6.1

Polygons

GOAL 1 DESCRIBING POLYGONS

A **polygon** is a plane figure that meets the following conditions.

1. It is formed by three or more segments called **sides,** such that no two sides with a common endpoint are collinear.

2. Each side intersects exactly two other sides, one at each endpoint.
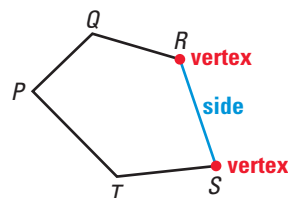
Each endpoint of a side is a **vertex** of the polygon. The plural of *vertex* is *vertices.* You can name a polygon by listing its vertices *consecutively.* For instance, *PQRST* and *QPTSR* are two correct names for the polygon above.

EXAMPLE 1 Identifying Polygons

State whether the figure is a polygon. If it is not, explain why.
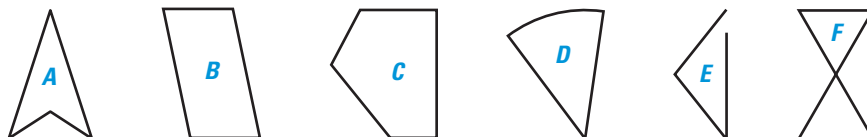
SOLUTION

Figures *A*, *B*, and *C are* polygons.

- Figure *D is not* a polygon because it has a side that is not a segment.

- Figure *E is not* a polygon because two of the sides intersect only one other side.

- Figure *F is not* a polygon because some of its sides intersect more than two other sides.

. . . .

Polygons are named by the number of sides they have.

Number of sides	Type of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon

Number of sides	Type of polygon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	n-gon

A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is called **nonconvex** or **concave**.
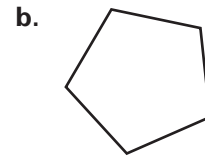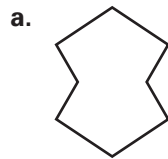
convex polygon   **concave polygon**
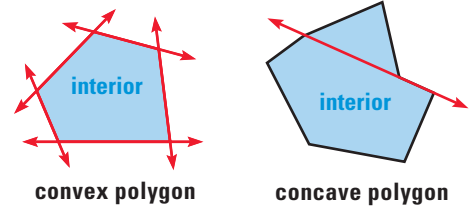
EXAMPLE 2 *Identifying Convex and Concave Polygons*

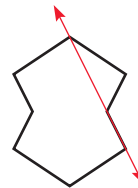Identify the polygon and state whether it is convex or concave.

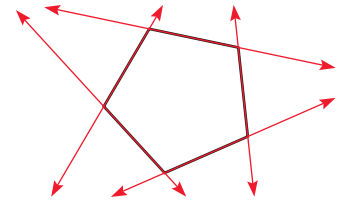**a.**   **b.**

SOLUTION

a. The polygon has 8 sides, so it is an octagon. When extended, some of the sides intersect the interior, so the polygon is concave.

b. The polygon has 5 sides, so it is a pentagon. When extended, none of the sides intersect the interior, so the polygon is convex.
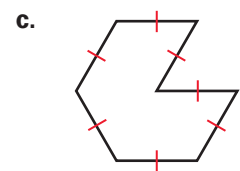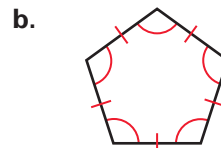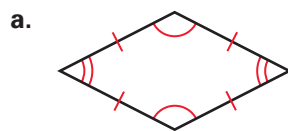
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A polygon is **equilateral** if all of its sides are congruent. A polygon is **equiangular** if all of its interior angles are congruent. A polygon is **regular** if it is equilateral and equiangular.

EXAMPLE 3 *Identifying Regular Polygons*

Decide whether the polygon is regular.

a.   **b.**   **c.**

SOLUTION

a. The polygon is an equilateral quadrilateral, but not equiangular. So, it is not a regular polygon.

b. This pentagon is equilateral and equiangular. So, it is a regular polygon.

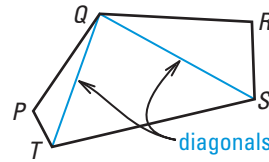**c.** This heptagon is equilateral, but not equiangular. So, it is not regular.

This tile pattern in Iran contains both convex and concave polygons.

INTERIOR ANGLES OF QUADRILATERALS

A **diagonal** of a polygon is a segment that joins two *nonconsecutive* vertices. Polygon *PQRST* has 2 diagonals from point *Q*, $\overline{QT}$ and $\overline{QS}$.

