CHAPTER 5

Chapter Summary

WHAT did you learn? / WHY did you learn it?

WHAT did you learn?	WHY did you learn it?
Use properties of perpendicular bisectors and angle bisectors. (5.1)	Decide where a hockey goalie should be positioned to defend the goal. (p. 270)
Use properties of perpendicular bisectors and angle bisectors of a triangle. (5.2)	Find the center of a mushroom ring. (p. 277)
Use properties of medians and altitudes of a triangle. (5.3)	Find points in a triangle used to measure a person's heart fitness. (p. 283)
Use properties of midsegments of a triangle. (5.4)	Determine the length of the crossbar of a swing set. (p. 292)
Compare the lengths of the sides or the measures of the angles of a triangle. (5.5)	Determine how the lengths of the boom lines of a crane affect the position of the boom. (p. 300)
Understand and write indirect proofs. (5.6)	Prove theorems that cannot be easily proved directly.
Use the Hinge Theorem and its converse to compare side lengths and angle measures of triangles. (5.6)	Decide which of two airplanes is farther from an airport. (p. 304)

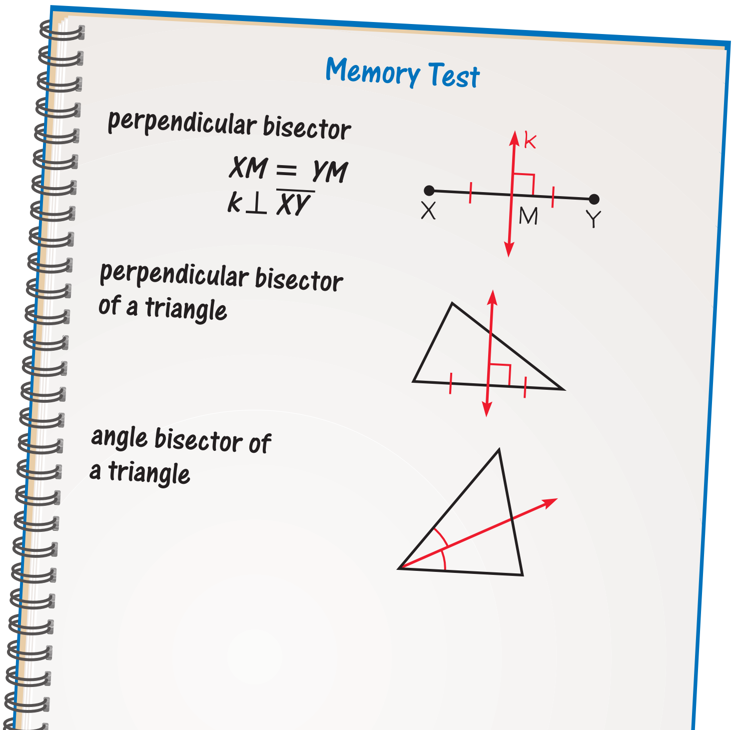
How does Chapter 5 fit into the BIGGER PICTURE of geometry?

In this chapter, you studied properties of special segments of triangles, which are an important building block for more complex figures that you will explore in later chapters. The special segments of a triangle have applications in many areas such as demographics (p. 280), medicine (p. 283), and room design (p. 299).

STUDY STRATEGY

Did you test your memory?

The list of important vocabulary terms and skills you made, following the **Study Strategy** on page 262, may resemble this one.



