4.4 Proving Triangles are Congruent: ASA and AAS

What *you should learn*

GOAL 1 Prove that triangles are congruent using the ASA Congruence Postulate and the AAS Congruence Theorem.

GOAL 2 Use congruence postulates and theorems in **real-life** problems, such as taking measurements for a map in **Exs. 24 and 25**.

Why *you should learn it*

▼ To solve **real-life** problems, such as finding the location of a meteorite in **Example 3**.

REAL LIFE

Lars Lindberg Christensen is an astronomer who participated in a search for a meteorite in Greenland.

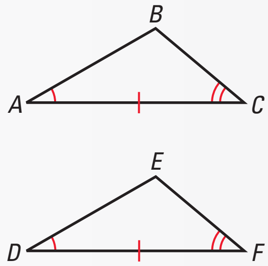
GOAL 1 USING THE ASA AND AAS CONGRUENCE METHODS

In Lesson 4.3, you studied the SSS and the SAS Congruence Postulates. Two additional ways to prove two triangles are congruent are listed below.

MORE WAYS TO PROVE TRIANGLES ARE CONGRUENT

POSTULATE 21 ***Angle-Side-Angle (ASA) Congruence Postulate***

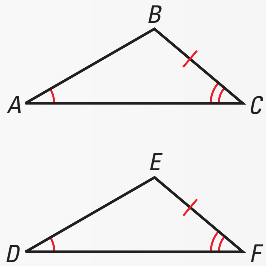
If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.



If **A**ngle $\angle A \cong \angle D$,
Side $\overline{AC} \cong \overline{DF}$, and
Angle $\angle C \cong \angle F$,
then $\triangle ABC \cong \triangle DEF$.

THEOREM 4.5 ***Angle-Angle-Side (AAS) Congruence Theorem***

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of a second triangle, then the two triangles are congruent.

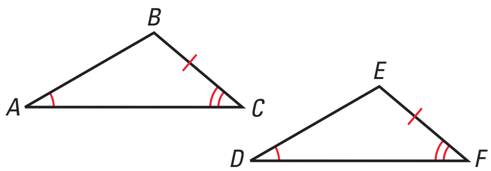


If **A**ngle $\angle A \cong \angle D$,
Angle $\angle C \cong \angle F$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.

A proof of the Angle-Angle-Side (AAS) Congruence Theorem is given below.

GIVEN ▶ $\angle A \cong \angle D$, $\angle C \cong \angle F$, $\overline{BC} \cong \overline{EF}$

PROVE ▶ $\triangle ABC \cong \triangle DEF$



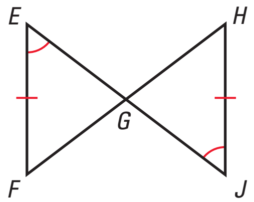
Paragraph Proof You are given that two angles of $\triangle ABC$ are congruent to two angles of $\triangle DEF$. By the Third Angles Theorem, the third angles are also congruent. That is, $\angle B \cong \angle E$. Notice that $\overline{BC}$ is the side included between $\angle B$ and $\angle C$, and $\overline{EF}$ is the side included between $\angle E$ and $\angle F$. You can apply the ASA Congruence Postulate to conclude that $\triangle ABC \cong \triangle DEF$.

Logical Reasoning

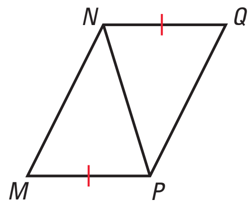
EXAMPLE 1 *Developing Proof*

Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use. Explain your reasoning.

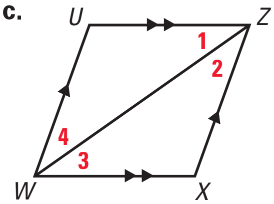
a.



b.



c.



> **STUDENT HELP**
>
> **Study Tip**
> In addition to the information that is marked on a diagram, you need to consider other pairs of angles or sides that may be congruent. For instance, look for vertical angles or a side that is shared by two triangles.

SOLUTION

a. In addition to the angles and segments that are marked, $\angle EGF \cong \angle JGH$ by the Vertical Angles Theorem. Two pairs of corresponding angles and one pair of corresponding sides are congruent. You can use the AAS Congruence Theorem to prove that $\triangle EFG \cong \triangle JHG$.

b. In addition to the congruent segments that are marked, $\overline{NP} \cong \overline{NP}$. Two pairs of corresponding sides are congruent. This is not enough information to prove that the triangles are congruent.

