4.3

Proving Triangles are Congruent: SSS and SAS

GOAL 1 SSS AND SAS CONGRUENCE POSTULATES

How much do you need to know about two triangles to prove that they are congruent? In Lesson 4.2, you learned that if all six pairs of corresponding parts (sides and angles) are congruent, then the triangles are congruent.
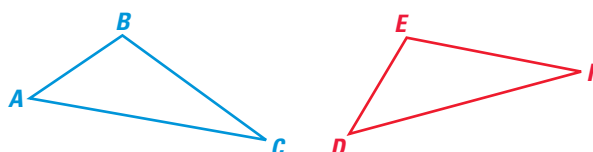
| If	Sides are congruent	and	Angles are congruent	then	Triangles are congruent
	1. $\overline{AB} \cong \overline{DE}$		4. $\angle A \cong \angle D$		$\triangle ABC \cong \triangle DEF$
	2. $\overline{BC} \cong \overline{EF}$		5. $\angle B \cong \angle E$		
	3. $\overline{AC} \cong \overline{DF}$		6. $\angle C \cong \angle F$		

In this lesson and the next, you will learn that you do not need all six of the pieces of information above to prove that the triangles are congruent. For example, if all three pairs of corresponding sides are congruent, then the *SSS Congruence Postulate* guarantees that the triangles are congruent.

POSTULATE

POSTULATE 19 *Side-Side-Side (SSS) Congruence Postulate*

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If **S**ide $\overline{MN} \cong \overline{QR}$,
  Side $\overline{NP} \cong \overline{RS}$, and
  Side $\overline{PM} \cong \overline{SQ}$,
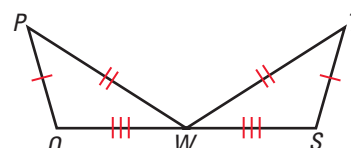then $\triangle MNP \cong \triangle QRS$.
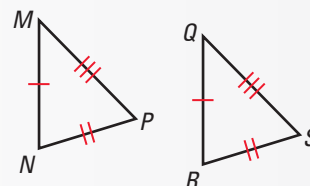
EXAMPLE 1 *Using the SSS Congruence Postulate*

Prove that $\triangle PQW \cong \triangle TSW$.

Paragraph Proof The marks on the diagram show that $\overline{PQ} \cong \overline{TS}$, $\overline{PW} \cong \overline{TW}$, and $\overline{QW} \cong \overline{SW}$.

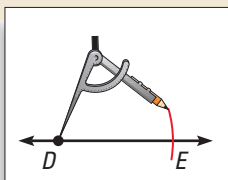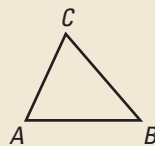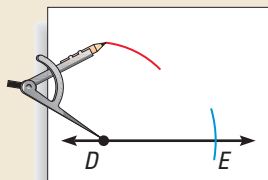▶ So, by the SSS Congruence Postulate, you know that $\triangle PQW \cong \triangle TSW$.
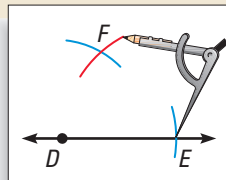
Construction   **Copying a Triangle**

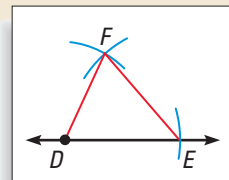**Follow the steps below to construct a triangle that is congruent to a given △ABC.**

① Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$. (See page 104 for the construction.)

② Open your compass to the length AC. Use this length to draw an arc with the compass point at D.

③ Draw an arc with radius BC and center E that intersects the arc from **Step 2**. Label the intersection point F.

④ Draw $\triangle DEF$. By the SSS Congruence Postulate, $\triangle ABC \cong \triangle DEF$.

The SSS Congruence Postulate is a shortcut for proving two triangles are congruent without using all six pairs of corresponding parts. The postulate below is a shortcut that uses two sides and the angle that is *included* between the sides.

