CHAPTER 4

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Classify triangles by their sides and angles. **(4.1)**	Lay the foundation for work with triangles.
Find angle measures in triangles. **(4.1)**	Find the angle measures in triangular objects, such as a wing deflector. **(p. 200)**
Identify congruent figures and corresponding parts. **(4.2)**	Analyze patterns, such as those made by the folds of an origami kite. **(p. 208)**
Prove that triangles are congruent	
• using corresponding sides and angles. **(4.2)**	Learn to work with congruent triangles.
• using the SSS and SAS Congruence Postulates. **(4.3)**	Explain why triangles are used in structural supports for buildings. **(p. 215)**
• using the ASA Congruence Postulate and the AAS Congruence Theorem. **(4.4)**	Understand how properties of triangles are applied in surveying. **(p. 225)**
• using the HL Congruence Theorem. **(4.6)**	Prove that right triangles are congruent.
• using coordinate geometry. **(4.7)**	Plan and write coordinate proofs.
Use congruent triangles to plan and write proofs. **(4.5)**	Prove that triangular parts of the framework of a bridge are congruent. **(p. 234)**
Prove that constructions are valid. **(4.5)**	Develop understanding of geometric constructions.
Use properties of isosceles, equilateral, and right triangles. **(4.6)**	Apply a law from physics, the law of reflection. **(p. 241)**

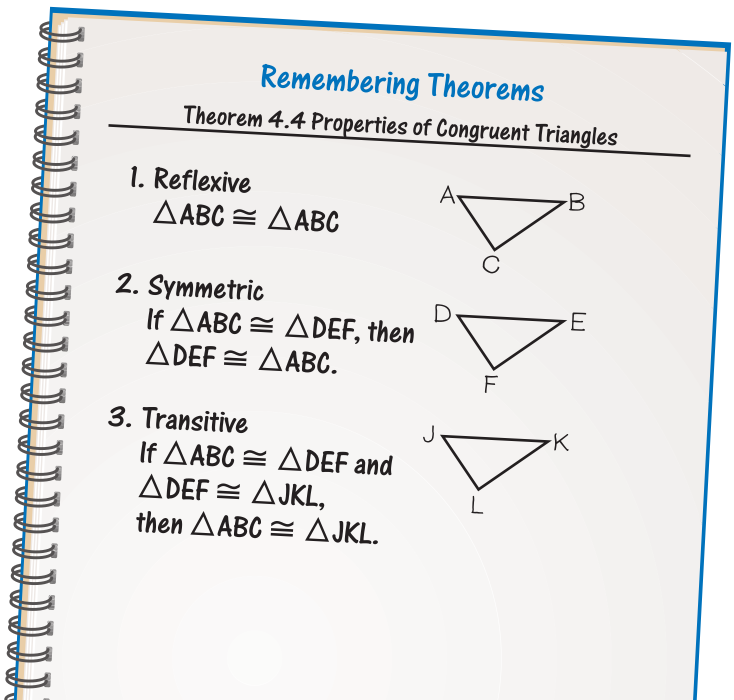
How does Chapter 4 fit into the BIGGER PICTURE of geometry?

The ways you have learned to prove triangles are congruent will be used to prove theorems about *polygons*, as well as in other topics throughout the book. Knowing the properties of triangles will help you solve real-life problems in fields such as art, architecture, and engineering.

STUDY STRATEGY

How did you use your list of theorems?

The list of theorems you made, following the **Study Strategy** on page 192, may resemble this one.