Like triangles, quadrilaterals have both *interior* and *exterior* angles. If you draw a diagonal in a quadrilateral, you divide it into two triangles, each of which has interior angles with measures that add up to 180°. So you can conclude that the sum of the measures of the interior angles of a quadrilateral is 2(180°), or 360°.
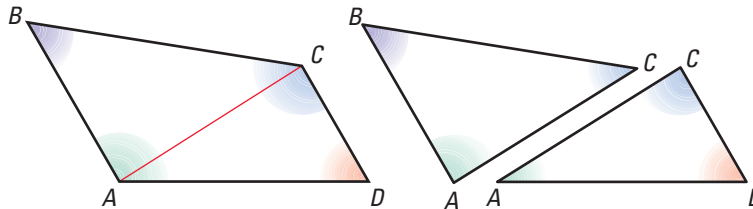
THEOREM

THEOREM 6.1 *Interior Angles of a Quadrilateral*
The sum of the measures of the interior angles of a quadrilateral is 360°.

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$$
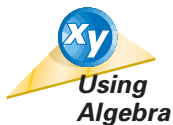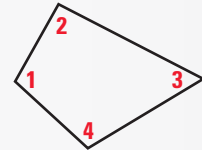
EXAMPLE 4 *Interior Angles of a Quadrilateral*

Using Algebra

Find $m\angle Q$ and $m\angle R$.

SOLUTION

Find the value of *x*. Use the sum of the measures of the interior angles to write an equation involving *x*. Then, solve the equation.

$$x° + 2x° + 70° + 80° = 360°$$ 　　**Sum of measures of int. ∡ of a quad. is 360°.**

$$3x + 150 = 360$$ 　　**Combine like terms.**

$$3x = 210$$ 　　**Subtract 150 from each side.**

$$x = 70$$ 　　**Divide each side by 3.**

Find $m\angle Q$ and $m\angle R$.

$$m\angle Q = x° = 70°$$

$$m\angle R = 2x° = 140°$$

▶ So, $m\angle Q = 70°$ and $m\angle R = 140°$.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is the plural of *vertex*?

2. What do you call a polygon with 8 sides? a polygon with 15 sides?

Concept Check ✓

3. Suppose you could tie a string tightly around a convex polygon. Would the length of the string be equal to the perimeter of the polygon? What if the polygon were concave? Explain.
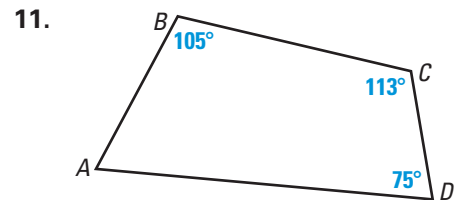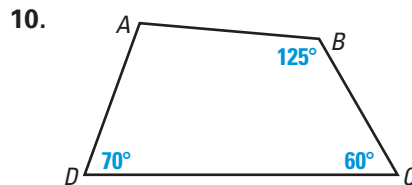
Skill Check ✓

Decide whether the figure is a polygon. If it is not, explain why.

4.

5.

6.
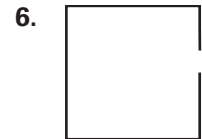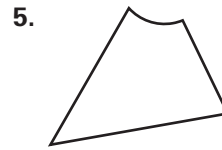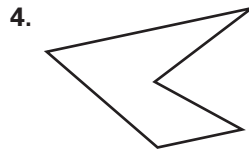
Tell whether the polygon is best described as *equiangular*, *equilateral*, *regular*, or *none of these*.

7.

8.

9.

Use the information in the diagram to find *m∠A*.

10.

11.
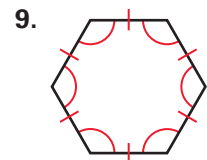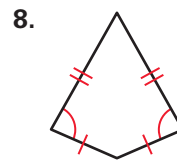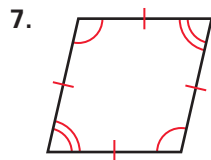
PRACTICE AND APPLICATIONS

RECOGNIZING POLYGONS **Decide whether the figure is a polygon.**
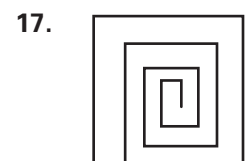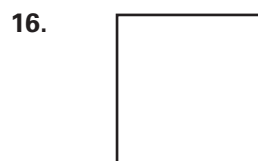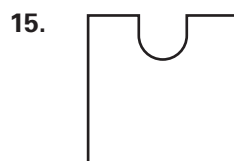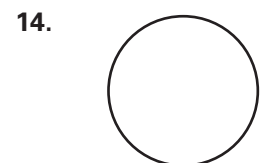
12.

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17.

CONVEX OR CONCAVE Use the number of sides to tell what kind of
polygon the shape is. Then state whether the polygon is *convex* or *concave*.

18.

19.