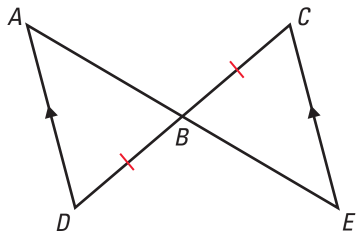
c. The two pairs of parallel sides can be used to show $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. Because the included side $\overline{WZ}$ is congruent to itself, $\triangle WUZ \cong \triangle ZXW$ by the ASA Congruence Postulate.

Proof

EXAMPLE 2 *Proving Triangles are Congruent*

GIVEN ▶ $\overline{AD} \parallel \overline{EC}$, $\overline{BD} \cong \overline{BC}$

PROVE ▶ $\triangle ABD \cong \triangle EBC$



Plan for Proof Notice that $\angle ABD$ and $\angle EBC$ are congruent. You are given that $\overline{BD} \cong \overline{BC}$. Use the fact that $\overline{AD} \parallel \overline{EC}$ to identify a pair of congruent angles.

Statements	Reasons
1. $\overline{BD} \cong \overline{BC}$	**1.** Given
2. $\overline{AD} \parallel \overline{EC}$	**2.** Given
3. $\angle D \cong \angle C$	**3.** Alternate Interior Angles Theorem
4. $\angle ABD \cong \angle EBC$	**4.** Vertical Angles Theorem
5. $\triangle ABD \cong \triangle EBC$	**5.** ASA Congruence Postulate

. . . .

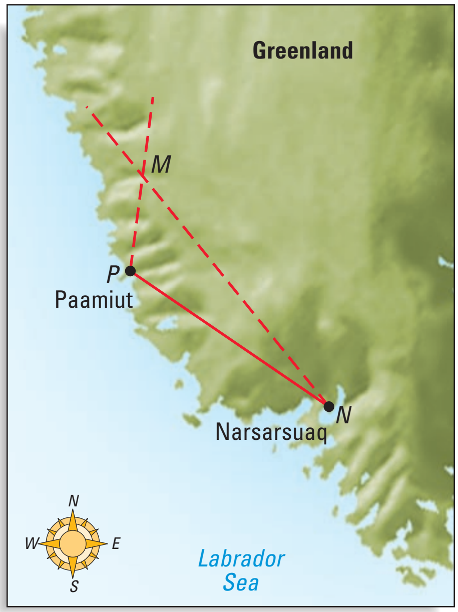
You can often use more than one method to prove a statement. In Example 2, you can use the parallel segments to show that $\angle D \cong \angle C$ and $\angle A \cong \angle E$. Then you can use the AAS Congruence Theorem to prove that the triangles are congruent.

GOAL 2 USING CONGRUENCE POSTULATES AND THEOREMS

EXAMPLE 3 *Using Properties of Congruent Triangles*

METEORITES On December 9, 1997, an extremely bright meteor lit up the sky above Greenland. Scientists attempted to find meteorite fragments by collecting data from eyewitnesses who had seen the meteor pass through the sky. As shown, the scientists were able to describe sightlines from observers in different towns. One sightline was from observers in Paamiut (Town P) and another was from observers in Narsarsuaq (Town N).

Assuming the sightlines were accurate, did the scientists have enough information to locate any meteorite fragments? Explain.



SOLUTION

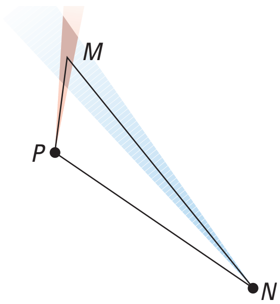
Think of Town P and Town N as two vertices of a triangle. The meteorite's position M is the other vertex. The scientists knew $m\angle P$ and $m\angle N$. They also knew the length of the included side $\overline{PN}$.

From the ASA Congruence Postulate, the scientists could conclude that any two triangles with these measurements are congruent. In other words, there is only one triangle with the given measurements and location.

▶ Assuming the sightlines were accurate, the scientists did have enough information to locate the meteorite fragments.

ACCURACY IN MEASUREMENT The conclusion in Example 3 depends on the assumption that the sightlines were accurate. If, however, the sightlines based on that information were only approximate, then the scientists could only narrow the meteorite's location to a region near point M.

