3.7 Perpendicular Lines in the Coordinate Plane

What you should learn

GOAL 1 Use slope to identify perpendicular lines in a coordinate plane.

GOAL 2 Write equations of perpendicular lines, as applied in **Ex. 46**.

Why you should learn it

▼ Equations of perpendicular lines are used by ray tracing software to create realistic reflections, as in the illustration below and in **Example 6**.

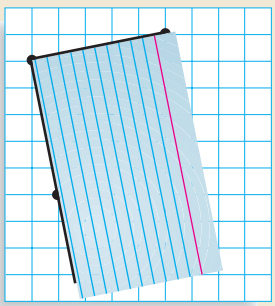
GOAL 1 SLOPE OF PERPENDICULAR LINES

In the activity below, you will trace a piece of paper to draw perpendicular lines on a coordinate grid. Points where grid lines cross are called *lattice points*.

ACTIVITY Developing Concepts

Investigating Slopes of Perpendicular Lines

1. Put the corner of a piece of paper on a lattice point. Rotate the corner so each edge passes through another lattice point but neither edge is vertical. Trace the edges.
2. Find the slope of each line.
3. Multiply the slopes.
4. Repeat Steps 1–3 with the paper at a different angle.



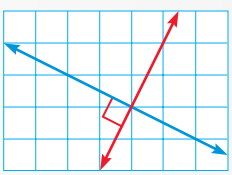
In the activity, you may have discovered the following.

POSTULATE

POSTULATE 18 ***Slopes of Perpendicular Lines***

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Vertical and horizontal lines are perpendicular.



product of slopes $= 2\left(-\frac{1}{2}\right) = -1$

EXAMPLE 1 ***Deciding Whether Lines are Perpendicular***

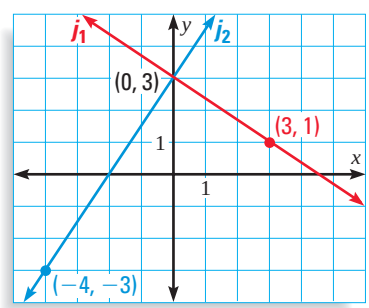
Find each slope.

$$\text{Slope of } j_1 = \frac{3-1}{0-3} = -\frac{2}{3}$$

$$\text{Slope of } j_2 = \frac{3-(-3)}{0-(-4)} = \frac{6}{4} = \frac{3}{2}$$

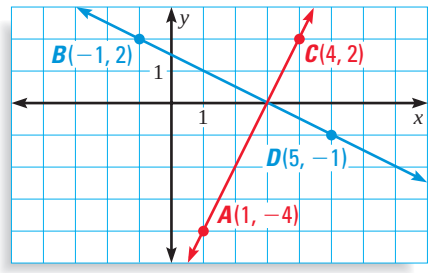
Multiply the slopes.

The product is $\left(-\frac{2}{3}\right)\left(\frac{3}{2}\right) = -1$, so $j_1 \perp j_2$.



EXAMPLE 2 Deciding Whether Lines are Perpendicular

Decide whether $\overleftrightarrow{AC}$ and $\overleftrightarrow{DB}$ are perpendicular.



SOLUTION

Slope of $\overleftrightarrow{AC} = \frac{2-(-4)}{4-1} = \frac{6}{3} = 2$

Slope of $\overleftrightarrow{DB} = \frac{2-(-1)}{-1-5} = \frac{3}{-6} = -\frac{1}{2}$

The product is $2\left(-\frac{1}{2}\right) = -1$, so $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$.

EXAMPLE 3 Deciding Whether Lines are Perpendicular

Decide whether the lines are perpendicular.

line *h*: $y = \frac{3}{4}x + 2$ **line *j*:** $y = -\frac{4}{3}x - 3$

SOLUTION

The slope of line h is $\frac{3}{4}$. The slope of line j is $-\frac{4}{3}$.

The product is $\left(\frac{3}{4}\right)\left(-\frac{4}{3}\right) = -1$, so the lines are perpendicular.

EXAMPLE 4 Deciding Whether Lines are Perpendicular

Decide whether the lines are perpendicular.

line *r*: $4x + 5y = 2$ **line *s*:** $5x + 4y = 3$

SOLUTION

Rewrite each equation in slope-intercept form to find the slope.

line *r*:

$4x + 5y = 2$

$5y = -4x + 2$

$y = -\frac{4}{5}x + \frac{2}{5}$

slope $= -\frac{4}{5}$

line *s*:

$5x + 4y = 3$

$4y = -5x + 3$

$y = -\frac{5}{4}x + \frac{3}{4}$

slope $= -\frac{5}{4}$

Multiply the slopes to see if the lines are perpendicular.

