4.5

Using Congruent Triangles

What *you should learn*

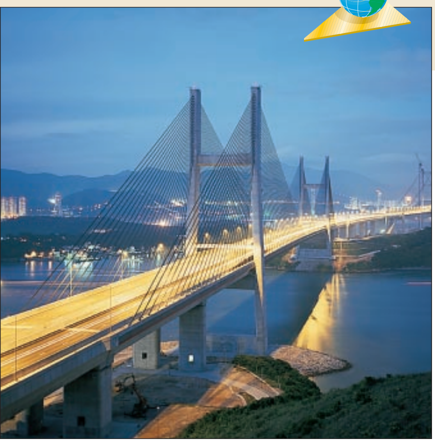
GOAL 1 Use congruent triangles to plan and write proofs.

GOAL 2 Use congruent triangles to prove constructions are valid.

Why *you should learn it*

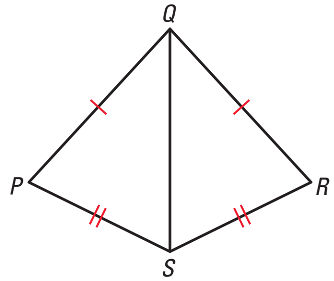
▼ Congruent triangles are important in **real-life** problems, such as designing and constructing bridges like the one in **Ex. 16**.

REAL LIFE



GOAL 1 PLANNING A PROOF

Knowing that all pairs of corresponding parts of congruent triangles are congruent can help you reach conclusions about congruent figures.

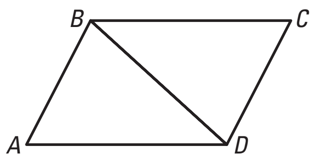


For instance, suppose you want to prove that $\angle PQS \cong \angle RQS$ in the diagram shown at the right. One way to do this is to show that $\triangle PQS \cong \triangle RQS$ by the SSS Congruence Postulate. Then you can use the fact that corresponding parts of congruent triangles are congruent to conclude that $\angle PQS \cong \angle RQS$.

EXAMPLE 1 *Planning and Writing a Proof*

GIVEN ▶ $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$

PROVE ▶ $\overline{AB} \cong \overline{CD}$

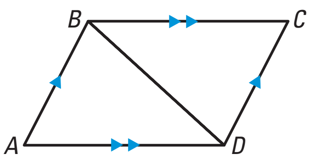


Plan for Proof Show that $\triangle ABD \cong \triangle CDB$. Then use the fact that corresponding parts of congruent triangles are congruent.

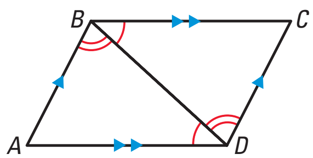
SOLUTION

First copy the diagram and mark it with the given information. Then mark any additional information that you can deduce. Because $\overline{AB}$ and $\overline{CD}$ are parallel segments intersected by a transversal, and $\overline{BC}$ and $\overline{DA}$ are parallel segments intersected by a transversal, you can deduce that two pairs of alternate interior angles are congruent.

Mark given information.



Add deduced information.



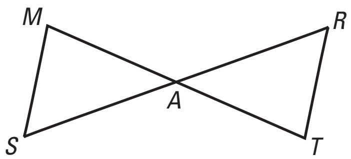
Paragraph Proof Because $\overline{AB} \parallel \overline{CD}$, it follows from the Alternate Interior Angles Theorem that $\angle ABD \cong \angle CDB$. For the same reason, $\angle ADB \cong \angle CBD$ because $\overline{BC} \parallel \overline{DA}$. By the Reflexive Property of Congruence, $\overline{BD} \cong \overline{BD}$. You can use the ASA Congruence Postulate to conclude that $\triangle ABD \cong \triangle CDB$. Finally, because corresponding parts of congruent triangles are congruent, it follows that $\overline{AB} \cong \overline{CD}$.

Proof

EXAMPLE 2 *Planning and Writing a Proof*

GIVEN ▶ A is the midpoint of $\overline{MT}$,
A is the midpoint of $\overline{SR}$.

PROVE ▶ $\overline{MS} \parallel \overline{TR}$



Plan for Proof Prove that $\triangle MAS \cong \triangle TAR$. Then use the fact that corresponding parts of congruent triangles are congruent to show that $\angle M \cong \angle T$. Because these angles are formed by two segments intersected by a transversal, you can conclude that $\overline{MS} \parallel \overline{TR}$.