STUDENT HELP

▶ **Study Tip**
In the triangle, $\angle B$ is the **included angle** between sides $\overline{AB}$ and $\overline{BC}$.
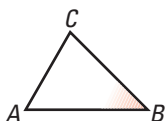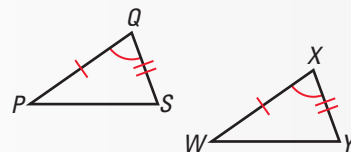
POSTULATE

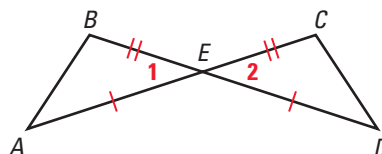**POSTULATE 20**   *Side-Angle-Side (SAS) Congruence Postulate*

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If   **S**ide $\overline{PQ} \cong \overline{WX}$,
     Angle $\angle Q \cong \angle X$, and
     Side $\overline{QS} \cong \overline{XY}$,
then $\triangle PQS \cong \triangle WXY$.

EXAMPLE 2   *Using the SAS Congruence Postulate*

Prove that $\triangle AEB \cong \triangle DEC$.

| Statements	Reasons
1. $\overline{AE} \cong \overline{DE}$, $\overline{BE} \cong \overline{CE}$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Vertical Angles Theorem
3. $\triangle AEB \cong \triangle DEC$	**3.** SAS Congruence Postulate

GOAL ② **MODELING A REAL-LIFE SITUATION**

Logical Reasoning

EXAMPLE 3 — *Choosing Which Congruence Postulate to Use*

Decide whether enough information is given in the diagram to prove that $\triangle PQR \cong \triangle PSR$. If there is enough information, state the congruence postulate you would use.
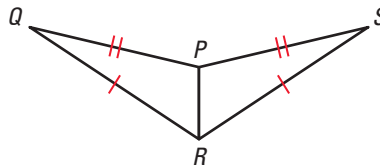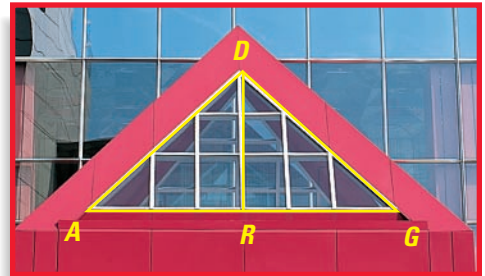
SOLUTION

Paragraph Proof The marks on the diagram show that $\overline{PQ} \cong \overline{PS}$ and $\overline{QR} \cong \overline{SR}$. By the Reflexive Property of Congruence, $\overline{RP} \cong \overline{RP}$. Because the sides of $\triangle PQR$ are congruent to the corresponding sides of $\triangle PSR$, you can use the SSS Congruence Postulate to prove that the triangles are congruent.

EXAMPLE 4 — *Proving Triangles Congruent*

REAL LIFE

ARCHITECTURE You are designing the window shown in the photo. You want to make $\triangle DRA$ congruent to $\triangle DRG$. You design the window so that $\overline{DR} \perp \overline{AG}$ and $\overline{RA} \cong \overline{RG}$. Can you conclude that $\triangle DRA \cong \triangle DRG$?
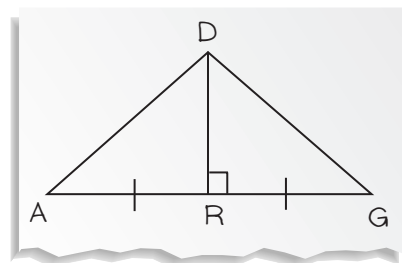
SOLUTION

To begin, copy the diagram and label it using the given information. Then write the given information and the statement you need to prove.