20.
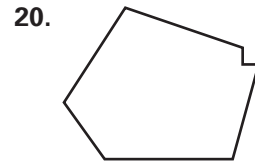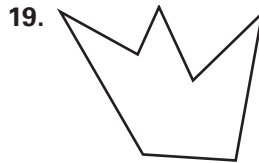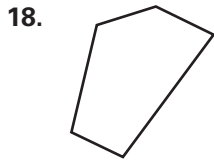
🌐 **PARACHUTES** Some gym classes use parachutes
that look like the polygon at the right.

21. Is the polygon a *heptagon*, *octagon*, or *nonagon*?

22. Polygon *LMNPQRST* is one name for the
polygon. State two other names.

23. Name all of the diagonals that have vertex *M* as an
endpoint. Not all of the diagonals are shown.
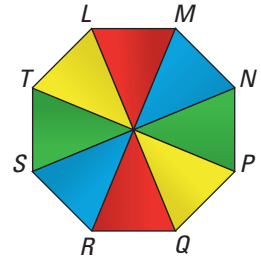
RECOGNIZING PROPERTIES State whether the polygon is best described as
equilateral, *equiangular*, *regular*, or *none of these*.

24.

25.

26.
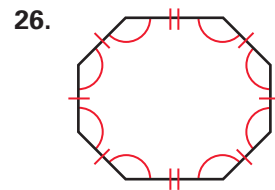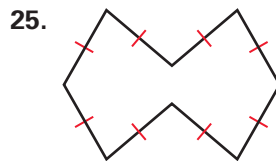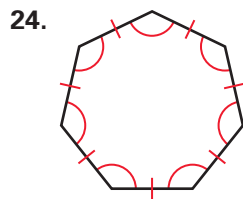
🌐 **TRAFFIC SIGNS** Use the number of sides of the traffic sign to tell what kind
of polygon it is. Is it *equilateral*, *equiangular*, *regular*, or *none of these*?

27.

28.

29.

30.

**DRAWING** Draw a figure that fits the description.

31. A convex heptagon

32. A concave nonagon

33. An equilateral hexagon that is not equiangular

34. An equiangular polygon that is not equilateral

35. 🧩 **LOGICAL REASONING** Is every triangle convex? Explain your reasoning.

36. 🧩 **LOGICAL REASONING** Quadrilateral *ABCD* is regular. What is the
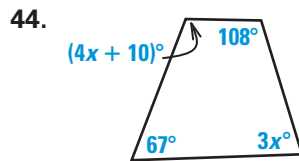measure of ∠*ABC*? How do you know?

ANGLE MEASURES Use the information in the diagram to find $m\angle A$.
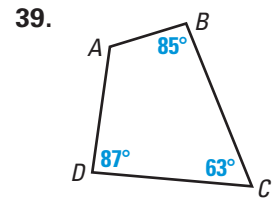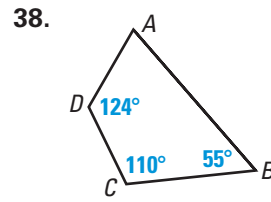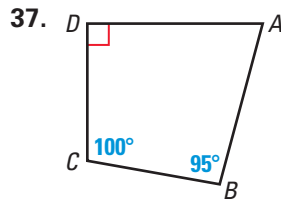
37.

38.

39.

40. **TECHNOLOGY** Use geometry software to draw a quadrilateral. Measure each interior angle and calculate the sum. What happens to the sum as you drag the vertices of the quadrilateral?

**USING ALGEBRA** Use the information in the diagram to solve for *x*.

41.

42.

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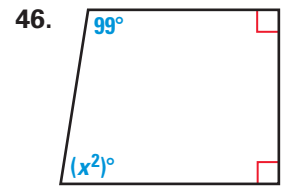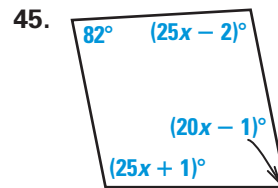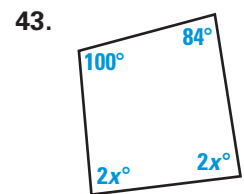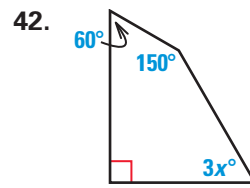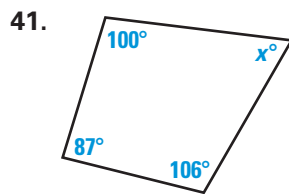
47. **LANGUAGE** **CONNECTION** A *deca*gon has ten sides and a *deca*de has ten years. The prefix *deca-* comes from Greek. It means *ten*. What does the prefix *tri-* mean? List four words that use *tri-* and explain what they mean.