STUDENT HELP

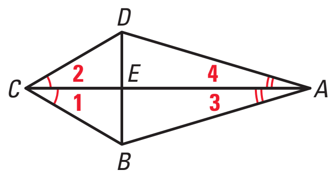
INTERNET HOMEWORK HELP
Visit our Web site at www.mcdougallittell.com for extra examples.

Statements	Reasons
1. A is the midpoint of $\overline{MT}$, A is the midpoint of $\overline{SR}$.	**1.** Given
2. $\overline{MA} \cong \overline{TA}$, $\overline{SA} \cong \overline{RA}$	**2.** Definition of midpoint
3. $\angle MAS \cong \angle TAR$	**3.** Vertical Angles Theorem
4. $\triangle MAS \cong \triangle TAR$	**4.** SAS Congruence Postulate
5. $\angle M \cong \angle T$	**5.** Corresp. parts of $\cong$ $\triangle$s are $\cong$.
6. $\overline{MS} \parallel \overline{TR}$	**6.** Alternate Interior Angles Converse

EXAMPLE 3 *Using More than One Pair of Triangles*

GIVEN ▶ $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$

PROVE ▶ $\triangle BCE \cong \triangle DCE$



Plan for Proof The only information you have about $\triangle BCE$ and $\triangle DCE$ is that $\angle 1 \cong \angle 2$ and that $\overline{CE} \cong \overline{CE}$. Notice, however, that sides $\overline{BC}$ and $\overline{DC}$ are also sides of $\triangle ABC$ and $\triangle ADC$. If you can prove that $\triangle ABC \cong \triangle ADC$, you can use the fact that corresponding parts of congruent triangles are congruent to get a third piece of information about $\triangle BCE$ and $\triangle DCE$.

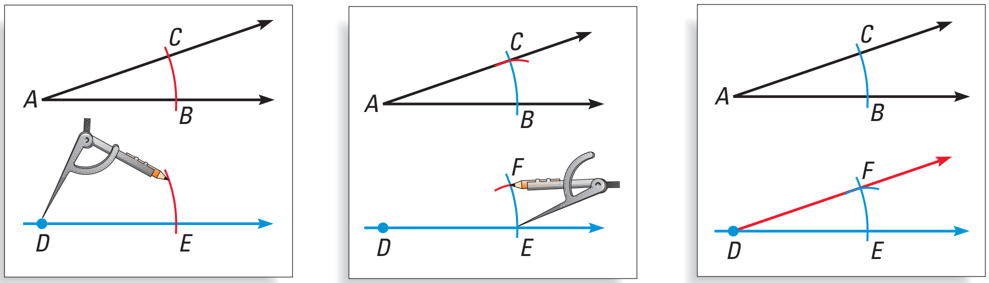
Statements	Reasons
1. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	**1.** Given
2. $\overline{AC} \cong \overline{AC}$	**2.** Reflexive Property of Congruence
3. $\triangle ABC \cong \triangle ADC$	**3.** ASA Congruence Postulate
4. $\overline{BC} \cong \overline{DC}$	**4.** Corresp. parts of $\cong$ $\triangle$s are $\cong$.
5. $\overline{CE} \cong \overline{CE}$	**5.** Reflexive Property of Congruence
6. $\triangle BCE \cong \triangle DCE$	**6.** SAS Congruence Postulate

GOAL 2 PROVING CONSTRUCTIONS ARE VALID

STUDENT HELP

Look Back
For help with copying an angle, see p. 159.

In Lesson 3.5, you learned how to copy an angle using a compass and a straightedge. The construction is summarized below. You can use congruent triangles to prove that this (and other) constructions are valid.



1 To copy $\angle A$, first draw a ray with initial point D. Then use the same compass setting to draw an arc with center A and an arc with center D. Label points B, C, and E.

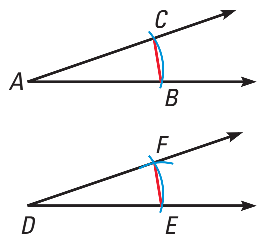
2 Draw an arc with radius BC and center E. Label the intersection F.

3 Draw $\overrightarrow{DF}$. $\angle FDE \cong \angle CAB$

EXAMPLE 4 *Proving a Construction*

Proof

Using the construction summarized above, you can copy $\angle CAB$ to form $\angle FDE$. Write a proof to verify that the construction is valid.



Plan for Proof Show that $\triangle CAB \cong \triangle FDE$. Then use the fact that corresponding parts of congruent triangles are congruent to conclude that $\angle CAB \cong \angle FDE$. By construction, you can assume the following statements as given.

