5.2 Bisectors of a Triangle

What you should learn

GOAL 1 Use properties of perpendicular bisectors of a triangle, as applied in **Example 1**.

GOAL 2 Use properties of angle bisectors of a triangle.

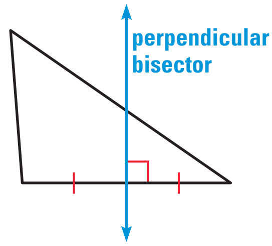
Why you should learn it

▼ To solve **real-life** problems, such as finding the center of a mushroom ring in **Exs. 24–26**.

REAL LIFE

GOAL 1 USING PERPENDICULAR BISECTORS OF A TRIANGLE

In Lesson 5.1, you studied properties of perpendicular bisectors of segments and angle bisectors. In this lesson, you will study the special cases in which the segments and angles being bisected are parts of a triangle.

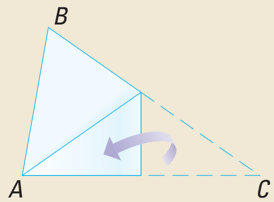


A **perpendicular bisector of a triangle** is a line (or ray or segment) that is perpendicular to a side of the triangle at the midpoint of the side.

ACTIVITY Developing Concepts — Perpendicular Bisectors of a Triangle

1. Cut four large acute scalene triangles out of paper. Make each one different.

2. Choose one triangle. Fold the triangle to form the perpendicular bisectors of the sides. Do the three bisectors intersect at the same point?

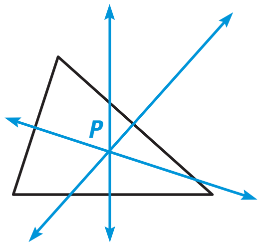


3. Repeat the process for the other three triangles. What do you observe? Write your observation in the form of a conjecture.

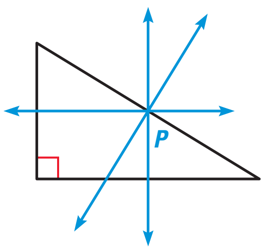
4. Choose one triangle. Label the vertices A, B, and C. Label the point of intersection of the perpendicular bisectors as P. Measure $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$. What do you observe?

When three or more lines (or rays or segments) intersect in the same point, they are called **concurrent lines** (or rays or segments). The point of intersection of the lines is called the **point of concurrency**.

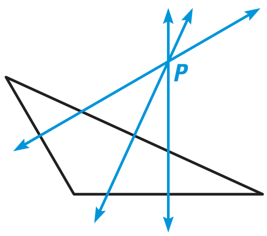
The three perpendicular bisectors of a triangle are concurrent. The point of concurrency can be *inside* the triangle, *on* the triangle, or *outside* the triangle.



acute triangle



right triangle



obtuse triangle

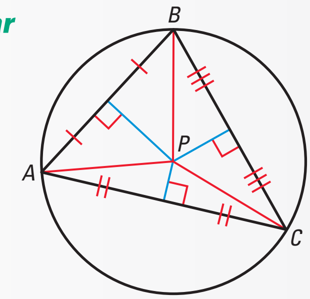
The point of concurrency of the perpendicular bisectors of a triangle is called the **circumcenter of the triangle.** In each triangle at the bottom of page 272, the circumcenter is at *P*. The circumcenter of a triangle has a special property, as described in Theorem 5.5. You will use coordinate geometry to illustrate this theorem in Exercises 29–31. A proof appears on page 835.

THEOREM

THEOREM 5.5 ***Concurrency of Perpendicular Bisectors of a Triangle***

The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

$$PA = PB = PC$$

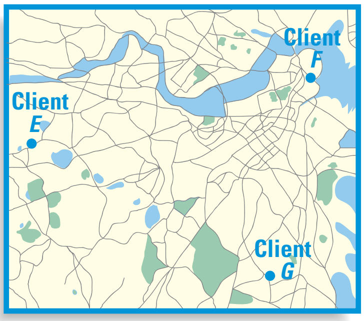


The diagram for Theorem 5.5 shows that the circumcenter is the center of the circle that passes through the vertices of the triangle. The circle is *circumscribed* about $\triangle ABC$. Thus, the radius of this circle is the distance from the center to any of the vertices.

