4.7 Triangles and Coordinate Proof

What *you should learn*

GOAL 1 Place geometric figures in a coordinate plane.

GOAL 2 Write a coordinate proof.

Why *you should learn it*

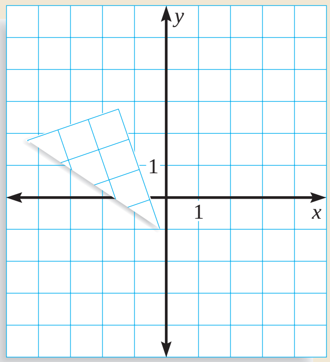
Sometimes a coordinate proof is the most efficient way to prove a statement.

GOAL 1 PLACING FIGURES IN A COORDINATE PLANE

So far, you have studied two-column proofs, paragraph proofs, and flow proofs. A **coordinate proof** involves placing geometric figures in a coordinate plane. Then you can use the Distance Formula and the Midpoint Formula, as well as postulates and theorems, to prove statements about the figures.

ACTIVITY — Developing Concepts: Placing Figures in a Coordinate Plane

1. Draw a right triangle with legs of 3 units and 4 units on a piece of grid paper. Cut out the triangle.
2. Use another piece of grid paper to draw a coordinate plane.
3. Sketch different ways that the triangle can be placed on the coordinate plane. Which of the ways that you placed the triangle is best for finding the length of the hypotenuse?

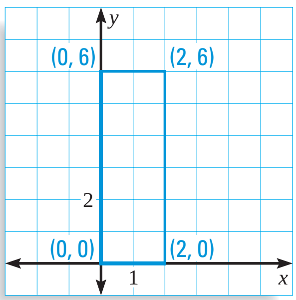


EXAMPLE 1 *Placing a Rectangle in a Coordinate Plane*

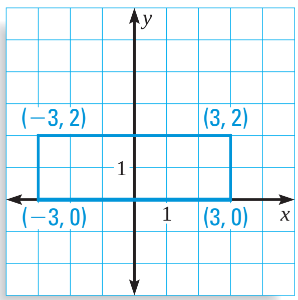
Place a 2-unit by 6-unit rectangle in a coordinate plane.

SOLUTION

Choose a placement that makes finding distances easy. Here are two possible placements.



One vertex is at the origin, and three of the vertices have at least one coordinate that is 0.



One side is centered at the origin, and the *x*-coordinates are opposites.

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Once a figure has been placed in a coordinate plane, you can use the Distance Formula or the Midpoint Formula to measure distances or locate points.

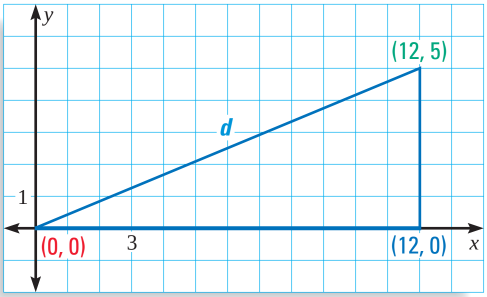
EXAMPLE 2 *Using the Distance Formula*

Using Algebra

A right triangle has legs of 5 units and 12 units. Place the triangle in a coordinate plane. Label the coordinates of the vertices and find the length of the hypotenuse.

SOLUTION

One possible placement is shown. Notice that one leg is vertical and the other leg is horizontal, which assures that the legs meet at right angles. Points on the same vertical segment have the same x-coordinate, and points on the same horizontal segment have the same y-coordinate.



You can use the Distance Formula to find the length of the hypotenuse.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \textbf{Distance Formula}$$

$$= \sqrt{(12 - 0)^2 + (5 - 0)^2} \quad \textbf{Substitute.}$$

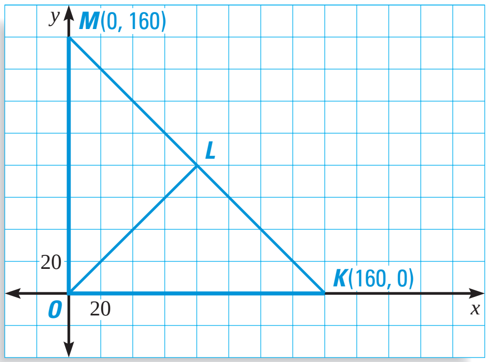
$$= \sqrt{169} \quad \textbf{Simplify.}$$

$$= 13 \quad \textbf{Evaluate square root.}$$

EXAMPLE 3 *Using the Midpoint Formula*

In the diagram, $\triangle MLO \cong \triangle KLO$.

