

CHAPTER 3

Chapter Summary

WHAT did you learn?

Identify relationships between lines. (3.1)

Identify angles formed by coplanar lines intersected by a transversal. (3.1)

Prove and use results about perpendicular lines. (3.2)

Write flow proofs and paragraph proofs. (3.2)

Prove and use results about parallel lines and transversals. (3.3)

Prove that lines are parallel. (3.4)

Use properties of parallel lines. (3.4, 3.5)

Use slope to decide whether lines in a coordinate plane are parallel. (3.6)

Write an equation of a line parallel to a given line in a coordinate plane. (3.6)

Use slope to decide whether lines in a coordinate plane are perpendicular. (3.7)

Write an equation of a line perpendicular to a given line. (3.7)

WHY did you learn it?

Describe lines and planes in real-life objects, such as escalators. (p. 133)

Lay the foundation for work with angles and proof.

Solve real-life problems, such as deciding how many angles of a window frame to measure. (p. 141)

Learn to write and use different types of proof.

Understand the world around you, such as how rainbows are formed. (p. 148)

Solve real-life problems, such as predicting paths of sailboats. (p. 152)

Analyze light passing through glass. (p. 163)

Use coordinate geometry to show that two segments are parallel. (p. 170)

Prepare to write coordinate proofs.

Solve real-life problems, such as deciding whether two stitched lines form a right angle. (p. 176)

Find the distance from a point to a line. (p. 177)

How does Chapter 3 fit into the BIGGER PICTURE of geometry?

In this chapter, you learned about properties of perpendicular and parallel lines. You also learned to write flow proofs and learned some important skills related to coordinate geometry. This work will prepare you to reach conclusions about triangles and other figures and to solve real-life problems in areas such as carpentry, engineering, and physics.

STUDY STRATEGY

How did your study questions help you learn?

The study questions you wrote, following the study strategy on page 128, may resemble this one.

Lines and Angles

1. If two lines do not intersect, can you conclude they are parallel?
2. What is the slope of a line perpendicular to $2x - 3y = 6$?
3. If a transversal intersects two parallel lines, which angles are supplementary?

CHAPTER 3

Chapter Review

VOCABULARY

- parallel lines, p. 129
- transversal, p. 131
- alternate exterior angles, p. 131
- same side interior angles, p. 131
- skew lines, p. 129
- corresponding angles, p. 131
- consecutive interior angles, p. 131
- flow proof, p. 136
- parallel planes, p. 129
- alternate interior angles, p. 131

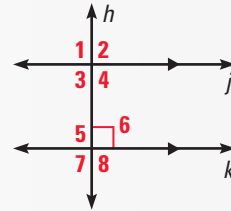
3.1

LINES AND ANGLES

Examples on
pp. 129–131

EXAMPLES In the figure, $j \parallel k$, h is a transversal, and $h \perp k$.

- $\angle 1$ and $\angle 5$ are corresponding angles.
- $\angle 3$ and $\angle 6$ are alternate interior angles.
- $\angle 1$ and $\angle 8$ are alternate exterior angles.
- $\angle 4$ and $\angle 6$ are consecutive interior angles.

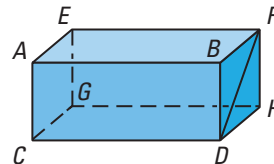


Complete the statement. Use the figure above.

- $\angle 2$ and $\angle 7$ are ? angles.
- $\angle 4$ and $\angle 5$ are ? angles.

Use the figure at the right.

- Name a line parallel to \overleftrightarrow{DH} .
- Name a line perpendicular to \overleftrightarrow{AE} .
- Name a line skew to \overleftrightarrow{FD} .



3.2

PROOF AND PERPENDICULAR LINES

Examples on
pp. 136–138

EXAMPLE **GIVEN** $\angle 1$ and $\angle 2$ are complements.

PROVE $\overleftrightarrow{GH} \perp \overleftrightarrow{GJ}$

$\angle 1$ and $\angle 2$ are
complements.

 ?

$$m\angle 1 + m\angle 2 = 90^\circ$$

 ?

$$m\angle 1 + m\angle 2 = m\angle HGJ$$

 ?

$$m\angle HGJ = 90^\circ$$

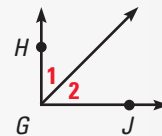
 ?

$\angle HGJ$ is a right \angle .

 ?

$\overleftrightarrow{GH} \perp \overleftrightarrow{GJ}$

 ?



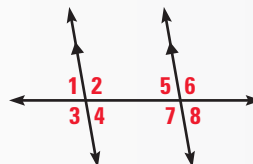
- Copy the flow proof and add a reason for each statement.