EXAMPLE 1 *Using Perpendicular Bisectors*

REAL LIFE

FACILITIES PLANNING A company plans to build a distribution center that is convenient to three of its major clients. The planners start by roughly locating the three clients on a sketch and finding the circumcenter of the triangle formed.



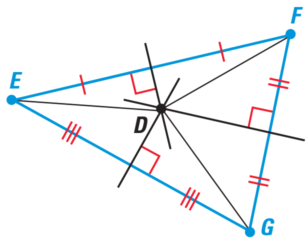
a. Explain why using the circumcenter as the location of a distribution center would be convenient for all the clients.

b. Make a sketch of the triangle formed by the clients. Locate the circumcenter of the triangle. Tell what segments are congruent.

SOLUTION

a. Because the circumcenter is equidistant from the three vertices, each client would be equally close to the distribution center.

b. Label the vertices of the triangle as *E*, *F*, and *G*. Draw the perpendicular bisectors. Label their intersection as *D*.



▶ By Theorem 5.5, $DE = DF = DG$.

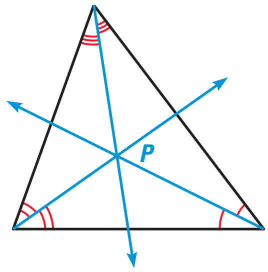
STUDENT HELP

INTERNET

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 USING ANGLE BISECTORS OF A TRIANGLE

An **angle bisector of a triangle** is a bisector of an angle of the triangle. The three angle bisectors are concurrent. The point of concurrency of the angle bisectors is called the **incenter of the triangle**, and it always lies inside the triangle. The incenter has a special property that is described below in Theorem 5.6. Exercise 22 asks you to write a proof of this theorem.

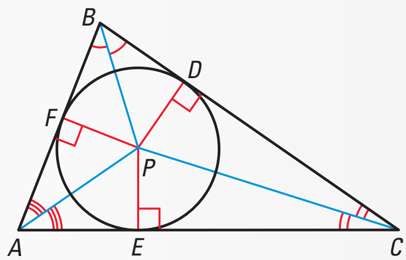


THEOREM

THEOREM 5.6 ***Concurrency of Angle Bisectors of a Triangle***

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

$$PD = PE = PF$$



The diagram for Theorem 5.6 shows that the incenter is the center of the circle that touches each side of the triangle once. The circle is *inscribed* within $\triangle ABC$. Thus, the radius of this circle is the distance from the center to any of the sides.

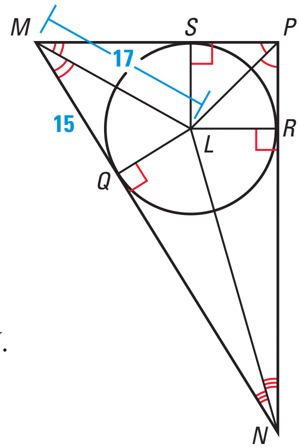
EXAMPLE 2 *Using Angle Bisectors*

Logical Reasoning

The angle bisectors of $\triangle MNP$ meet at point L.

a. What segments are congruent?

b. Find LQ and LR.



SOLUTION

a. By Theorem 5.6, the three angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle. So, $\overline{LR} \cong \overline{LQ} \cong \overline{LS}$.

b. Use the Pythagorean Theorem to find LQ in $\triangle LQM$.

$$(LQ)^2 + (MQ)^2 = (LM)^2$$

$$(LQ)^2 + 15^2 = 17^2 \quad \text{Substitute.}$$

$$(LQ)^2 + 225 = 289 \quad \text{Multiply.}$$

$$(LQ)^2 = 64 \quad \text{Subtract 225 from each side.}$$

$$LQ = 8 \quad \text{Find the positive square root.}$$

▶ So, $LQ = 8$ units. Because $\overline{LR} \cong \overline{LQ}$, $LR = 8$ units.

STUDENT HELP

Look Back
For help with the Pythagorean Theorem, see p. 20.