Find the coordinates of point L.



SOLUTION

Because the triangles are congruent, it follows that $\overline{ML} \cong \overline{KL}$. So, point L must be the midpoint of $\overline{MK}$. This means you can use the Midpoint Formula to find the coordinates of point L.

$$L(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \textbf{Midpoint Formula}$$

$$= \left(\frac{160 + 0}{2}, \frac{0 + 160}{2}\right) \quad \textbf{Substitute.}$$

$$= (80, 80) \quad \textbf{Simplify.}$$

▶ The coordinates of L are (80, 80).

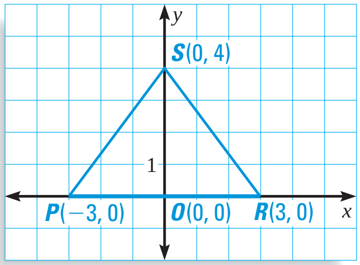
GOAL 2 WRITING COORDINATE PROOFS

Once a figure is placed in a coordinate plane, you may be able to prove statements about the figure.

EXAMPLE 4 *Writing a Plan for a Coordinate Proof*

Proof

Write a plan to prove that $\overrightarrow{SO}$ bisects $\angle PSR$.



GIVEN ▶ Coordinates of vertices of $\triangle POS$ and $\triangle ROS$

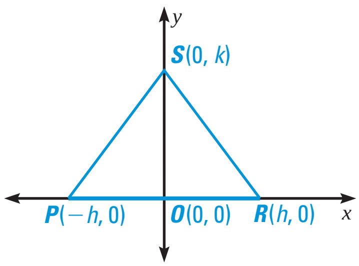
PROVE ▶ $\overrightarrow{SO}$ bisects $\angle PSR$

SOLUTION

Plan for Proof Use the Distance Formula to find the side lengths of $\triangle POS$ and $\triangle ROS$. Then use the SSS Congruence Postulate to show that $\triangle POS \cong \triangle ROS$. Finally, use the fact that corresponding parts of congruent triangles are congruent to conclude that $\angle PSO \cong \angle RSO$, which implies that $\overrightarrow{SO}$ bisects $\angle PSR$.

. . . .

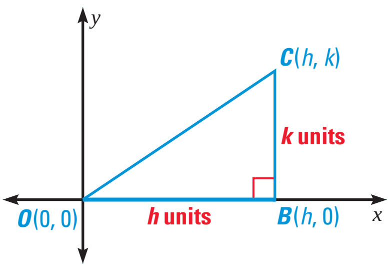
The coordinate proof in Example 4 applies to a specific triangle. When you want to prove a statement about a more general set of figures, it is helpful to use variables as coordinates.



For instance, you can use variable coordinates to duplicate the proof in Example 4. Once this is done, you can conclude that $\overrightarrow{SO}$ bisects $\angle PSR$ for *any* triangle whose coordinates fit the given pattern.

EXAMPLE 5 *Using Variables as Coordinates*

Right $\triangle OBC$ has leg lengths of h units and k units. You can find the coordinates of points B and C by considering how the triangle is placed in the coordinate plane.



Point B is h units horizontally from the origin, so its coordinates are $(h, 0)$. Point C is h units horizontally from the origin and k units vertically from the origin, so its coordinates are (h, k).

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You can use the Distance Formula to find the length of the hypotenuse $\overline{OC}$.