CHAPTER 4

Chapter Review

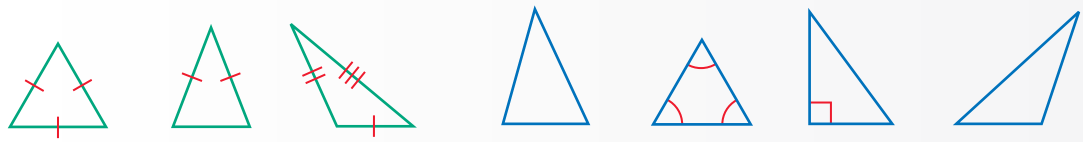
VOCABULARY

- equilateral triangle, p. 194
- isosceles triangle, p. 194
- scalene triangle, p. 194
- acute triangle, p. 194
- equiangular triangle, p. 194
- right triangle, p. 194
- obtuse triangle, p. 194
- vertex of a triangle, p. 195
- adjacent sides of a triangle, p. 195
- legs of a right triangle, p. 195
- hypotenuse, p. 195
- legs of an isosceles triangle, p. 195
- base of an isosceles triangle, p. 195
- interior angle, p. 196
- exterior angle, p. 196
- corollary, p. 197
- congruent, p. 202
- corresponding angles, p. 202
- corresponding sides, p. 202
- base angles, p. 236
- vertex angle, p. 236
- coordinate proof, p. 243

4.1 TRIANGLES AND ANGLES

Examples on pp. 194–197

EXAMPLES You can classify triangles by their sides and by their angles.



equilateral | isosceles | scalene | acute | equiangular | right | obtuse

Note that an equilateral triangle is also isosceles and acute.

You can apply the Triangle Sum Theorem to find unknown angle measures in triangles.

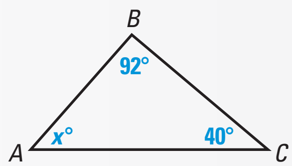
$m\angle A + m\angle B + m\angle C = 180°$ **Triangle Sum Theorem**

$x° + 92° + 40° = 180°$ **Substitute.**

$x + 132 = 180$ **Simplify.**

$x = 48$ **Subtract 132 from each side.**

$m\angle A = 48°$



In Exercises 1–4, classify the triangle by its angles and by its sides.

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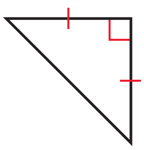

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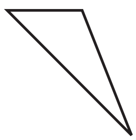

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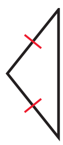

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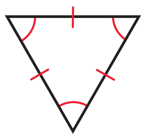


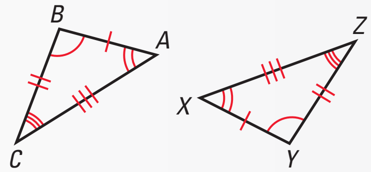
5. One acute angle of a right triangle measures 37°. Find the measure of the other acute angle.

6. In $\triangle MNP$, the measure of $\angle M$ is 24°. The measure of $\angle N$ is five times the measure of $\angle P$. Find $m\angle N$ and $m\angle P$.

4.2 CONGRUENCE AND TRIANGLES

Examples on pp. 202–205

EXAMPLE When two figures are congruent, their corresponding sides and corresponding angles are congruent. In the diagram, $\triangle ABC \cong \triangle XYZ$.



Use the diagram above of $\triangle ABC$ and $\triangle XYZ$.

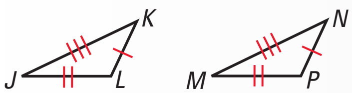
7. Identify the congruent corresponding parts of the triangles.

8. Given $m\angle A = 48°$ and $m\angle Z = 37°$, find $m\angle Y$.

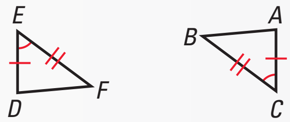
4.3 & 4.4 PROVING TRIANGLES ARE CONGRUENT: SSS, SAS, ASA, AND AAS

Examples on pp. 212–215, 220–222

EXAMPLES You can prove triangles are congruent using congruence postulates and theorems.



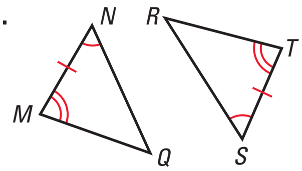
$\overline{JK} \cong \overline{MN}$, $\overline{KL} \cong \overline{NP}$, $\overline{JL} \cong \overline{MP}$, so $\triangle JKL \cong \triangle MNP$ by the SSS Congruence Postulate.