PLANT SHAPES In Exercises 48–51, use the following information.
Cross sections of seeds and fruits often resemble polygons. Next to each cross section is the polygon it resembles. Describe each polygon. Tell what kind of polygon it is, whether it is *concave* or *convex*, and whether it appears to be *equilateral*, *equiangular*, *regular*, or *none of these*. ▶ Source: The History and Folklore of N. American Wildflowers
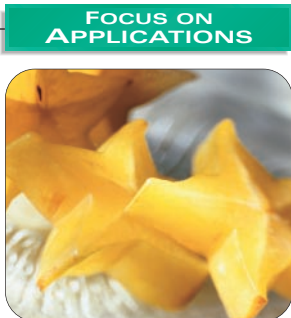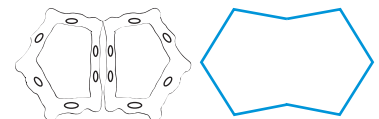
48. Virginia Snakeroot

49. Caraway

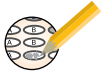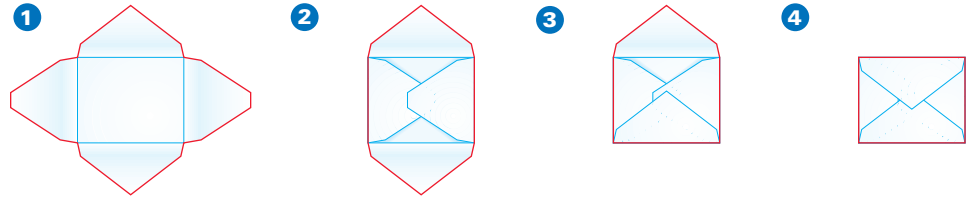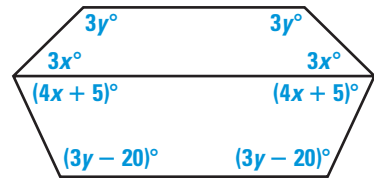**50.** Fennel

51. Poison Hemlock

52. MULTI-STEP PROBLEM Envelope manufacturers fold a specially-shaped piece of paper to make an envelope, as shown below.

a. What type of polygon is formed by the outside edges of the paper before it is folded? Is the polygon convex?

b. Tell what type of polygon is formed at each step. Which of the polygons are convex?

c. *Writing* Explain the reason for the V-shaped notches that are at the ends of the folds.

★ **Challenge**

53. FINDING VARIABLES Find the values of x and y in the diagram at the right. Check your answer. Then copy the shape and write the measure of each angle on your diagram.
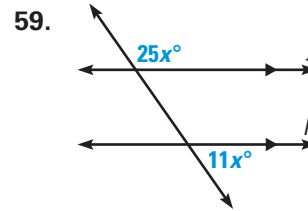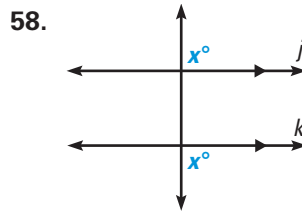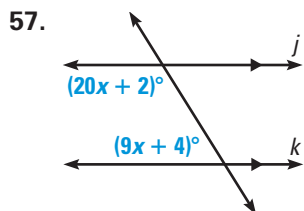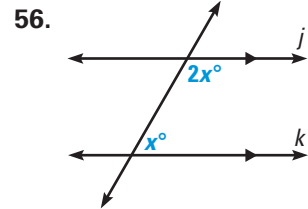
MIXED REVIEW

PARALLEL LINES In the diagram, $j \parallel k$. Find the value of x. **(Review 3.3 for 6.2)**

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55.

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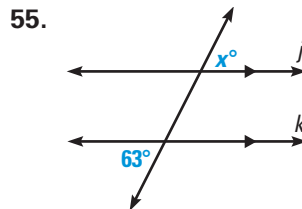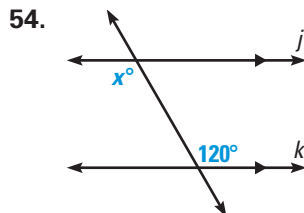
COORDINATE GEOMETRY You are given the midpoints of the sides of a triangle. Find the coordinates of the vertices of the triangle. **(Review 5.4)**

60. $L(-3, 7)$, $M(-5, 1)$, $N(-8, 8)$

61. $L(-4, -1)$, $M(3, 6)$, $N(-2, -8)$

62. $L(2, 4)$, $M(-1, 2)$, $N(0, 7)$

63. $L(-1, 3)$, $M(6, 7)$, $N(3, -5)$

64. **USING ALGEBRA** Use the Distance Formula to find the lengths of the diagonals of a polygon with vertices $A(0, 3)$, $B(3, 3)$, $C(4, 1)$, $D(0, -1)$, and $E(-2, 1)$. **(Review 1.3)**