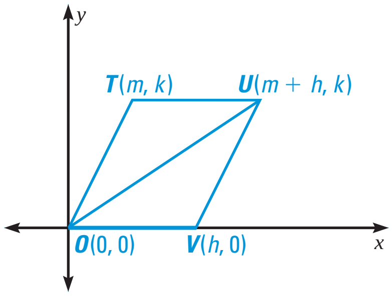
$$OC = \sqrt{(h - 0)^2 + (k - 0)^2} = \sqrt{h^2 + k^2}$$

Proof

EXAMPLE 6 *Writing a Coordinate Proof*

GIVEN ▶ Coordinates of figure $OTUV$

PROVE ▶ $\triangle OTU \cong \triangle UVO$



SOLUTION

COORDINATE PROOF Segments $\overline{OV}$ and $\overline{UT}$ have the same length.

$$OV = \sqrt{(h-0)^2 + (0-0)^2} = h$$

$$UT = \sqrt{(m+h-m)^2 + (k-k)^2} = h$$

Horizontal segments $\overline{UT}$ and $\overline{OV}$ each have a slope of 0, which implies that they are parallel. Segment $\overline{OU}$ intersects $\overline{UT}$ and $\overline{OV}$ to form congruent alternate interior angles $\angle TUO$ and $\angle VOU$. Because $\overline{OU} \cong \overline{OU}$, you can apply the SAS Congruence Postulate to conclude that $\triangle OTU \cong \triangle UVO$.

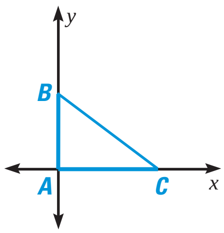
GUIDED PRACTICE

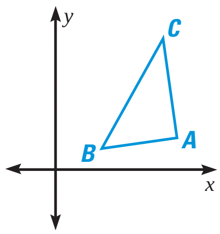
Vocabulary Check ✓

1. Prior to this section, you have studied two-column proofs, paragraph proofs, and flow proofs. How is a *coordinate proof* different from these other types of proof? How is it the same?

Concept Check ✓

2. Two different ways to place the same right triangle in a coordinate plane are shown. Which placement is more convenient for finding the side lengths? Explain your thinking. Then sketch a third placement that also makes it convenient to find the side lengths.





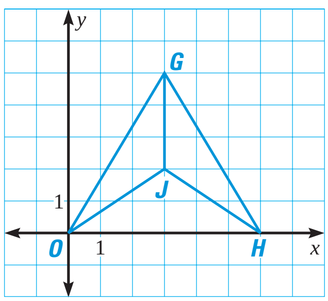
Skill Check ✓

3. A right triangle with legs of 7 units and 4 units has one vertex at (0, 0) and another at (0, 7). Give possible coordinates of the third vertex.

DEVELOPING PROOF **Describe a plan for the proof.**

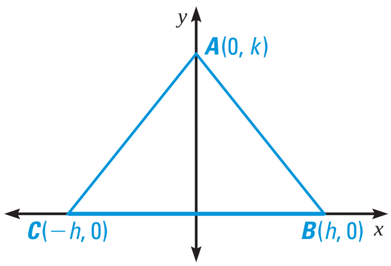
4. **GIVEN** ▶ $\overrightarrow{GJ}$ bisects $\angle OGH$.

   PROVE ▶ $\triangle GJO \cong \triangle GJH$



5. **GIVEN** ▶ Coordinates of vertices of $\triangle ABC$

   PROVE ▶ $\triangle ABC$ is isosceles.



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 810.

PLACING FIGURES IN A COORDINATE PLANE **Place the figure in a coordinate plane. Label the vertices and give the coordinates of each vertex.**

6. A 5-unit by 8-unit rectangle with one vertex at (0, 0)

7. An 8-unit by 6-unit rectangle with one vertex at (0, −4)

8. A square with side length s and one vertex at (s, 0)

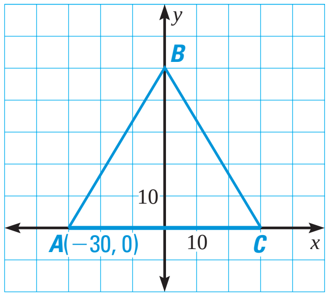
CHOOSING A GOOD PLACEMENT **Place the figure in a coordinate plane. Label the vertices and give the coordinates of each vertex. Explain the advantages of your placement.**