Proof

**GIVEN** ▶ $\overline{DR} \perp \overline{AG}$, $\overline{RA} \cong \overline{RG}$

PROVE ▶ $\triangle DRA \cong \triangle DRG$

Statements	Reasons
1. $\overline{DR} \perp \overline{AG}$	**1.** Given
2. $\angle DRA$ and $\angle DRG$ are right angles.	**2.** If 2 lines are $\perp$, then they form 4 rt. $\angle$s.
3. $\angle DRA \cong \angle DRG$	**3.** Right Angle Congruence Theorem
4. $\overline{RA} \cong \overline{RG}$	**4.** Given
5. $\overline{DR} \cong \overline{DR}$	**5.** Reflexive Property of Congruence
6. $\triangle DRA \cong \triangle DRG$	**6.** SAS Congruence Postulate

**EXAMPLE 5** *Triangular Frameworks are Rigid*

STRUCTURAL SUPPORT To prevent a doorway from collapsing after an earthquake, you can reinforce it. Explain why the doorway with the diagonal brace is more stable, while the one without the brace can collapse.
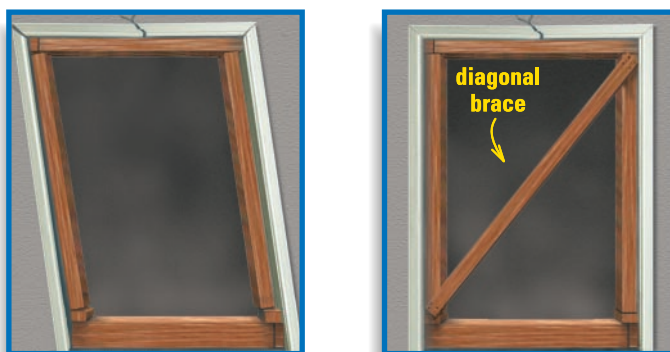
SOLUTION

In the doorway with the diagonal brace, the wood forms triangles whose sides have fixed lengths. The SSS Congruence Postulate guarantees that these triangles are rigid, because a triangle with given side lengths has only one possible size and shape. The doorway without the brace is unstable because there are many possible shapes for a four-sided figure with the given side lengths.

EXAMPLE 6 *Congruent Triangles in a Coordinate Plane*

Using Algebra

Use the SSS Congruence Postulate to show that $\triangle ABC \cong \triangle FGH$.
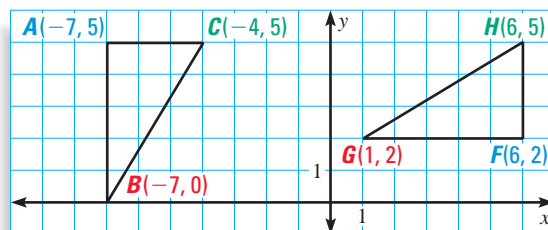
SOLUTION

Because $AC = 3$ and $FH = 3$, $\overline{AC} \cong \overline{FH}$. Because $AB = 5$ and $FG = 5$, $\overline{AB} \cong \overline{FG}$. Use the Distance Formula to find the lengths BC and GH.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BC = \sqrt{(-4 - (-7))^2 + (5 - 0)^2} \qquad GH = \sqrt{(6 - 1)^2 + (5 - 2)^2}$$

$$= \sqrt{3^2 + 5^2} \qquad\qquad\qquad = \sqrt{5^2 + 3^2}$$

$$= \sqrt{34} \qquad\qquad\qquad\qquad = \sqrt{34}$$

▶ Because $BC = \sqrt{34}$ and $GH = \sqrt{34}$, $\overline{BC} \cong \overline{GH}$. All three pairs of corresponding sides are congruent, so $\triangle ABC \cong \triangle FGH$ by the SSS Congruence Postulate.

STUDENT HELP

▶ **Look Back**
   For help with the Distance Formula, see page 19.

GUIDED PRACTICE

Vocabulary Check ✓

1. Sketch a triangle and label its vertices. Name two sides and the included angle between the sides.

Concept Check ✓

2. ERROR ANALYSIS Henry believes he can use the information given in the diagram and the SAS Congruence Postulate to prove the two triangles are congruent. Explain Henry's mistake.
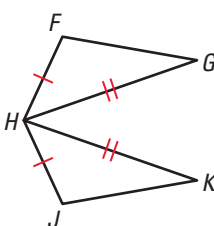
Skill Check ✓

🧩 **LOGICAL REASONING** Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, tell which congruence postulate you would use.