GUIDED PRACTICE

Vocabulary Check ✓

1. If three or more lines intersect at the same point, the lines are __?__.

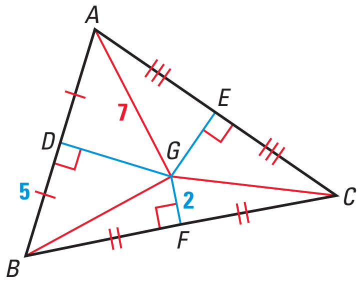
Concept Check ✓

2. Think of something about the words *incenter* and *circumcenter* that you can use to remember which special parts of a triangle meet at each point.

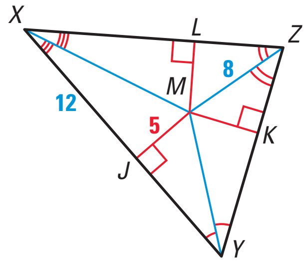
Skill Check ✓

Use the diagram and the given information to find the indicated measure.

3. The perpendicular bisectors of $\triangle ABC$ meet at point G. Find GC.



4. The angle bisectors of $\triangle XYZ$ meet at point M. Find MK.



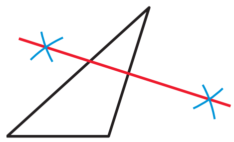
PRACTICE AND APPLICATIONS

STUDENT HELP

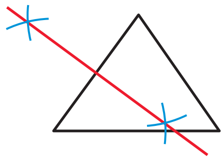
Extra Practice to help you master skills is on p. 811.

CONSTRUCTION **Draw a large example of the given type of triangle. Construct perpendicular bisectors of the sides. (See page 264.) For the type of triangle, do the bisectors intersect *inside, on,* or *outside* the triangle?**

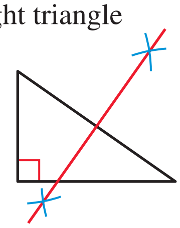
5. obtuse triangle



6. acute triangle



7. right triangle



DRAWING CONCLUSIONS **Draw a large $\triangle ABC$.**

8. Construct the angle bisectors of $\triangle ABC$. Label the point where the angle bisectors meet as D.

9. Construct perpendicular segments from D to each of the sides of the triangle. Measure each segment. What do you notice? Which theorem have you just confirmed?

LOGICAL REASONING **Use the results of Exercises 5–9 to complete the statement using *always, sometimes,* or *never*.**

10. A perpendicular bisector of a triangle __?__ passes through the midpoint of a side of the triangle.

11. The angle bisectors of a triangle __?__ intersect at a single point.

12. The angle bisectors of a triangle __?__ meet at a point outside the triangle.

13. The circumcenter of a triangle __?__ lies outside the triangle.

STUDENT HELP

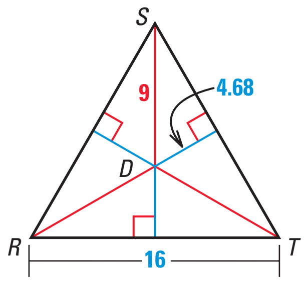
HOMEWORK HELP

Example 1: Exs. 5–7, 10–13, 14, 17, 20, 21

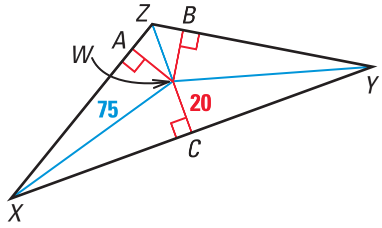
Example 2: Exs. 8, 9, 10–13, 15, 16, 22

BISECTORS In each case, find the indicated measure.

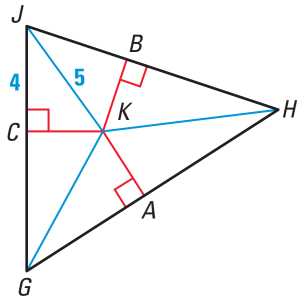
14. The perpendicular bisectors of $\triangle RST$ meet at point D. Find DR.



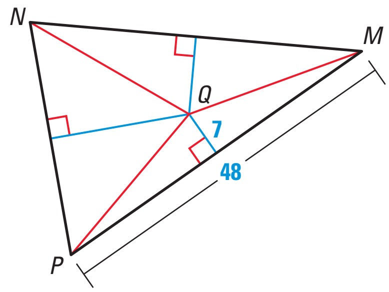
15. The angle bisectors of $\triangle XYZ$ meet at point W. Find WB.