For instance, if the angle measures for the sightlines were off by 2° in either direction, the meteorite's location would be known to lie within a region of about 25 square miles, which is a very large area to search.



In fact, the scientists looking for the meteorite searched over 1150 square miles of rough, icy terrain without finding any meteorite fragments.

FOCUS ON APPLICATIONS

REAL LIFE

METEORITES When a *meteoroid* (a piece of rocky or metallic matter from space) enters Earth's atmosphere, it heats up, leaving a trail of burning gases called a *meteor*. Meteoroid fragments that reach Earth without burning up are called *meteorites*.

Guided Practice

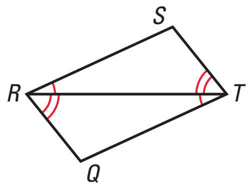
Vocabulary Check ✓

1. Name the four methods you have learned for proving triangles congruent. Only one of these is called a *theorem*. Why is it called a theorem?

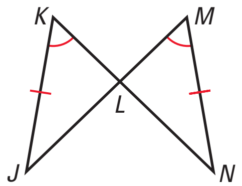
Concept Check ✓

Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use. Explain your reasoning.

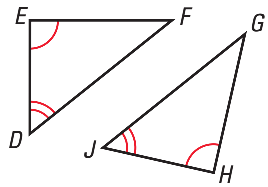
2. $\triangle RST$ and $\triangle TQR$



3. $\triangle JKL$ and $\triangle NML$



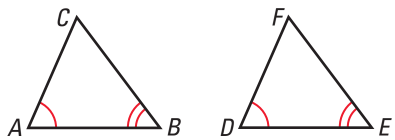
4. $\triangle DFE$ and $\triangle JGH$



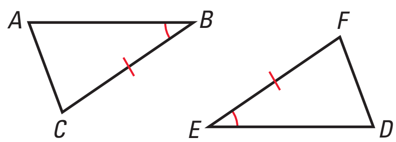
Skill Check ✓

State the third congruence that must be given to prove that $\triangle ABC \cong \triangle DEF$ using the indicated postulate or theorem.

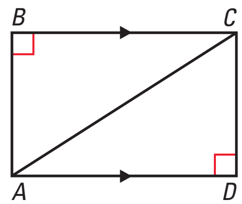
5. ASA Congruence Postulate



6. AAS Congruence Theorem



7. **RELAY RACE** A course for a relay race is marked on the gymnasium floor. Your team starts at A, goes to B, then C, then returns to A. The other team starts at C, goes to D, then A, then returns to C. Given that $\overline{AD} \parallel \overline{BC}$ and $\angle B$ and $\angle D$ are right angles, explain how you know the two courses are the same length.



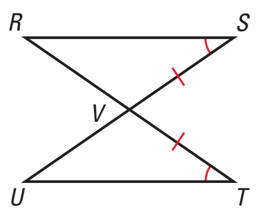
Practice and Applications

STUDENT HELP

Extra Practice to help you master skills is on pp. 809 and 810.

LOGICAL REASONING **Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use. Explain your reasoning.**

8.



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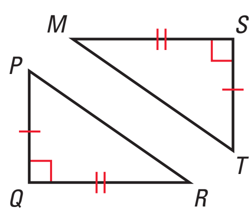

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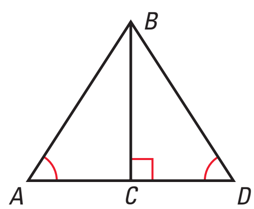

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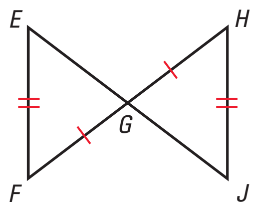

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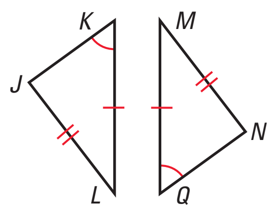

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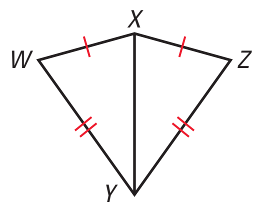


STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 8–13
Example 2: Exs. 14–22
Example 3: Exs. 23–25, 28