CHAPTER 5

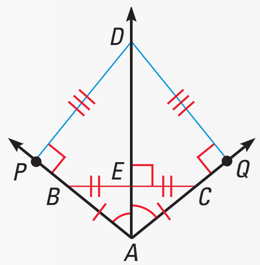
Chapter Review

- perpendicular bisector, p. 264
- equidistant from two points, p. 264
- distance from a point to a line, p. 266
- equidistant from two lines, p. 266
- perpendicular bisector of a triangle, p. 272
- concurrent lines, p. 272
- point of concurrency, p. 272
- circumcenter of a triangle, p. 273
- angle bisector of a triangle, p. 274
- incenter of a triangle, p. 274
- median of a triangle, p. 279
- centroid of a triangle, p. 279
- altitude of a triangle, p. 281
- orthocenter of a triangle, p. 281
- midsegment of a triangle, p. 287
- indirect proof, p. 302

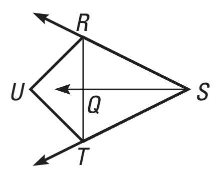
5.1 PERPENDICULARS AND BISECTORS

Examples on pp. 264–267

EXAMPLES In the figure, $\overrightarrow{AD}$ is the angle bisector of $\angle BAC$ and the perpendicular bisector of $\overline{BC}$. You know that $BE = CE$ by the definition of perpendicular bisector and that $AB = AC$ by the Perpendicular Bisector Theorem. Because $\overline{DP} \perp \overrightarrow{AP}$ and $\overline{DQ} \perp \overrightarrow{AQ}$, then DP and DQ are the distances from D to the sides of $\angle PAQ$ and you know that $DP = DQ$ by the Angle Bisector Theorem.



In Exercises 1–3, use the diagram.



1. If $\overrightarrow{SQ}$ is the perpendicular bisector of $\overline{RT}$, explain how you know that $\overline{RQ} \cong \overline{TQ}$ and $\overline{RS} \cong \overline{TS}$.

2. If $\overline{UR} \cong \overline{UT}$, what can you conclude about U?

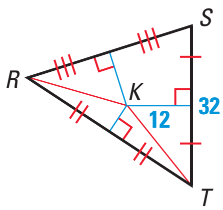
3. If Q is equidistant from $\overrightarrow{SR}$ and $\overrightarrow{ST}$, what can you conclude about Q?

5.2 BISECTORS OF A TRIANGLE

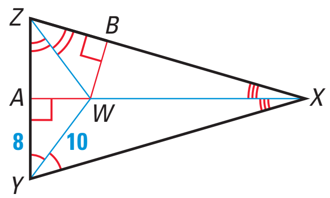
Examples on pp. 272–274

EXAMPLES The perpendicular bisectors of a triangle intersect at the *circumcenter*, which is equidistant from the vertices of the triangle. The angle bisectors of a triangle intersect at the *incenter*, which is equidistant from the sides of the triangle.

4. The perpendicular bisectors of $\triangle RST$ intersect at K. Find KR.



5. The angle bisectors of $\triangle XYZ$ intersect at W. Find WB.

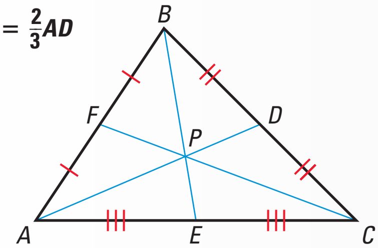


5.3 MEDIANS AND ALTITUDES OF A TRIANGLE

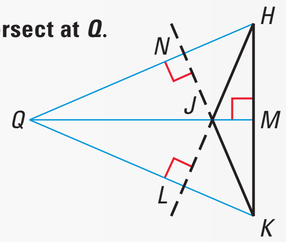
Examples on pp. 279–281

EXAMPLES The medians of a triangle intersect at the centroid. The lines containing the altitudes of a triangle intersect at the orthocenter.

$AP = \frac{2}{3}AD$

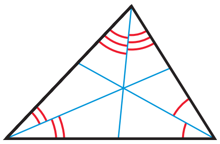


$\overleftrightarrow{HN}$, $\overleftrightarrow{JM}$, and $\overleftrightarrow{KL}$ intersect at Q.



Name the special segments and point of concurrency of the triangle.

6.