16. The angle bisectors of $\triangle GHJ$ meet at point K. Find KB.

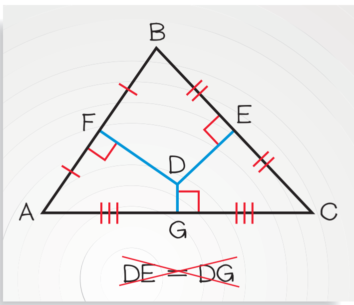


17. The perpendicular bisectors of $\triangle MNP$ meet at point Q. Find QN.

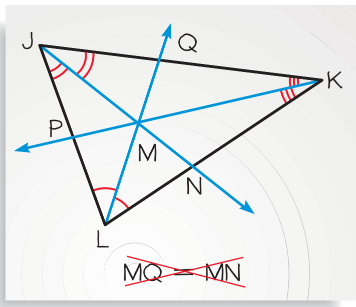


ERROR ANALYSIS Explain why the student's conclusion is *false*. Then state a correct conclusion that can be deduced from the diagram.

18.

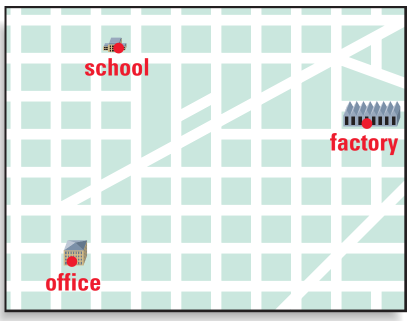


19.



LOGICAL REASONING In Exercises 20 and 21, use the following information and map.

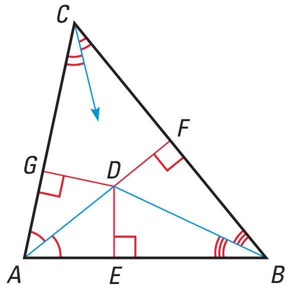
Your family is considering moving to a new home. The diagram shows the locations of where your parents work and where you go to school. The locations form a triangle.



20. In the diagram, how could you find a point that is equidistant from each location? Explain your answer.

21. Make a sketch of the situation. Find the best location for the new home.

22. **DEVELOPING PROOF** Complete the proof of Theorem 5.6, the Concurrency of Angle Bisectors.



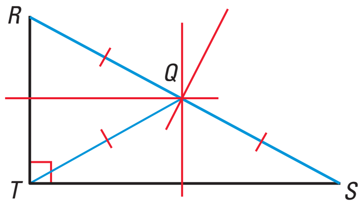
GIVEN ▶ $\triangle ABC$, the bisectors of $\angle A$, $\angle B$, and $\angle C$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, $\overline{DG} \perp \overline{CA}$

PROVE ▶ The angle bisectors intersect at a point that is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

Plan for Proof Show that D, the point of intersection of the bisectors of $\angle A$ and $\angle B$, also lies on the bisector of $\angle C$. Then show that D is equidistant from the sides of the triangle.

Statements	Reasons
1. $\triangle ABC$, the bisectors of $\angle A$, $\angle B$, and $\angle C$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, $\overline{DG} \perp \overline{CA}$	**1.** Given
2. __?__ $= DG$	**2.** $\overrightarrow{AD}$ bisects $\angle BAC$, so D is __?__ from the sides of $\angle BAC$.
3. $DE = DF$	**3.** __?__
4. $DF = DG$	**4.** __?__
5. D is on the __?__ of $\angle C$.	**5.** Converse of the Angle Bisector Theorem
6. __?__	**6.** Givens and Steps __?__

23. *Writing* Joannie thinks that the midpoint of the hypotenuse of a right triangle is equidistant from the vertices of the triangle. Explain how she could use perpendicular bisectors to verify her conjecture.

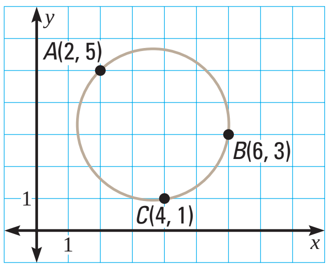


FOCUS ON APPLICATIONS

REAL LIFE **MUSHROOMS** live for only a few days. As the mycelium spreads outward, new mushroom rings are formed. A mushroom ring in France is almost half a mile in diameter and is about 700 years old.