3. △ABC, △DEC
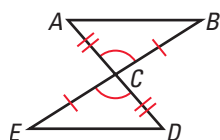
4. △FGH, △JKH

5. △PQR, △SRQ
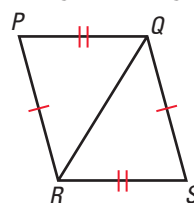
PRACTICE AND APPLICATIONS

NAMING SIDES AND INCLUDED ANGLES Use the diagram. Name the included angle between the pair of sides given.
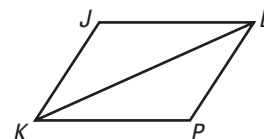
6. $\overline{JK}$ and $\overline{KL}$

7. $\overline{PK}$ and $\overline{LK}$

8. $\overline{LP}$ and $\overline{LK}$

9. $\overline{JL}$ and $\overline{JK}$

10. $\overline{KL}$ and $\overline{JL}$

11. $\overline{KP}$ and $\overline{PL}$

🧩 **LOGICAL REASONING** Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate you would use.

12. △UVT, △WVT
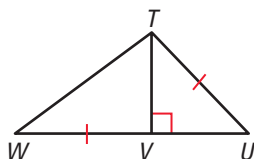
13. △LMN, △TNM
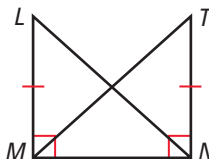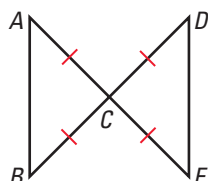
14. △YZW, △YXW
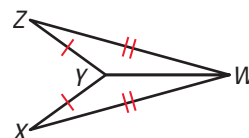
15. △ACB, △ECD

16. △RST, △WVU
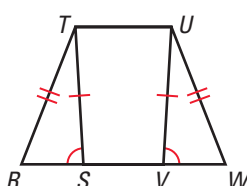
17. △GJH, △HLK
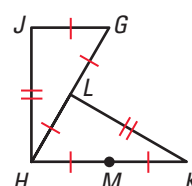
DEVELOPING PROOF In Exercises 18 and 19, use the photo of the Navajo rug. Assume that $\overline{BC} \cong \overline{DE}$ and $\overline{AC} \cong \overline{CE}$.
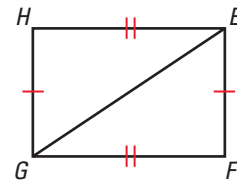
18. What other piece of information is needed to prove that $\triangle ABC \cong \triangle CDE$ using the SSS Congruence Postulate?

19. What other piece of information is needed to prove that $\triangle ABC \cong \triangle CDE$ using the SAS Congruence Postulate?
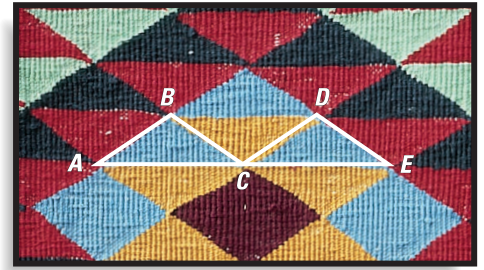
20. **DEVELOPING PROOF** Complete the proof by supplying the reasons.

GIVEN ▶ $\overline{EF} \cong \overline{GH}$,
$\overline{FG} \cong \overline{HE}$
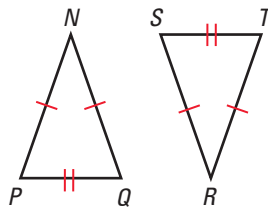
PROVE ▶ $\triangle EFG \cong \triangle GHE$

Statements	Reasons
1. $\overline{EF} \cong \overline{GH}$	**1.** ___?___
2. $\overline{FG} \cong \overline{HE}$	**2.** ___?___
3. $\overline{GE} \cong \overline{GE}$	**3.** ___?___
4. $\triangle EFG \cong \triangle GHE$	**4.** ___?___

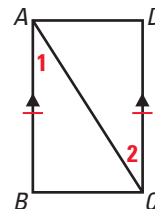**TWO-COLUMN PROOF** Write a two-column proof.