9. A right triangle with legs of 3 units and 8 units

10. An isosceles right triangle with legs of 20 units

11. A rectangle with length h and width k

FINDING AND USING COORDINATES **In the diagram, $\triangle ABC$ is isosceles. Its base is 60 units and its height is 50 units.**



12. Give the coordinates of points B and C.

13. Find the length of a leg of $\triangle ABC$. Round your answer to the nearest hundredth.

USING THE DISTANCE FORMULA **Place the figure in a coordinate plane and find the given information.**

14. A right triangle with legs of 7 and 9 units; find the length of the hypotenuse.

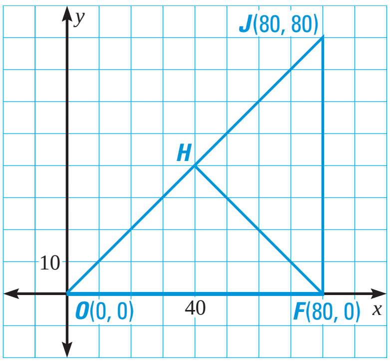
15. A rectangle with length 5 units and width 4 units; find the length of a diagonal.

16. An isosceles right triangle with legs of 3 units; find the length of the hypotenuse.

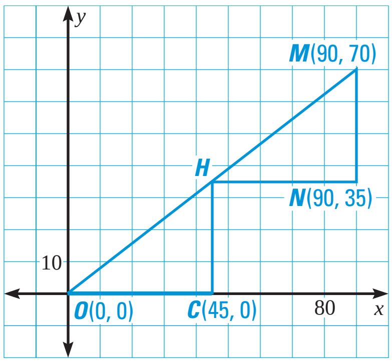
17. A 3-unit by 3-unit square; find the length of a diagonal.

USING THE MIDPOINT FORMULA **Use the given information and diagram to find the coordinates of H.**

18. $\triangle FOH \cong \triangle FJH$



19. $\triangle OCH \cong \triangle HNM$



STUDENT HELP

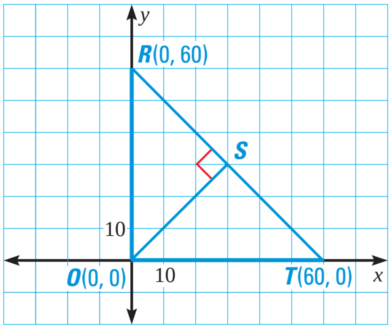
HOMEWORK HELP

Example 1: Exs. 6–11
Example 2: Exs. 12–17
Example 3: Exs. 18, 19
Example 4: Exs. 20, 21
Example 5: Exs. 22–25
Example 6: Exs. 26, 27

DEVELOPING PROOF Write a plan for a proof.

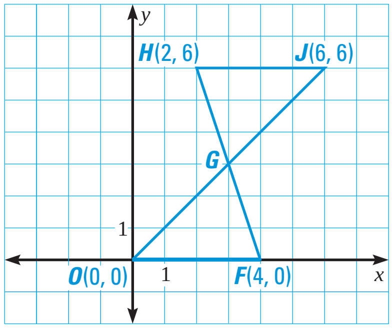
20. **GIVEN** ▶ $\overline{OS} \perp \overline{RT}$

PROVE ▶ $\overrightarrow{OS}$ bisects $\angle TOR$.



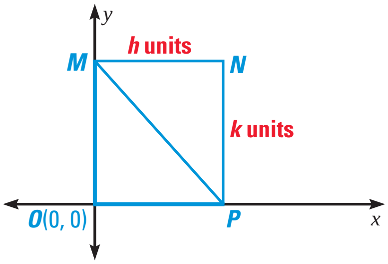
21. **GIVEN** ▶ G is the midpoint of $\overline{HF}$.

PROVE ▶ $\triangle GHJ \cong \triangle GFO$

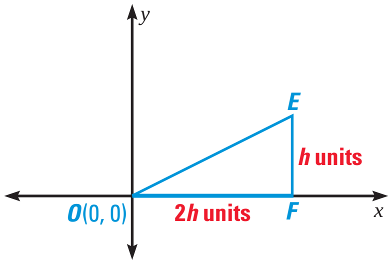


USING VARIABLES AS COORDINATES Find the coordinates of any unlabeled points. Then find the requested information.