SCIENCE CONNECTION **In Exercises 24–26, use the following information.**
A *mycelium* fungus grows underground in all directions from a central point. Under certain conditions, mushrooms sprout up in a ring at the edge. The radius of the mushroom ring is an indication of the mycelium's age.

24. Suppose three mushrooms in a mushroom ring are located as shown. Make a large copy of the diagram and draw $\triangle ABC$. Each unit on your coordinate grid should represent 1 foot.



25. Draw perpendicular bisectors on your diagram to find the center of the mushroom ring. Estimate the radius of the ring.

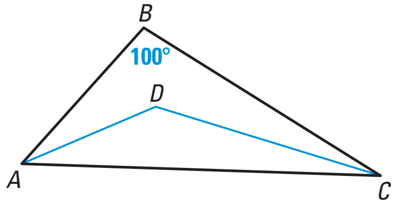
26. Suppose the radius of the mycelium increases at a rate of about 8 inches per year. Estimate its age.

Test Preparation

MULTIPLE CHOICE **Choose the correct answer from the list given.**

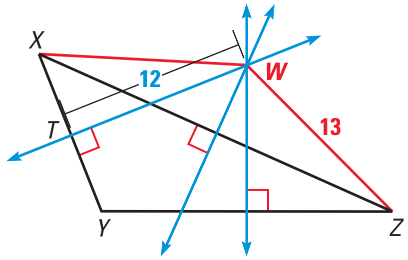
27. $\overline{AD}$ and $\overline{CD}$ are angle bisectors of $\triangle ABC$ and $m\angle ABC = 100°$. Find $m\angle ADC$.

Ⓐ 80° Ⓑ 90° Ⓒ 100°
Ⓓ 120° Ⓔ 140°



28. The perpendicular bisectors of $\triangle XYZ$ intersect at point W, $WT = 12$, and $WZ = 13$. Find XY.

Ⓐ 5 Ⓑ 8 Ⓒ 10
Ⓓ 12 Ⓔ 13



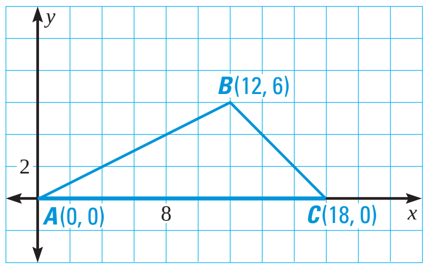
★ Challenge

USING ALGEBRA **Use the graph of $\triangle ABC$ to illustrate Theorem 5.5, the Concurrency of Perpendicular Bisectors.**

29. Find the midpoint of each side of $\triangle ABC$. Use the midpoints to find the equations of the perpendicular bisectors of $\triangle ABC$.

30. Using your equations from Exercise 29, find the intersection of two of the lines. Show that the point is on the third line.

31. Show that the point in Exercise 30 is equidistant from the vertices of $\triangle ABC$.



EXTRA CHALLENGE
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MIXED REVIEW

FINDING AREAS **Find the area of the triangle described.** (Review 1.7 for 5.3)

32. base = 9, height = 5

33. base = 22, height = 7

WRITING EQUATIONS **The line with the given equation is perpendicular to line *j* at point *P*. Write an equation of line *j*.** (Review 3.7)

34. $y = 3x - 2$, $P(1, 4)$

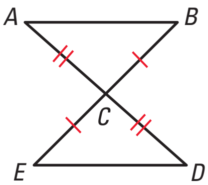
35. $y = -2x + 5$, $P(7, 6)$

36. $y = -\frac{2}{3}x - 1$, $P(2, 8)$

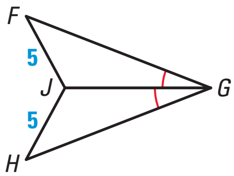
37. $y = \frac{10}{11}x + 3$, $P(-2, -9)$

LOGICAL REASONING **Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, tell which congruence postulate or theorem you would use.** (Review 4.3, 4.4, and 4.6)

38.



39.



40.