21. **GIVEN** ▶ $\overline{NP} \cong \overline{QN} \cong \overline{RS} \cong \overline{TR}$,
$\overline{PQ} \cong \overline{ST}$

PROVE ▶ $\triangle NPQ \cong \triangle RST$

22. **GIVEN** ▶ $\overline{AB} \cong \overline{CD}, \overline{AB} \parallel \overline{CD}$
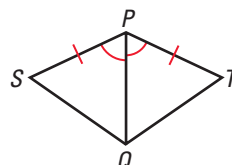
PROVE ▶ $\triangle ABC \cong \triangle CDA$

PARAGRAPH PROOF Write a paragraph proof.

23. **GIVEN** ▶ $\overrightarrow{PQ}$ bisects $\angle SPT$,
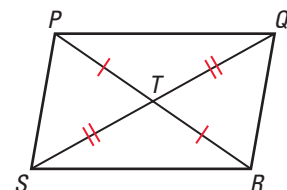$\overline{SP} \cong \overline{TP}$

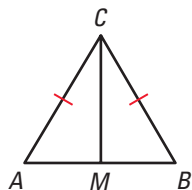**PROVE** ▶ $\triangle SPQ \cong \triangle TPQ$

24. **GIVEN** ▶ $\overline{PT} \cong \overline{RT}, \overline{QT} \cong \overline{ST}$

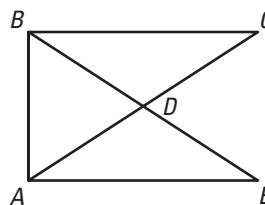**PROVE** ▶ $\triangle PQT \cong \triangle RST$

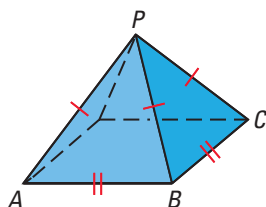⊙ **PROOF** Write a two-column proof or a paragraph proof.

25. GIVEN ▶ $\overline{AC} \cong \overline{BC}$,
M is the midpoint of $\overline{AB}$.
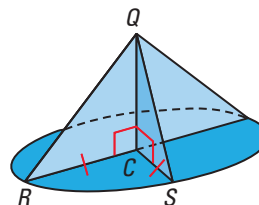**PROVE** ▶ $\triangle ACM \cong \triangle BCM$

26. GIVEN ▶ $\overline{BC} \cong \overline{AE}$, $\overline{BD} \cong \overline{AD}$,
$\overline{DE} \cong \overline{DC}$
PROVE ▶ $\triangle ABC \cong \triangle BAE$

27. GIVEN ▶ $\overline{PA} \cong \overline{PB} \cong \overline{PC}$,
$\overline{AB} \cong \overline{BC}$
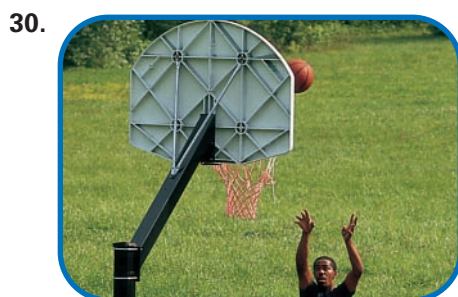**PROVE** ▶ $\triangle PAB \cong \triangle PBC$

28. GIVEN ▶ $\overline{CR} \cong \overline{CS}$, $\overline{QC} \perp \overline{CR}$,
$\overline{QC} \perp \overline{CS}$
**PROVE** ▶ $\triangle QCR \cong \triangle QCS$

29. 🖳 **TECHNOLOGY** Use geometry software to draw a triangle. Draw a line and reflect the triangle across the line. Measure the sides and the angles of the new triangle and tell whether it is congruent to the original one.

Writing **Explain how triangles are used in the object shown to make it more stable.**

30.

31.

32. 📐 **CONSTRUCTION** Draw an isosceles triangle with vertices *A*, *B*, and *C*. Use a compass and straightedge to construct $\triangle DEF$ so that $\triangle DEF \cong \triangle ABC$.