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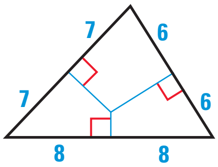

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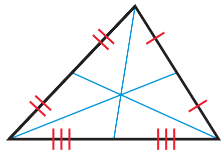

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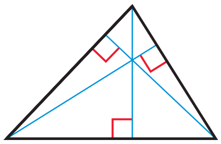


$\triangle XYZ$ has vertices $X(0, 0)$, $Y(-4, 0)$, and $Z(0, 6)$. Find the coordinates of the indicated point.

10. the centroid of $\triangle XYZ$

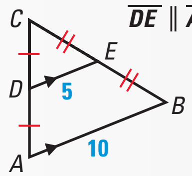
11. the orthocenter of $\triangle XYZ$

5.4 MIDSEGMENT THEOREM

Examples on pp. 287–289

EXAMPLES A midsegment of a triangle connects the midpoints of two sides of the triangle. By the Midsegment Theorem, a midsegment of a triangle is parallel to the third side and its length is half the third side.

$\overline{DE} \parallel \overline{AB}$, $DE = \frac{1}{2}AB$

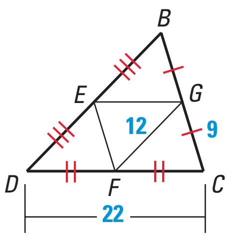


In Exercises 12 and 13, the midpoints of the sides of $\triangle HJK$ are $L(4, 3)$, $M(8, 3)$, and $N(6, 1)$.

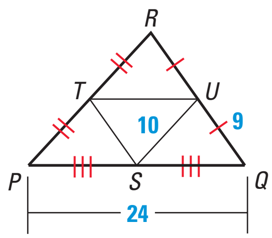
12. Find the coordinates of the vertices of the triangle.

13. Show that each midsegment is parallel to a side of the triangle.

14. Find the perimeter of $\triangle BCD$.



15. Find the perimeter of $\triangle STU$.



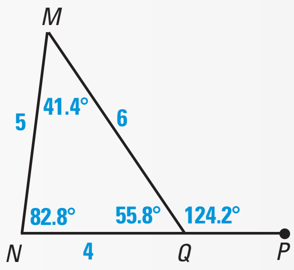
5.5 Inequalities in One Triangle

Examples on pp. 295–297

EXAMPLES In a triangle, the side and the angle of greatest measurement are always opposite each other. In the diagram, the largest angle, $\angle MNQ$, is opposite the longest side, $\overline{MQ}$.

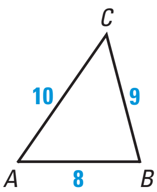
By the Exterior Angle Inequality, $m\angle MQP > m\angle N$ and $m\angle MQP > m\angle M$.

By the Triangle Inequality, $MN + NQ > MQ$, $NQ + MQ > MN$, and $MN + MQ > NQ$.



In Exercises 16–19, write the angle and side measurements in order from least to greatest.

16.



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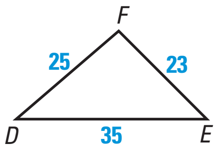

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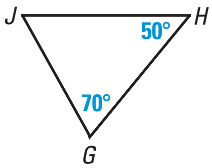

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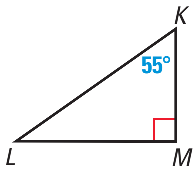


20. **FENCING A GARDEN** You are enclosing a triangular garden region with a fence. You have measured two sides of the garden to be 100 feet and 200 feet. What is the maximum length of fencing you need? Explain.

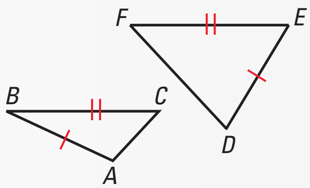
5.6 Indirect Proof and Inequalities in Two Triangles

Examples on pp. 302–304

EXAMPLES $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$

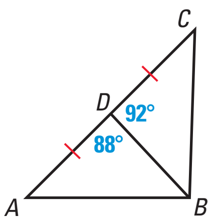
Hinge Theorem: If $m\angle E > m\angle B$, then $DF > AC$.