$\left(-\frac{4}{5}\right)\left(-\frac{5}{4}\right) = 1$

▶ The product of the slopes is *not* -1. So, r and s are *not* perpendicular.

GOAL 2 WRITING EQUATIONS OF PERPENDICULAR LINES

STUDENT HELP

Study Tip

You can check m_2 by multiplying $m_1 \cdot m_2$.

$(-2)\left(\frac{1}{2}\right) = -1$ ✓

EXAMPLE 5 *Writing the Equation of a Perpendicular Line*

Line ℓ_1 has equation $y = -2x + 1$. Find an equation of the line ℓ_2 that passes through $P(4, 0)$ and is perpendicular to ℓ_1. First you must find the slope, m_2.

$m_1 \cdot m_2 = -1$ — The product of the slopes of $\perp$ lines is -1.

$-2 \cdot m_2 = -1$ — The slope of ℓ_1 is -2.

$m_2 = \frac{1}{2}$ — Divide both sides by -2.

Then use $m = \frac{1}{2}$ and $(x, y) = (4, 0)$ to find b.

$y = mx + b$ — Slope-intercept form

$0 = \frac{1}{2}(4) + b$ — Substitute 0 for y, $\frac{1}{2}$ for m, and 4 for x.

$-2 = b$ — Simplify.

▶ So, an equation of ℓ_2 is $y = \frac{1}{2}x - 2$.

RAY TRACING Computer illustrators use *ray tracing* to make accurate reflections. To figure out what to show in the mirror, the computer traces a ray of light as it reflects off the mirror. This calculation has many steps. One of the first steps is to find the equation of a line perpendicular to the mirror.

EXAMPLE 6 *Writing the Equation of a Perpendicular Line*

The equation $y = \frac{3}{2}x + 3$ represents a mirror. A ray of light hits the mirror at $(-2, 0)$. What is the equation of the line p that is perpendicular to the mirror at this point?

SOLUTION

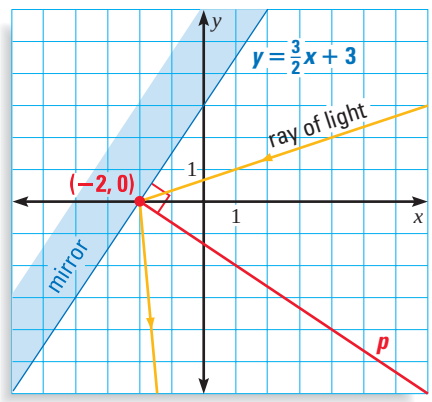
The mirror's slope is $\frac{3}{2}$, so the slope of p is $-\frac{2}{3}$.

Use $m = -\frac{2}{3}$ and $(x, y) = (-2, 0)$ to find b.

$0 = -\frac{2}{3}(-2) + b$

$-\frac{4}{3} = b$

▶ So, an equation for p is $y = -\frac{2}{3}x - \frac{4}{3}$.



Top view of mirror

FOCUS ON CAREERS

REAL LIFE GRAPHIC ARTS

Many graphic artists use computer software to design images.

INTERNET CAREER LINK

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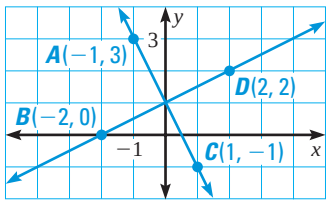
GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

Skill Check ✓

1. Define *slope of a line*.
2. The slope of line m is $-\frac{1}{5}$. What is the slope of a line perpendicular to m?
3. In the coordinate plane shown at the right, is $\overleftrightarrow{AC}$ perpendicular to $\overleftrightarrow{BD}$? Explain.



4. Decide whether the lines with the equations $y = 2x - 1$ and $y = -2x + 1$ are perpendicular.
5. Decide whether the lines with the equations $5y - x = 15$ and $y + 5x = 2$ are perpendicular.
6. The line ℓ_1 has the equation $y = 3x$. The line ℓ_2 is perpendicular to ℓ_1 and passes through the point $P(0, 0)$. Write an equation of ℓ_2.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 808.