⊗y **USING ALGEBRA** Use the Distance Formula and the SSS Congruence Postulate to show that $\triangle ABC \cong \triangle DEF$.
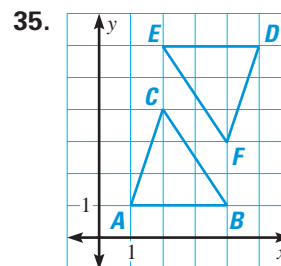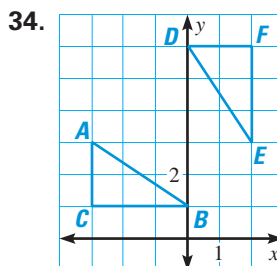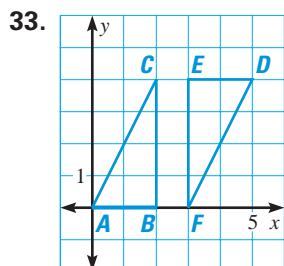
33.

34.

35.

**36. MULTIPLE CHOICE** In △RST and △ABC, $\overline{RS} \cong \overline{AB}$, $\overline{ST} \cong \overline{BC}$, and $\overline{TR} \cong \overline{CA}$. Which angle is congruent to ∠T?

Ⓐ ∠R   Ⓑ ∠A   Ⓒ ∠C   Ⓓ cannot be determined

37. MULTIPLE CHOICE In equilateral △DEF, a segment is drawn from point F to G, the midpoint of $\overline{DE}$. Which of the statements below is *not* true?

Ⓐ $\overline{DF} \cong \overline{EF}$ Ⓑ $\overline{DG} \cong \overline{DF}$ Ⓒ $\overline{DG} \cong \overline{EG}$ Ⓓ △DFG ≅ △EFG

38. CHOOSING A METHOD Describe how to show that △PMO ≅ △PMN using the SSS Congruence Postulate. Then find a way to show that the triangles are congruent using the SAS Congruence Postulate. You may not use a protractor to measure any angles. Compare the two methods. Which do you prefer? Why?
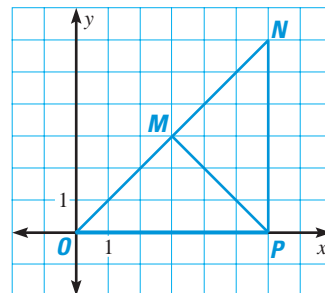
MIXED REVIEW

SCIENCE CONNECTION **Find an important angle in the photo. Copy the angle, extend its sides, and use a protractor to measure it to the nearest degree.** (Review 1.4)
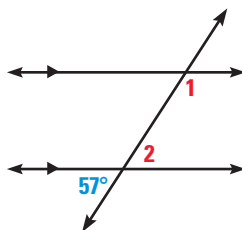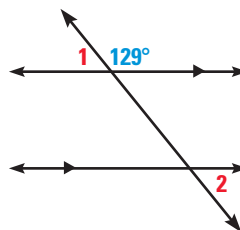
39.

40.

USING PARALLEL LINES **Find m∠1 and m∠2. Explain your reasoning.** (Review 3.3 for 4.4)
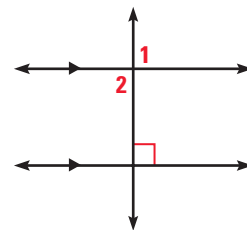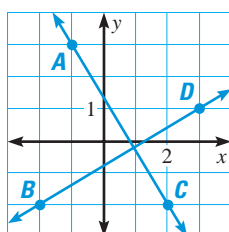
41.
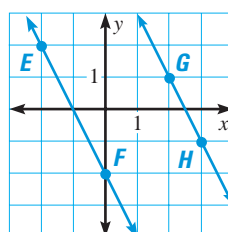
57°

42.

1 129°

43.

LINE RELATIONSHIPS **Find the slope of each line. Identify any parallel or perpendicular lines.** (Review 3.7)

44.

45.

46.