$\overline{AB} \cong \overline{DE}$ **Same compass setting is used.**

$\overline{AC} \cong \overline{DF}$ **Same compass setting is used.**

$\overline{BC} \cong \overline{EF}$ **Same compass setting is used.**

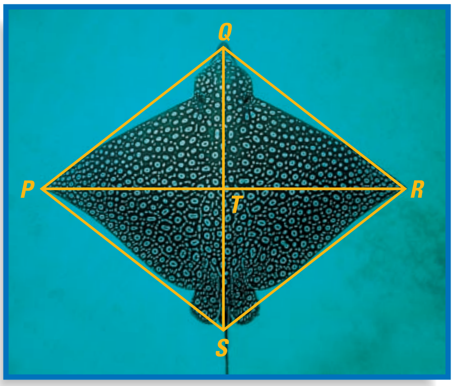
SOLUTION

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	**1.** Given
2. $\overline{AC} \cong \overline{DF}$	**2.** Given
3. $\overline{BC} \cong \overline{EF}$	**3.** Given
4. $\triangle CAB \cong \triangle FDE$	**4.** SSS Congruence Postulate
5. $\angle CAB \cong \angle FDE$	**5.** Corresp. parts of $\cong$ $\triangle$s are $\cong$.

GUIDED PRACTICE

Concept Check ✓

In Exercises 1–3, use the photo of the eagle ray.



1. To prove that $\angle PQT \cong \angle RQT$, which triangles might you prove to be congruent?
2. If you know that the opposite sides of figure $PQRS$ are parallel, can you prove that $\triangle PQT \cong \triangle RST$? Explain.

Skill Check ✓

3. The statements listed below are not in order. Use the photo to order them as statements in a two-column proof. Write a reason for each statement.

GIVEN ▶ $\overline{QS} \perp \overline{RP}$, $\overline{PT} \cong \overline{RT}$

PROVE ▶ $\overline{PS} \cong \overline{RS}$

A. $\overline{QS} \perp \overline{RP}$ **B.** $\triangle PTS \cong \triangle RTS$ **C.** $\angle PTS \cong \angle RTS$

D. $\overline{PS} \cong \overline{RS}$ **E.** $\overline{PT} \cong \overline{RT}$ **F.** $\overline{TS} \cong \overline{TS}$

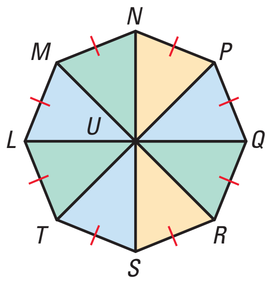
G. $\angle PTS$ and $\angle RTS$ are right angles.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 810.

STAINED GLASS WINDOW **The eight window panes in the diagram are isosceles triangles. The bases of the eight triangles are congruent.**



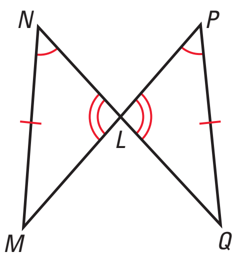
4. Explain how you know that $\triangle NUP \cong \triangle PUQ$.
5. Explain how you know that $\triangle NUP \cong \triangle QUR$.
6. Do you have enough information to prove that all the triangles are congruent? Explain.
7. Explain how you know that $\angle UNP \cong \angle UPQ$.

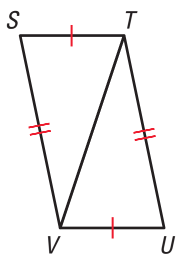
DEVELOPING PROOF **State which postulate or theorem you can use to prove that the triangles are congruent. Then explain how proving that the triangles are congruent proves the given statement.**

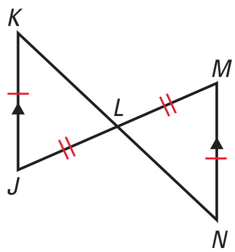
8. PROVE ▶ $\overline{ML} \cong \overline{QL}$

9. PROVE ▶ $\angle STV \cong \angle UVT$

10. PROVE ▶ $KL = NL$







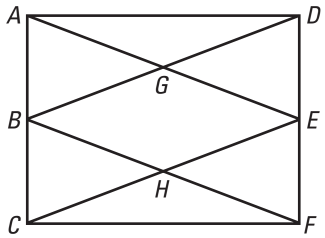
STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 4–14, 17, 18
Example 2: Exs. 14, 17, 18
Example 3: Exs. 15, 16
Example 4: Exs. 19–21

CAT'S CRADLE **Use the diagram of the string game Cat's Cradle and the information given below.**

GIVEN ▶ $\triangle EDA \cong \triangle BCF$
$\triangle AGD \cong \triangle FHC$
$\triangle BFC \cong \triangle ECF$



11. PROVE ▶ $\overline{GD} \cong \overline{HC}$

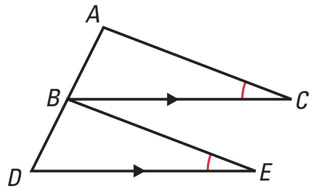
12. PROVE ▶ $\angle CBH \cong \angle FEH$

13. PROVE ▶ $\overline{AE} \cong \overline{FB}$

14. **DEVELOPING PROOF** Complete the proof that $\angle BAC \cong \angle DBE$.