DEVELOPING PROOF **State the third congruence that must be given to prove that $\triangle PQR \cong \triangle STU$ using the indicated postulate or theorem. (*Hint:* First sketch $\triangle PQR$ and $\triangle STU$. Mark the triangles with the given information.)**

14. GIVEN ▶ $\angle Q \cong \angle T$, $\overline{PQ} \cong \overline{ST}$
Use the AAS Congruence Theorem.

15. GIVEN ▶ $\angle R \cong \angle U$, $\overline{PR} \cong \overline{SU}$
Use the ASA Congruence Postulate.

16. GIVEN ▶ $\angle R \cong \angle U$, $\angle P \cong \angle S$
Use the ASA Congruence Postulate.

17. GIVEN ▶ $\overline{PR} \cong \overline{SU}$, $\angle R \cong \angle U$
Use the SAS Congruence Postulate.

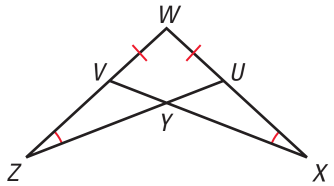
STUDENT HELP

Study Tip
When a proof involves overlapping triangles, such as the ones in Exs. 18 and 22, you may find it helpful to sketch the triangles separately.

18. **DEVELOPING PROOF** Complete the proof that $\triangle XWV \cong \triangle ZWU$.

GIVEN ▶ $\overline{VW} \cong \overline{UW}$
$\angle X \cong \angle Z$

PROVE ▶ $\triangle XWV \cong \triangle ZWU$

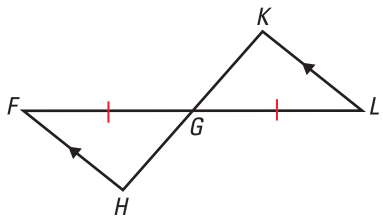


Statements	Reasons
1. $\overline{VW} \cong \overline{UW}$	**1.** ?
2. $\angle X \cong \angle Z$	**2.** ?
3. ?	**3.** Reflexive Property of Congruence
4. $\triangle XWV \cong \triangle ZWU$	**4.** ?

PROOF **Write a two-column proof or a paragraph proof.**

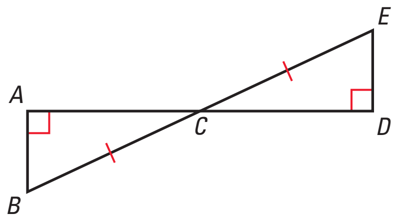
19. GIVEN ▶ $\overline{FH} \parallel \overline{LK}$,
$\overline{GF} \cong \overline{GL}$

PROVE ▶ $\triangle FGH \cong \triangle LGK$



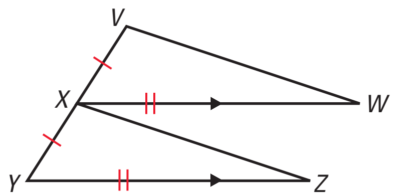
20. GIVEN ▶ $\overline{AB} \perp \overline{AD}$, $\overline{DE} \perp \overline{AD}$,
$\overline{BC} \cong \overline{EC}$

PROVE ▶ $\triangle ABC \cong \triangle DEC$



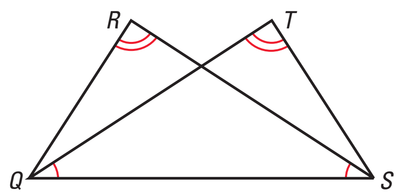
21. GIVEN ▶ $\overline{VX} \cong \overline{XY}$, $\overline{XW} \cong \overline{YZ}$,
$\overline{XW} \parallel \overline{YZ}$

