MLK \cong$ __?__
7. $\angle JKL \cong$ __?__
8. $\overline{JN} \cong$ __?__
9. $\overline{KL} \cong$ __?__
10. $\angle MNL \cong$ __?__
11. $\angle MKL \cong$ __?__



Find the measure in parallelogram *LMNQ*. Explain your reasoning.

12. LM
13. LP
14. LQ
15. QP
16. $m\angle LMN$
17. $m\angle NQL$
18. $m\angle MNQ$
19. $m\angle LMQ$

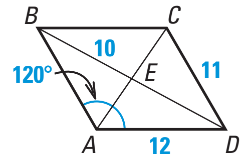


PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master skills is on p. 813.

FINDING MEASURES **Find the measure in parallelogram *ABCD*. Explain your reasoning.**



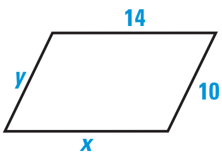
20. DE **21.** BA

22. BC **23.** $m\angle CDA$

24. $m\angle ABC$ **25.** $m\angle BCD$

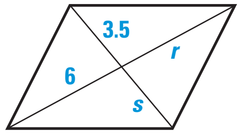
USING ALGEBRA **Find the value of each variable in the parallelogram.**

26.



27.

28.



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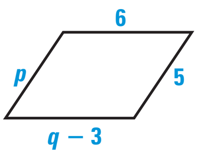

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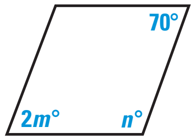

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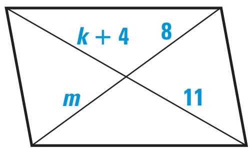
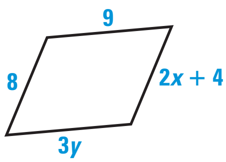


USING ALGEBRA **Find the value of each variable in the parallelogram.**

32.



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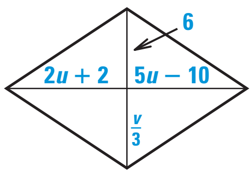

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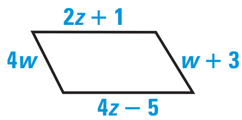

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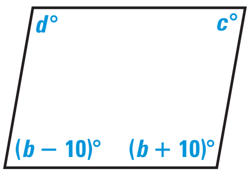

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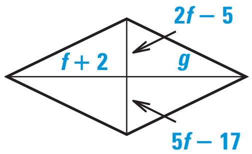
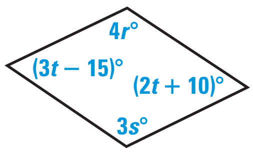


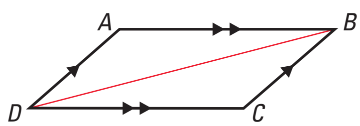
37.



38. **PROVING THEOREM 6.3** Copy and complete the proof of Theorem 6.3: If a quadrilateral is a parallelogram, then its opposite angles are congruent.

GIVEN ▶ $ABCD$ is a ▱.

PROVE ▶ $\angle A \cong \angle C$,
$\angle B \cong \angle D$



Paragraph Proof Opposite sides of a parallelogram are congruent, so ___**a.**___ and ___**b.**___. By the Reflexive Property of Congruence, ___**c.**___. $\triangle ABD \cong \triangle CDB$ because of the ___**d.**___ Congruence Postulate. Because ___**e.**___ parts of congruent triangles are congruent, $\angle A \cong \angle C$.

To prove that $\angle B \cong \angle D$, draw ___**f.**___ and use the same reasoning.

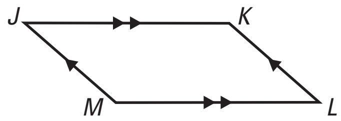
STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 20–22
Example 2: Exs. 23–25
Example 3: Exs. 26–37
Example 4: Exs. 55–58
Example 5: Exs. 38–44
Example 6: Exs. 45–54

39. **PROVING THEOREM 6.4** Copy and complete the two-column proof of Theorem 6.4: If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

GIVEN ▶ *JKLM* is a ▱.

PROVE ▶ ∠*J* and ∠*K* are supplementary.