3.3

PARALLEL LINES AND TRANSVERSALS

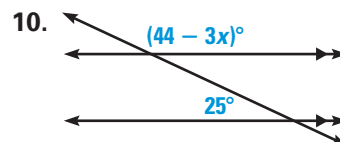
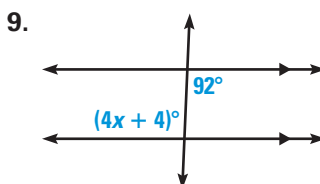
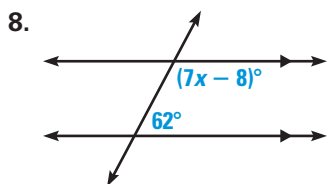
Examples on
pp. 143–145

EXAMPLE In the diagram, $m\angle 1 = 75^\circ$.
By the Alternate Exterior Angles Theorem,
 $m\angle 8 = m\angle 1 = 75^\circ$. Because $\angle 8$ and $\angle 7$
are a linear pair, $m\angle 8 + m\angle 7 = 180^\circ$.
So, $m\angle 7 = 180^\circ - 75^\circ = 105^\circ$.



7. Find the measures of the other five angles in the diagram above.

Find the value of x . Explain your reasoning.



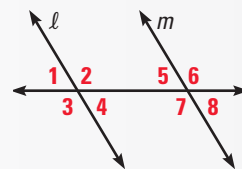
3.4

PROVING LINES ARE PARALLEL

Examples on
pp. 150–152

EXAMPLE **GIVEN** $\triangleright m\angle 3 = 125^\circ, m\angle 6 = 125^\circ$
PROVE $\triangleright \ell \parallel m$

Plan for Proof: $m\angle 3 = 125^\circ = m\angle 6$, so $\angle 3 \cong \angle 6$.
So, $\ell \parallel m$ by the Alternate Exterior Angles Converse.



Use the diagram above to write a proof.

11. **GIVEN** $\triangleright m\angle 4 = 60^\circ, m\angle 7 = 120^\circ$

PROVE $\triangleright \ell \parallel m$

12. **GIVEN** $\triangleright \angle 1$ and $\angle 7$ are supplementary.

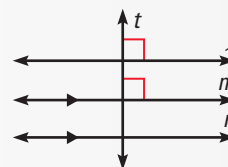
PROVE $\triangleright \ell \parallel m$

3.5

USING PROPERTIES OF PARALLEL LINES

Examples on
pp. 157–159

EXAMPLE In the diagram, $\ell \perp t$, $m \perp t$, and $m \parallel n$.
Because ℓ and m are coplanar and perpendicular to the
same line, $\ell \parallel m$. Then, because $\ell \parallel m$ and $m \parallel n$, $\ell \parallel n$.



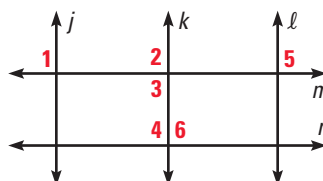
Which lines must be parallel? Explain.

13. $\angle 1$ and $\angle 2$ are right angles.

14. $\angle 3 \cong \angle 6$

15. $\angle 3$ and $\angle 4$ are supplements.

16. $\angle 1 \cong \angle 2, \angle 3 \cong \angle 5$



PARALLEL LINES IN THE COORDINATE PLANE

Examples on
pp. 165–167**EXAMPLES** slope of $\ell_1 = \frac{2-0}{1-0} = 2$

$$\text{slope of } \ell_2 = \frac{3-(-1)}{5-3} = \frac{4}{2} = 2$$

The slopes are the same, so $\ell_1 \parallel \ell_2$.

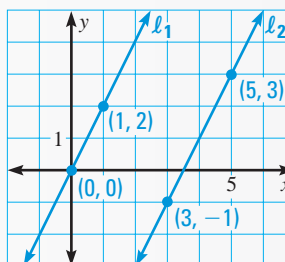
To write an equation for ℓ_2 , substitute $(x, y) = (5, 3)$ and $m = 2$ into the slope-intercept form.

$$y = mx + b \quad \text{Slope-intercept form.}$$

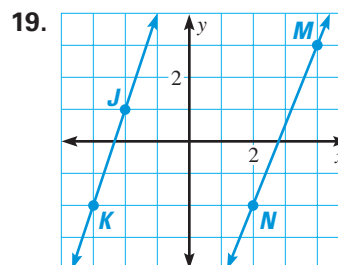
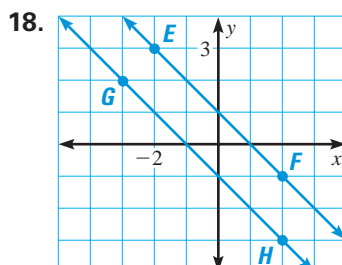
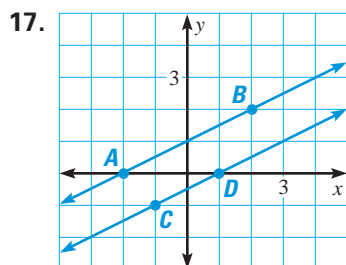
$$3 = (2)(5) + b \quad \text{Substitute 5 for } x, 3 \text{ for } y, \text{ and } 2 \text{ for } m.$$

$$-7 = b \quad \text{Solve for } b.$$

► So, an equation for ℓ_2 is $y = 2x - 7$.