PROVE ▶ $\triangle VXW \cong \triangle XYZ$



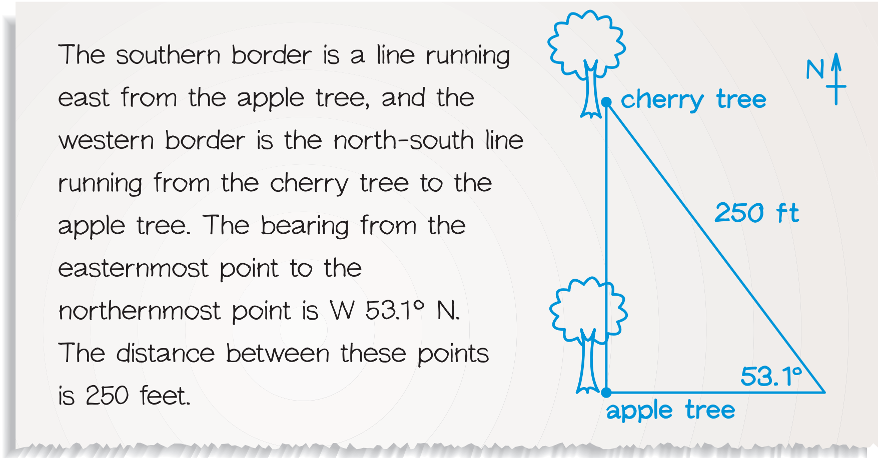
22. GIVEN ▶ $\angle TQS \cong \angle RSQ$,
$\angle R \cong \angle T$

PROVE ▶ $\triangle TQS \cong \triangle RSQ$



BEARINGS Use the information about bearings in Exercises 23–25. In surveying and orienteering, bearings convey information about direction. For example, the bearing W 53.1° N means 53.1° to the north of west. To find this bearing, face west. Then turn 53.1° to the north.

23. You want to describe the boundary lines of a triangular piece of property to a friend. You fax the note and the sketch below to your friend. Have you provided enough information to determine the boundary lines of the property? Explain.



24. A surveyor wants to make a map of several streets in a village. The surveyor finds that Green Street is on an east-west line. Plain Street is at a bearing of E 55° N from its intersection with Green Street. It runs 120 yards before intersecting Ellis Avenue. Ellis Avenue runs 100 yards between Green Street and Plain Street.

Assuming these measurements are accurate, what additional measurements, if any, does the surveyor need to make to draw Ellis Avenue correctly? Explain your reasoning.

25. You are creating a map for an orienteering race. Participants start out at a large oak tree, find a boulder that is 250 yards east of the oak tree, and then find an elm tree that is W 50° N of the boulder and E 35° N of the oak tree. Use this information to sketch a map. Do you have enough information to mark the position of the elm tree? Explain.

USING ALGEBRA Graph the equations in the same coordinate plane. Label the vertices of the two triangles formed by the lines. Show that the triangles are congruent.

26. $y = 0;\ y = x;\ y = -x + 3;\ y = 3$

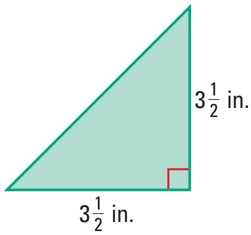
27. $y = 2;\ y = 6;\ x = 3;\ x = 5;\ y = 2x - 4$

FOCUS ON APPLICATIONS

REAL LIFE

ORIENTEERING In the sport of orienteering, participants use a map and a compass to navigate a course. Along the way, they travel to various points marked on the map.

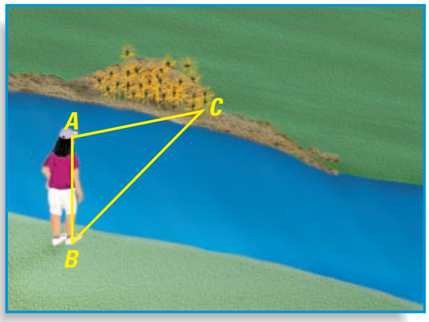
28. **QUILTING** You are making a quilt block out of congruent right triangles. Before cutting out each fabric triangle, you mark a right angle and the length of each leg, as shown. What theorem or postulate guarantees that the fabric triangles are congruent?

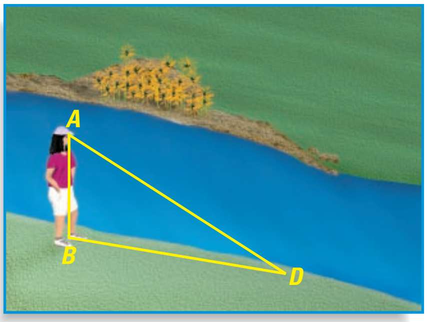


Test Preparation

29. **MULTI-STEP PROBLEM** You can use the method described below to approximate the distance across a stream without getting wet. As shown in the diagrams, you need a cap with a visor.