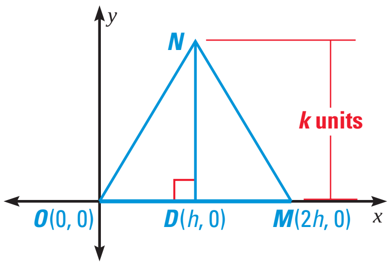
22. Find MP.



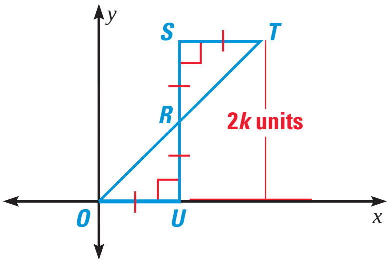
23. Find OE.



24. Find ON and MN.



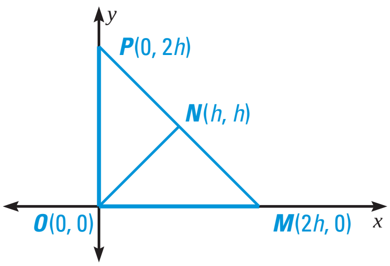
25. Find OT.



COORDINATE PROOF Write a coordinate proof.

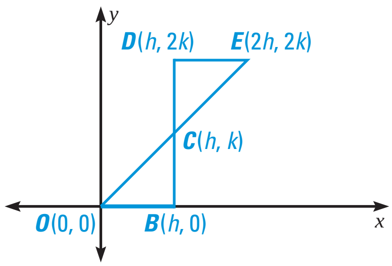
26. **GIVEN** ▶ Coordinates of $\triangle NPO$ and $\triangle NMO$

PROVE ▶ $\triangle NPO \cong \triangle NMO$

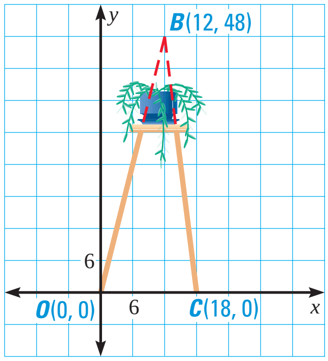


27. **GIVEN** ▶ Coordinates of $\triangle OBC$ and $\triangle EDC$

PROVE ▶ $\triangle OBC \cong \triangle EDC$

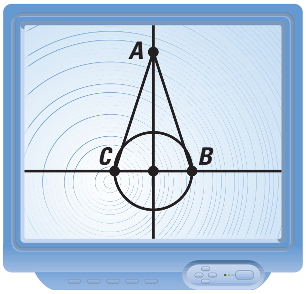


28. **PLANT STAND** You buy a tall, three-legged plant stand. When you place a plant on the stand, the stand appears to be unstable under the weight of the plant. The diagram at the right shows a coordinate plane superimposed on one pair of the plant stand's legs. The legs are extended to form $\triangle OBC$. Is $\triangle OBC$ an isosceles triangle? Explain why the plant stand may be unstable.



TECHNOLOGY **Use geometry software for Exercises 29–31. Follow the steps below to construct $\triangle ABC$.**

- Create a pair of axes. Construct point A on the y-axis so that the y-coordinate is positive. Construct point B on the x-axis.
- Construct a circle with a center at the origin that contains point B. Label the other point where the circle intersects the x-axis C.
- Connect points A, B, and C to form $\triangle ABC$. Find the coordinates of each vertex.



29. What type of triangle does $\triangle ABC$ appear to be? Does your answer change if you drag point A? If you drag point B?

30. Measure and compare AB and AC. What happens to these lengths as you drag point A? What happens as you drag point B?

31. Look back at the proof described in Exercise 5 on page 246. How does that proof help explain your answers to Exercises 29 and 30?

Test Preparation

32. **MULTIPLE CHOICE** A square with side length 4 has one vertex at (0, 2). Which of the points below *could* be a vertex of the square?