Converse of the Hinge Theorem: If $DF > AC$, then $m\angle E > m\angle B$.

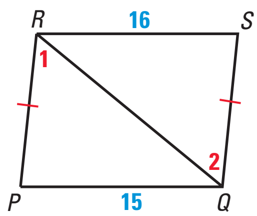


In Exercises 21–23, complete with <, >, or =.

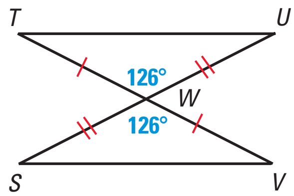
21. AB __?__ CB



22. $m\angle 1$ __?__ $m\angle 2$



23. TU __?__ VS



24. Write the first statement for an indirect proof of this situation: In a $\triangle MPQ$, if $\angle M \cong \angle Q$, then $\triangle MPQ$ is isosceles.

25. Write an indirect proof to show that no triangle has two right angles.

CHAPTER 5

Chapter Test

In Exercises 1–5, complete the statement with the word *always*, *sometimes*, or *never*.

1. If P is the circumcenter of $\triangle RST$, then PR, PS, and PT are __?__ equal.

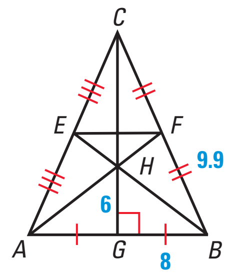
2. If $\overrightarrow{BD}$ bisects $\angle ABC$, then $\overline{AD}$ and $\overline{CD}$ are __?__ congruent.

3. The incenter of a triangle __?__ lies outside the triangle.

4. The length of a median of a triangle is __?__ equal to the length of a midsegment.

5. If $\overline{AM}$ is the altitude to side $\overline{BC}$ of $\triangle ABC$, then $\overline{AM}$ is __?__ shorter than $\overline{AB}$.

In Exercises 6–10, use the diagram.



6. Find each length.

a. HC **b.** HB **c.** HE **d.** BC

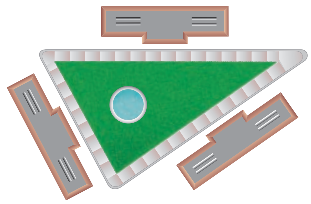
7. Point H is the __?__ of the triangle.

8. $\overline{CG}$ is a(n) __?__, __?__, __?__, and __?__ of $\triangle ABC$.

9. $EF =$ __?__ and $\overline{EF} \parallel$ __?__ by the __?__ Theorem.

10. Compare the measures of $\angle ACB$ and $\angle BAC$. Justify your answer.

11. **LANDSCAPE DESIGN** You are designing a circular swimming pool for a triangular lawn surrounded by apartment buildings. You want the center of the pool to be equidistant from the three sidewalks. Explain how you can locate the center of the pool.



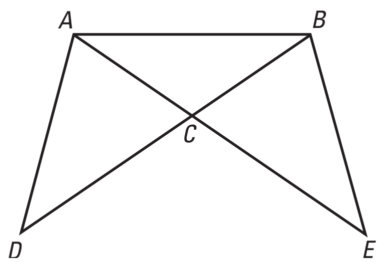
In Exercises 12–14, use the photo of the three-legged tripod.

12. As the legs of a tripod are spread apart, which theorem guarantees that the angles between each pair of legs get larger?

13. Each leg of a tripod can extend to a length of 5 feet. What is the maximum possible distance between the ends of two legs?

14. Let $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$ represent the legs of a tripod. Draw and label a sketch. Suppose the legs are congruent and $m\angle AOC > m\angle BOC$. Compare the lengths of $\overline{AC}$ and $\overline{BC}$.

In Exercises 15 and 16, use the diagram at the right.



15. Write a two-column proof.

GIVEN ▶ $AC = BC$

PROVE ▶ $BE < AE$

16. Write an indirect proof.

GIVEN ▶ $AD \neq AB$

PROVE ▶ $m\angle D \neq m\angle ABC$