- Stand on the edge of the stream and look straight across to a point on the other edge of the stream. Adjust the visor of your cap so that it is in line with that point.
- Without changing the inclination of your neck and head, turn sideways until the visor is in line with a point on your side of the stream.
- Measure the distance *BD* between your feet and that point.





a. From the description of the measuring method, what corresponding parts of the two triangles can you assume are congruent?

b. What theorem or postulate can be used to show that the two triangles are congruent?

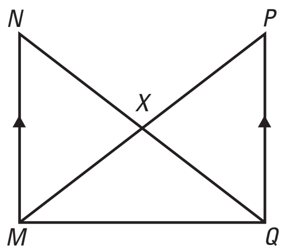
c. *Writing* Explain why the length of $\overline{BD}$ is also the distance across the stream.

★ **Challenge**

PROOF **Use the diagram.**

30. Alicia thinks that she can prove that $\triangle MNQ \cong \triangle QPM$ based on the information in the diagram. Explain why she cannot.

31. Suppose you are given that $\angle XMQ \cong \angle XQM$ and that $\angle N \cong \angle P$. Prove that $\triangle MNQ \cong \triangle QPM$.



EXTRA CHALLENGE
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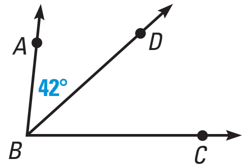
MIXED REVIEW

FINDING ENDPOINTS **Find the coordinates of the other endpoint of a segment with the given endpoint and midpoint *M*.** **(Review 1.5)**

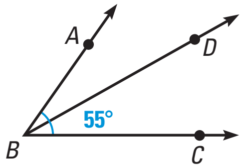
32. $B(5, 7), M(-1, 0)$ **33.** $C(0, 9), M(6, -2)$ **34.** $F(8, -5), M(-1, -3)$

USING ANGLE BISECTORS $\overrightarrow{BD}$ **is the angle bisector of** $\angle ABC$**. Find the two angle measures not given in the diagram.** **(Review 1.5 for 4.5)**

35.



36.



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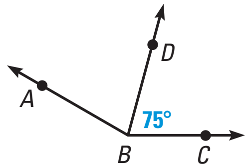
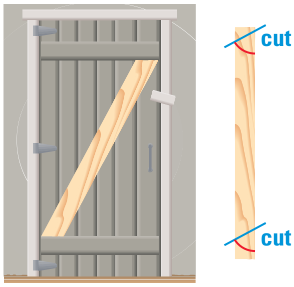


38. **BARN DOOR** You are making a brace for a barn door, as shown. The top and bottom pieces are parallel. To make the middle piece, you cut off the ends of a board at the same angle. What postulate or theorem guarantees that the cuts are parallel? **(Review 3.4)**

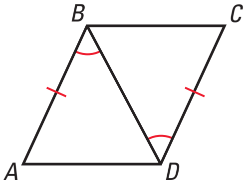


QUIZ 2

Self-Test for Lessons 4.3 and 4.4

In Exercises 1–6, decide whether it is possible to prove that the triangles are congruent. If it is possible, state the theorem or postulate you would use. Explain your reasoning. **(Lessons 4.3 and 4.4)**

1.



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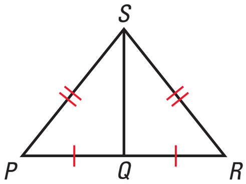

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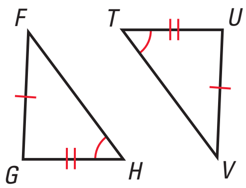

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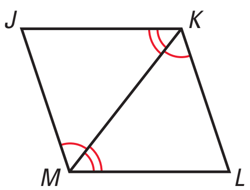
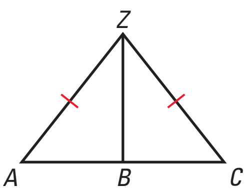
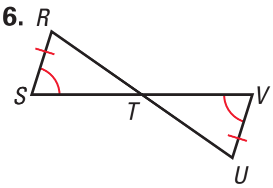


5.



6.



7. **PROOF** Write a two-column proof. **(Lesson 4.4)**

GIVEN ▶ M is the midpoint of $\overline{NL}$, $\overline{NL} \perp \overline{NQ}$, $\overline{NL} \perp \overline{MP}$, $\overline{QM} \parallel \overline{PL}$

PROVE ▶ $\triangle NQM \cong \triangle MPL$