GIVEN ▶ B is the midpoint of $\overline{AD}$,
$\angle C \cong \angle E$, $\overline{BC} \parallel \overline{DE}$

PROVE ▶ $\angle BAC \cong \angle DBE$

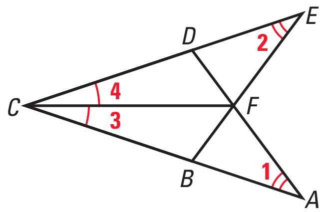


Statements	Reasons
1. B is the midpoint of $\overline{AD}$.	**1.** Given
2. $\overline{AB} \cong \overline{BD}$	**2.** ?
3. $\angle C \cong \angle E$	**3.** Given
4. $\overline{BC} \parallel \overline{DE}$	**4.** Given
5. $\angle EDB \cong \angle CBA$	**5.** ?
6. ?	**6.** AAS Congruence Theorem
7. $\angle BAC \cong \angle DBE$	**7.** ?

15. **DEVELOPING PROOF** Complete the proof that $\triangle AFB \cong \triangle EFD$.

GIVEN ▶ $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$

PROVE ▶ $\angle AFB \cong \angle EFD$



Statements	Reasons
1. $\angle 1 \cong \angle 2$	**1.** ?
2. $\angle 3 \cong \angle 4$	**2.** ?
3. ?	**3.** Reflexive Property of Congruence
4. $\triangle AFC \cong \triangle EFC$	**4.** ?
5. $\overline{AF} \cong \overline{EF}$	**5.** ?
6. ?	**6.** Vertical Angles Theorem
7. $\triangle AFB \cong \triangle EFD$	**7.** ?

FOCUS ON CAREER

REAL LIFE

CONSTRUCTION MANAGER

A construction manager plans and directs the work at a building site. Among other things, the manager reviews engineering specifications and architectural drawings to make sure that a project is proceeding according to plan.

INTERNET CAREER LINK
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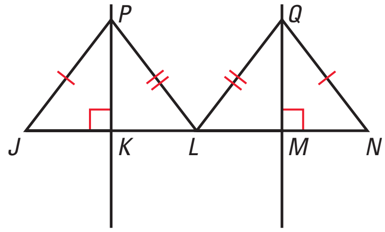
16. **BRIDGES** The diagram represents a section of the framework of the Kap Shui Mun Bridge shown in the photo on page 229. Write a two-column proof to show that $\triangle PKJ \cong \triangle QMN$.

GIVEN ▶ L is the midpoint of $\overline{JN}$,
$\overline{PJ} \cong \overline{QN}$, $\overline{PL} \cong \overline{QL}$,
$\angle PKJ$ and $\angle QMN$ are right angles.

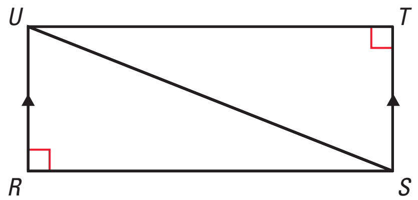
PROVE ▶ $\triangle PKJ \cong \triangle QMN$



PROOF **Write a two-column proof or a paragraph proof.**

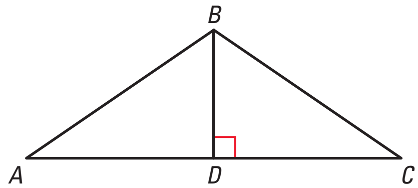
17. **GIVEN ▶** $\overline{UR} \parallel \overline{ST}$,
$\angle R$ and $\angle T$ are right angles.