Find the slope of each line. Are the lines parallel?



20. Find an equation of the line that is parallel to the line with equation $y = -2x + 5$ and passes through the point $(-1, -4)$.

PERPENDICULAR LINES IN THE COORDINATE PLANE

Examples on
pp. 172–174

EXAMPLE The slope of line j is 3. The slope of line k is $-\frac{1}{3}$.

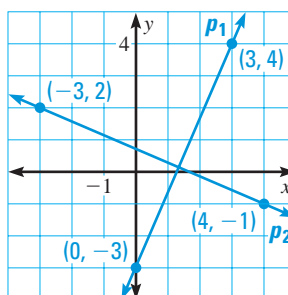
$$3\left(-\frac{1}{3}\right) = -1, \text{ so } j \perp k.$$

In Exercises 21–23, decide whether lines p_1 and p_2 are perpendicular.

21. Lines p_1 and p_2 in the diagram.

22. $p_1: y = \frac{3}{5}x + 2$; $p_2: y = \frac{5}{3}x - 1$

23. $p_1: 2y - x = 2$; $p_2: y + 2x = 4$

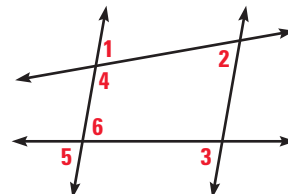


24. Line ℓ_1 has equation $y = -3x + 5$. Write an equation of line ℓ_2 which is perpendicular to ℓ_1 and passes through $(-3, 6)$.

CHAPTER 3

Chapter Test

In Exercises 1–6, identify the relationship between the angles in the diagram at the right.

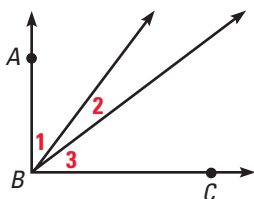


1. $\angle 1$ and $\angle 2$
2. $\angle 1$ and $\angle 4$
3. $\angle 2$ and $\angle 3$
4. $\angle 1$ and $\angle 5$
5. $\angle 4$ and $\angle 2$
6. $\angle 5$ and $\angle 6$

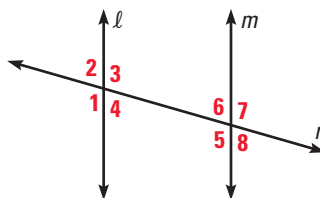
7. Write a flow proof.

GIVEN $m\angle 1 = m\angle 3 = 37^\circ$, $\overrightarrow{BA} \perp \overrightarrow{BC}$

PROVE $m\angle 2 = 16^\circ$

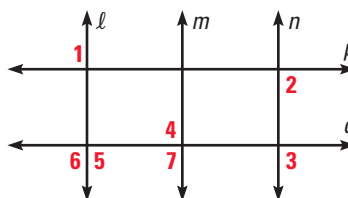


8. If $\ell \parallel m$, which angles are supplementary to $\angle 1$?



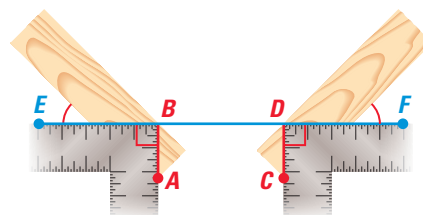
Use the given information and the diagram at the right to determine which lines must be parallel.

9. $\angle 1 \cong \angle 2$
10. $\angle 3$ and $\angle 4$ are right angles.
11. $\angle 1 \cong \angle 5$; $\angle 5$ and $\angle 7$ are supplementary.



In Exercises 12 and 13, write an equation of the line described.

12. The line parallel to $y = -\frac{1}{3}x + 5$ and with a y-intercept of 1
13. The line perpendicular to $y = -2x + 4$ and that passes through the point $(-1, 2)$
14. **Writing** Describe a real-life object that has edges that are straight lines. Are any of the lines skew? If so, describe a pair.
15. A carpenter wants to cut two boards to fit snugly together. The carpenter's squares are aligned along \overline{EF} , as shown. Are \overline{AB} and \overline{CD} parallel? State the theorem that justifies your answer.



16. Use the diagram to write a proof.

GIVEN $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

PROVE $n \parallel p$