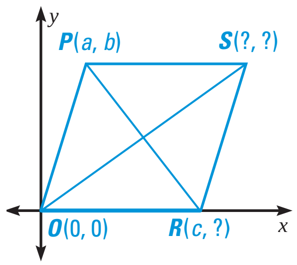
Statements	Reasons
1. ___?___	**1.** Given
2. $m\angle J = m\angle L$, $m\angle K = m\angle M$	**2.** ___?___
3. $m\angle J + m\angle L + m\angle K + m\angle M =$ ___?___	**3.** Sum of measures of int. ∠s of a quad. is 360°.
4. $m\angle J + m\angle J + m\angle K + m\angle K = 360°$	**4.** ___?___
5. 2(___?___ + ___?___) = 360°	**5.** Distributive property
6. $m\angle J + m\angle K = 180°$	**6.** ___?___ prop. of equality
7. ∠*J* and ∠*K* are supplementary.	**7.** ___?___

You can use the same reasoning to prove any other pair of consecutive angles in ▱*JKLM* are supplementary.

DEVELOPING COORDINATE PROOF **Copy and complete the coordinate proof of Theorem 6.5.**

GIVEN ▶ *PORS* is a ▱.

PROVE ▶ $\overline{PR}$ and $\overline{OS}$ bisect each other.



Plan for Proof Find the coordinates of the midpoints of the diagonals of ▱*PORS* and show that they are the same.

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with the coordinate proof in Exs. 40–44.

40. Point *R* is on the *x*-axis, and the length of $\overline{OR}$ is *c* units. What are the coordinates of point *R*?

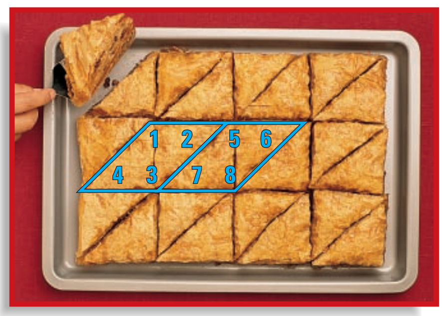
41. The length of $\overline{PS}$ is also *c* units, and $\overline{PS}$ is horizontal. What are the coordinates of point *S*?

42. What are the coordinates of the midpoint of $\overline{PR}$?

43. What are the coordinates of the midpoint of $\overline{OS}$?

44. *Writing* How do you know that $\overline{PR}$ and $\overline{OS}$ bisect each other?

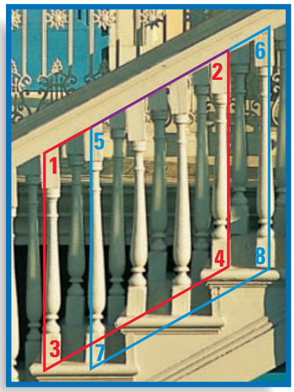
BAKING **In Exercises 45 and 46, use the following information.**
In a recipe for baklava, the pastry should be cut into triangles that form congruent parallelograms, as shown. Write a paragraph proof to prove the statement.



45. ∠3 is supplementary to ∠6.

46. ∠4 is supplementary to ∠5.

STAIR BALUSTERS **In Exercises 47–50, use the following information.** In the diagram at the right, the slope of the handrail is equal to the slope of the stairs. The balusters (vertical posts) support the handrail.



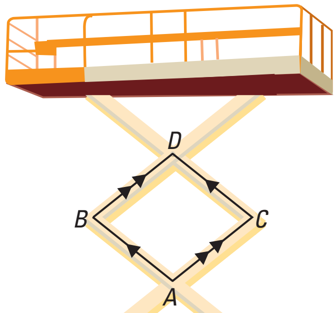
47. Which angle in the red parallelogram is congruent to $\angle 1$?

48. Which angles in the blue parallelogram are supplementary to $\angle 6$?

49. Which postulate can be used to prove that $\angle 1 \cong \angle 5$?

50. *Writing* Is the red parallelogram congruent to the blue parallelogram? Explain your reasoning.

SCISSORS LIFT **Photographers can use scissors lifts for overhead shots, as shown at the left. The crossing beams of the lift form parallelograms that move together to raise and lower the platform. In Exercises 51–54, use the diagram of parallelogram *ABDC* at the right.**



51. What is $m\angle B$ when $m\angle A = 120°$?

52. Suppose you decrease $m\angle A$. What happens to $m\angle B$?

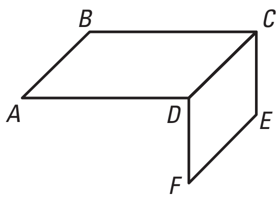
53. Suppose you decrease $m\angle A$. What happens to AD?

54. Suppose you decrease $m\angle A$. What happens to the overall height of the scissors lift?

TWO-COLUMN PROOF **Write a two-column proof.**

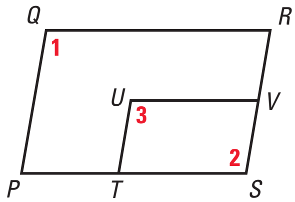
55. GIVEN ▶ $ABCD$ and $CEFD$ are ▱s.