PROVE ▶ $\angle RSU \cong \angle TUS$

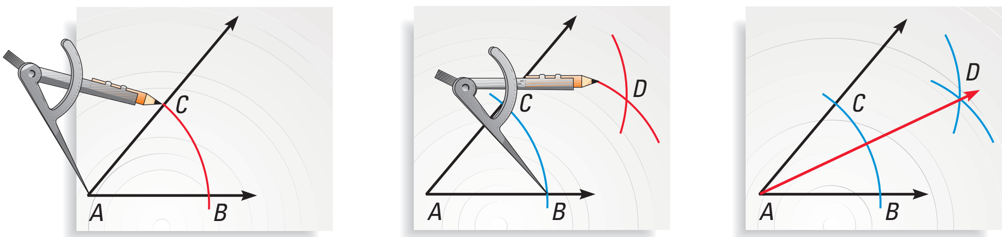


18. **GIVEN ▶** $\overline{BD} \perp \overline{AC}$,
$\overline{BD}$ bisects $\overline{AC}$.

PROVE ▶ $\angle ABD$ and $\angle BCD$ are complementary angles.



19. **PROVING A CONSTRUCTION** The diagrams below summarize the construction used to bisect $\angle A$. By construction, you can assume that $\overline{AB} \cong \overline{AC}$ and $\overline{BD} \cong \overline{CD}$. Write a proof to verify that $\overrightarrow{AD}$ bisects $\angle A$.



1 First draw an arc with center A. Label the points where the arc intersects the sides of the angle points B and C.

2 Draw an arc with center C. Using the same compass setting, draw an arc with center B. Label the intersection point D.

3 Draw $\overrightarrow{AD}$.
$\angle CAD \cong \angle BAD$

STUDENT HELP

Look Back
For help with bisecting an angle, see p. 36.

PROVING A CONSTRUCTION **Use a straightedge and a compass to perform the construction. Label the important points of your construction. Then write a flow proof to verify the results.**

20. Bisect an obtuse angle.

21. Copy an obtuse angle.

Test Preparation

22. **MULTIPLE CHOICE** Suppose $\overline{PQ} \parallel \overline{RS}$. You want to prove that $\overline{PR} \cong \overline{SQ}$. Which of the reasons below would *not* appear in your two-column proof?

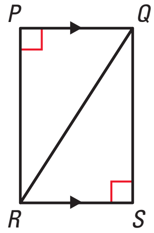
Ⓐ SAS Congruence Postulate

Ⓑ Reflexive Property of Congruence

Ⓒ AAS Congruence Theorem

Ⓓ Right Angle Congruence Theorem

Ⓔ Alternate Interior Angles Theorem



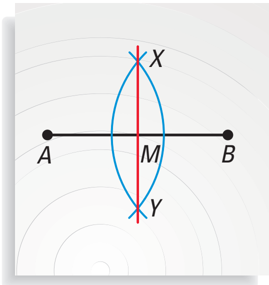
23. **MULTIPLE CHOICE** Which statement correctly describes the congruence of the triangles in the diagram in Exercise 22?

Ⓐ $\triangle SRQ \cong \triangle RQP$ Ⓑ $\triangle PRQ \cong \triangle SRQ$

Ⓒ $\triangle QRS \cong \triangle PQR$ Ⓓ $\triangle SRQ \cong \triangle PQR$

★ Challenge

24. **PROVING A CONSTRUCTION** Use a straightedge and a compass to bisect a segment. (For help with this construction, look back at page 34.) Then write a proof to show that the construction is valid.



EXTRA CHALLENGE
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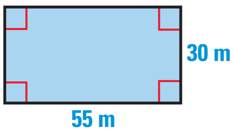
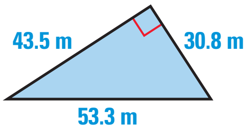
MIXED REVIEW

FINDING PERIMETER, CIRCUMFERENCE, AND AREA **Find the perimeter (or circumference) and area of the figure. (Where necessary, use $\pi \approx 3.14$.)** (Review 1.7)

25.


26.


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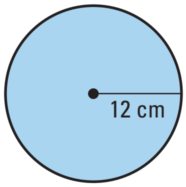


SOLVING EQUATIONS **Solve the equation and state a reason for each step.** (Review 2.4)

28. $x - 2 = 10$

29. $x + 11 = 21$

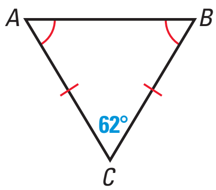
30. $9x + 2 = 29$

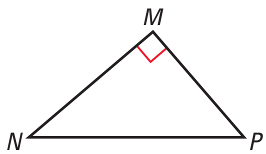
31. $8x + 13 = 3x + 38$

32. $3(x - 1) = 16$

33. $6(2x - 1) + 15 = 69$

IDENTIFYING PARTS OF TRIANGLES **Classify the triangle by its angles and by its sides. Identify the legs and the hypotenuse of any right triangles. Identify the legs and the base of any isosceles triangles.** (Review 4.1 for 4.6)

34.


35.


36.