$\overline{DE} \cong \overline{AC}$, $\angle E \cong \angle C$, and $\overline{EF} \cong \overline{CB}$, so $\triangle DEF \cong \triangle ACB$ by the SAS Congruence Postulate.

Decide whether it is possible to prove that the triangles are congruent. If it is possible, tell which postulate or theorem you would use. Explain your reasoning.

9.



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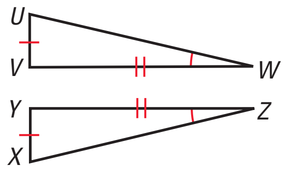

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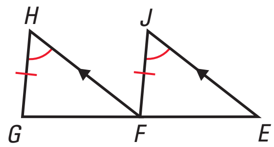
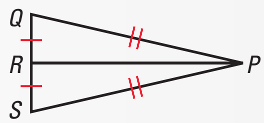


4.5 USING CONGRUENT TRIANGLES

Examples on pp. 229–231

EXAMPLE You can use congruent triangles to write proofs.



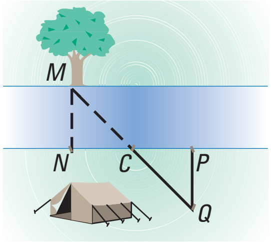
GIVEN ▶ $\overline{PQ} \cong \overline{PS}$, $\overline{RQ} \cong \overline{RS}$

PROVE ▶ $\overline{PR} \perp \overline{QS}$

Plan for Proof Use the SSS Congruence Postulate to show that $\triangle PRQ \cong \triangle PRS$. Because corresponding parts of congruent triangles are congruent, you can conclude that $\angle PRQ \cong \angle PRS$. These angles form a linear pair, so $\overline{PR} \perp \overline{QS}$.

4.5 continued

SURVEYING You want to determine the width of a river beside a camp. You place stakes so that $\overline{MN} \perp \overline{NP}$, $\overline{PQ} \perp \overline{NP}$, and C is the midpoint of $\overline{NP}$.



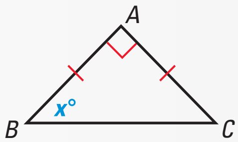
12. Are $\triangle MCN$ and $\triangle QCP$ congruent? If so, state the postulate or theorem that can be used to prove they are congruent.

13. Which segment should you measure to find the width of the river?

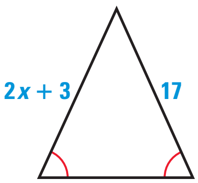
4.6 ISOSCELES, EQUILATERAL, AND RIGHT TRIANGLES

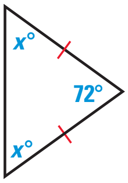
Examples on pp. 236–238

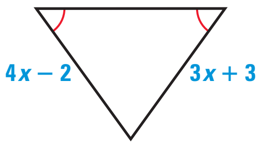
EXAMPLE To find the value of x, notice that $\triangle ABC$ is an isosceles right triangle. By the Base Angles Theorem, $\angle B \cong \angle C$. Because $\angle B$ and $\angle C$ are complementary, their sum is 90°. The measure of each must be 45°. So $x = 45°$.

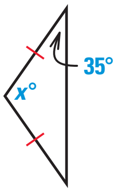


Find the value of *x*.

14. 

15. 

16. 

17. 

4.7 TRIANGLES AND COORDINATE PROOF

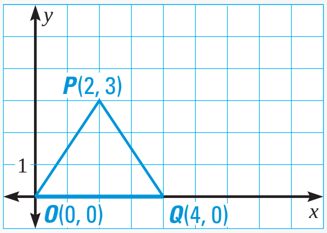
Examples on pp. 243–246

EXAMPLE You can use a coordinate proof to prove that $\triangle OPQ$ is isosceles. Use the Distance Formula to show that $\overline{OP} \cong \overline{QP}$.

$$OP = \sqrt{(2-0)^2 + (3-0)^2} = \sqrt{13}$$

$$QP = \sqrt{(2-4)^2 + (3-0)^2} = \sqrt{13}$$

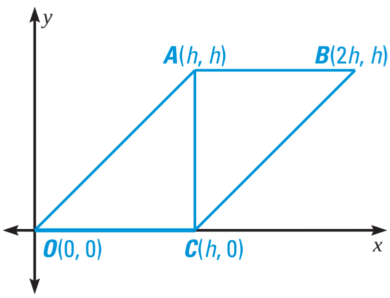
Because $\overline{OP} \cong \overline{QP}$, $\triangle OPQ$ is isosceles.