PROVE ▶ $\overline{AB} \cong \overline{FE}$



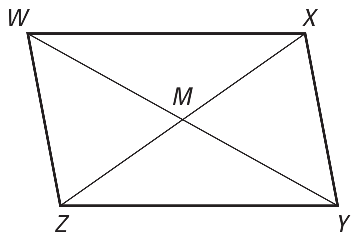
56. GIVEN ▶ $PQRS$ and $TUVS$ are ▱s.

PROVE ▶ $\angle 1 \cong \angle 3$



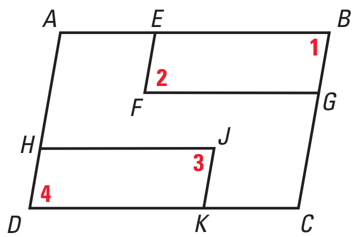
57. GIVEN ▶ $WXYZ$ is a ▱.

PROVE ▶ $\triangle WMZ \cong \triangle YMX$

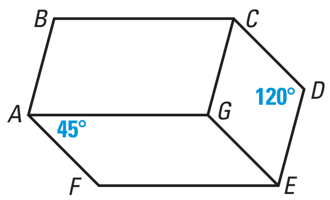


58. GIVEN ▶ $ABCD$, $EBGF$, $HJKD$ are ▱s.

PROVE ▶ $\angle 2 \cong \angle 3$



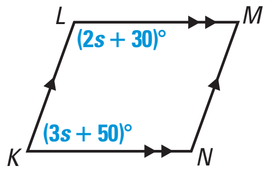
59. *Writing* In the diagram, *ABCG, CDEG,* and *AGEF* are parallelograms. Copy the diagram and add as many other angle measures as you can. Then describe how you know the angle measures you added are correct.



Test Preparation

60. **MULTIPLE CHOICE** In ▱*KLMN*, what is the value of *s*?

Ⓐ 5 Ⓑ 20 Ⓒ 40 Ⓓ 52 Ⓔ 70

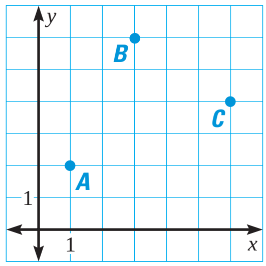


61. **MULTIPLE CHOICE** In ▱*ABCD*, point *E* is the intersection of the diagonals. Which of the following is *not* necessarily true?

Ⓐ $AB = CD$ Ⓑ $AC = BD$ Ⓒ $AE = CE$ Ⓓ $AD = BC$ Ⓔ $DE = BE$

★ Challenge

USING ALGEBRA Suppose points $A(1, 2)$, $B(3, 6)$, and $C(6, 4)$ are three vertices of a parallelogram.



62. Give the coordinates of a point that could be the fourth vertex. Sketch the parallelogram in a coordinate plane.

63. Explain how to check to make sure the figure you drew in Exercise 62 is a parallelogram.

64. How many different parallelograms can be formed using *A*, *B*, and *C* as vertices? Sketch each parallelogram and label the coordinates of the fourth vertex.

EXTRA CHALLENGE
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MIXED REVIEW

USING ALGEBRA Use the Distance Formula to find *AB*. (Review 1.3 for 6.3)

65. $A(2, 1), B(6, 9)$ 66. $A(-4, 2), B(2, -1)$ 67. $A(-8, -4), B(-1, -3)$

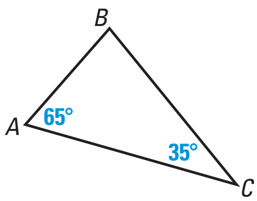
USING ALGEBRA Find the slope of $\overline{AB}$. (Review 3.6 for 6.3)

68. $A(2, 1), B(6, 9)$ 69. $A(-4, 2), B(2, -1)$ 70. $A(-8, -4), B(-1, -3)$

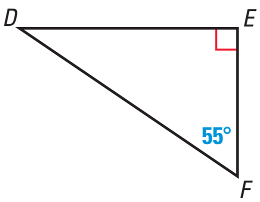
71. **PARKING CARS** In a parking lot, two guidelines are painted so that they are both perpendicular to the line along the curb. Are the guidelines parallel? Explain why or why not. (Review 3.5)

Name the shortest and longest sides of the triangle. Explain. (Review 5.5)

72.



73.



74.