(A) (0, −2) (B) (2, −2) (C) (0, 0) (D) (2, 2)

33. **MULTIPLE CHOICE** A rectangle with side lengths $2h$ and k has one vertex at $(-h, k)$. Which of the points below *could not* be a vertex of the rectangle?

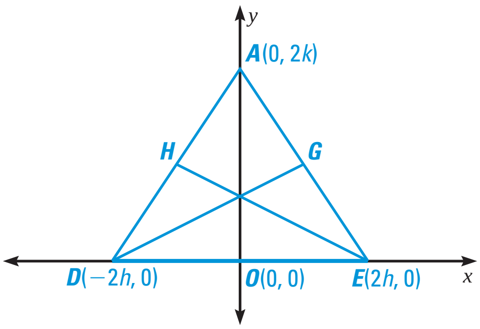
(A) $(0, k)$ (B) $(-h, 0)$ (C) (h, k) (D) $(h, 0)$

★ Challenge

34. **COORDINATE PROOF** Use the diagram and the given information to write a proof.

GIVEN ▶ Coordinates of $\triangle DEA$, H is the midpoint of $\overline{DA}$, G is the midpoint of $\overline{EA}$.

PROVE ▶ $\overline{DG} \cong \overline{EH}$



EXTRA CHALLENGE
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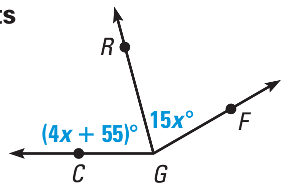
MIXED REVIEW

USING ALGEBRA **In the diagram, $\overrightarrow{GR}$ bisects $\angle CGF$.** (Review 1.5 for 5.1)

35. Find the value of x.

36. Find $m\angle CGF$.



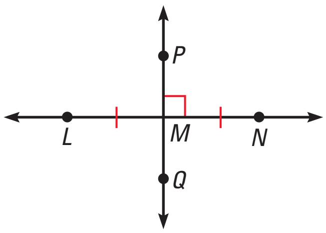
PERPENDICULAR LINES AND SEGMENT BISECTORS **Use the diagram to determine whether the statement is *true* or *false*.** (Review 1.5, 2.2 for 5.1)

37. $\overleftrightarrow{PQ}$ is perpendicular to $\overleftrightarrow{LN}$.

38. Points L, Q, and N are collinear.

39. $\overleftrightarrow{PQ}$ bisects $\overline{LN}$.

40. $\angle LMQ$ and $\angle PMN$ are supplementary.



WRITING STATEMENTS **Let p be "two triangles are congruent" and let q be "the corresponding angles of the triangles are congruent." Write the symbolic statement in words. Decide whether the statement is true.** (Review 2.3)

41. $p \to q$ **42.** $q \to p$ **43.** $\sim p \to \sim q$

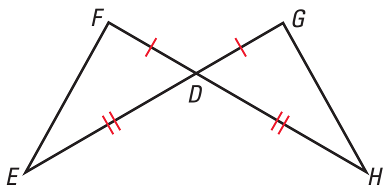
QUIZ 3

Self-Test for Lessons 4.5–4.7

PROOF **Write a two-column proof or a paragraph proof.** (Lessons 4.5 and 4.6)

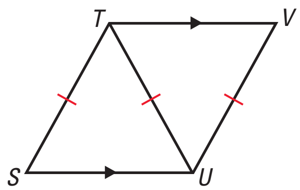
1. GIVEN ▶ $\overline{DF} \cong \overline{DG}$, $\overline{ED} \cong \overline{HD}$

PROVE ▶ $\angle EFD \cong \angle HGD$



2. GIVEN ▶ $\overline{ST} \cong \overline{UT} \cong \overline{VU}$, $\overline{SU} \parallel \overline{TV}$

PROVE ▶ $\triangle STU \cong \triangle TUV$



3. **COORDINATE PROOF** Write a plan for a coordinate proof. (Lesson 4.7)

GIVEN ▶ Coordinates of vertices of $\triangle OPM$ and $\triangle ONM$

PROVE ▶ $\triangle OPM$ and $\triangle ONM$ are congruent isosceles triangles.