18. Write a coordinate proof.

GIVEN ▶ Coordinates of vertices of $\triangle OAC$ and $\triangle BCA$

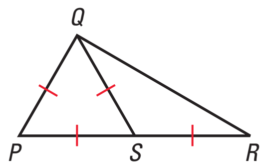
PROVE ▶ $\triangle OAC \cong \triangle BCA$



CHAPTER 4

Chapter Test

In Exercises 1–6, identify all triangles in the figure that fit the given description.



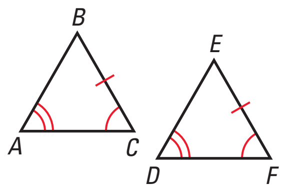
1. isosceles **2.** equilateral **3.** scalene

4. acute **5.** obtuse **6.** right

7. In $\triangle ABC$, the measure of $\angle A$ is 116°. The measure of $\angle B$ is three times the measure of $\angle C$. Find $m\angle B$ and $m\angle C$.

Decide whether it is possible to prove that the triangles are congruent. If it is possible, tell which congruence postulate or theorem you would use. Explain your reasoning.

8.



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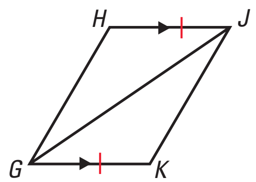

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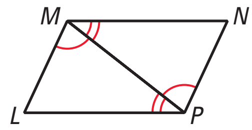

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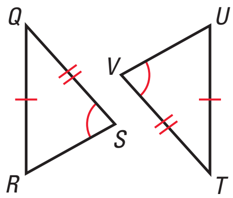

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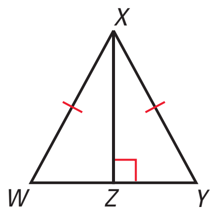

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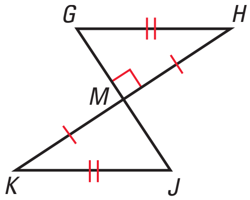
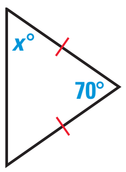


Find the value of *x*.

14.



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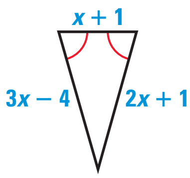

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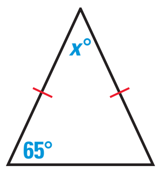
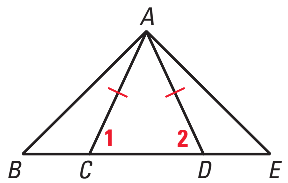


PROOF **Write a two-column proof or a paragraph proof.**

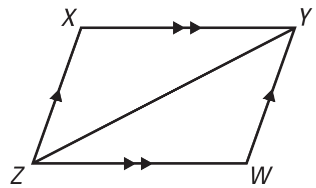
17. GIVEN ▶ $\overline{BD} \cong \overline{EC}$, $\overline{AC} \cong \overline{AD}$

PROVE ▶ $\overline{AB} \cong \overline{AE}$



18. GIVEN ▶ $\overline{XY} \parallel \overline{WZ}$, $\overline{XZ} \parallel \overline{WY}$

PROVE ▶ $\angle X \cong \angle W$



Place the figure in a coordinate plane and find the requested information.

19. A right triangle with leg lengths of 4 units and 7 units; find the length of the hypotenuse.

20. A square with side length s and vertices at $(0, 0)$ and (s, s); find the coordinates of the midpoint of a diagonal.