SLOPES OF PERPENDICULAR LINES **The slopes of two lines are given. Are the lines perpendicular?**

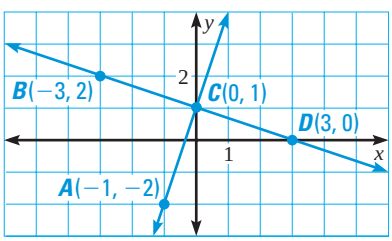
7. $m_1 = 2, m_2 = -\frac{1}{2}$
8. $m_1 = \frac{2}{3}, m_2 = \frac{3}{2}$
9. $m_1 = \frac{1}{4}, m_2 = -4$
10. $m_1 = \frac{5}{7}, m_2 = -\frac{7}{5}$
11. $m_1 = -\frac{1}{2}, m_2 = -\frac{1}{2}$
12. $m_1 = -1, m_2 = 1$

SLOPES OF PERPENDICULAR LINES **Lines *j* and *n* are perpendicular. The slope of line *j* is given. What is the slope of line *n*? Check your answer.**

13. 2
14. 5
15. -3
16. -7
17. $\frac{2}{3}$
18. $\frac{1}{5}$
19. $-\frac{1}{3}$
20. $-\frac{4}{3}$

IDENTIFYING PERPENDICULAR LINES **Find the slope of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$. Decide whether $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{BD}$.**

21.



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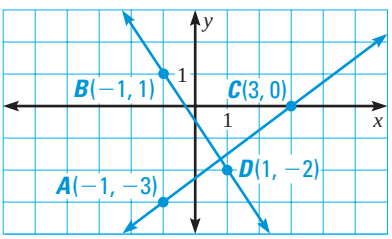

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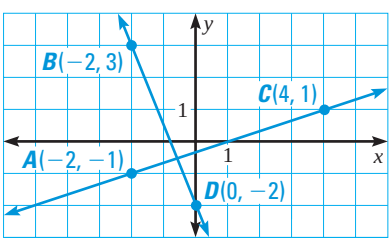

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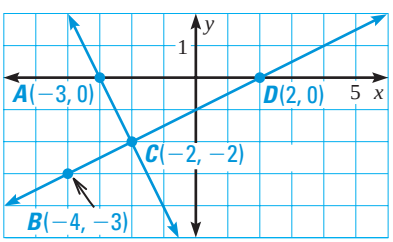


STUDENT HELP

HOMEWORK HELP

Example 1:	Exs. 7–20
Example 2:	Exs. 21–24, 33–37
Example 3:	Exs. 25–28, 47–50
Example 4:	Exs. 29–32
Example 5:	Exs. 38–41
Example 6:	Exs. 42–46

USING ALGEBRA **Decide whether lines k_1 and k_2 are perpendicular. Then graph the lines to check your answer.**

25. line k_1: $y = 3x$
line k_2: $y = -\frac{1}{3}x - 2$

26. line k_1: $y = -\frac{4}{5}x - 2$
line k_2: $y = \frac{1}{5}x + 4$

27. line k_1: $y = -\frac{3}{4}x + 2$
line k_2: $y = \frac{4}{3}x + 5$

28. line k_1: $y = \frac{1}{3}x - 10$
line k_2: $y = 3x$

USING ALGEBRA **Decide whether lines p_1 and p_2 are perpendicular.**

29. line p_1: $3y - 4x = 3$
line p_2: $4y + 3x = -12$

30. line p_1: $y - 6x = 2$
line p_2: $6y - x = 12$

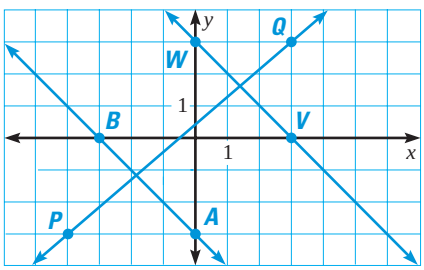
31. line p_1: $3y + 2x = -36$
line p_2: $4y - 3x = 16$

32. line p_1: $5y + 3x = -15$
line p_2: $3y - 5x = -33$

STUDENT HELP
INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 33–36.

LINE RELATIONSHIPS **Find the slope of each line. Identify any parallel or perpendicular lines.**

33.



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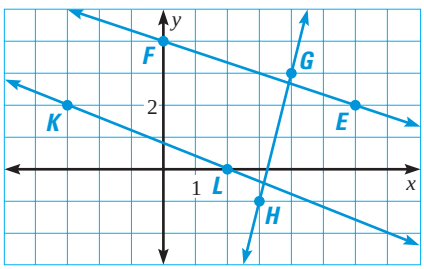

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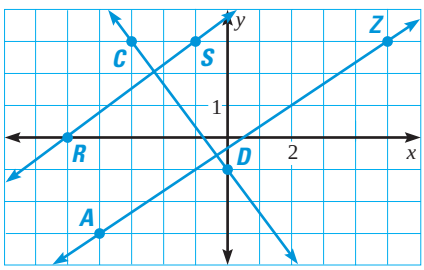

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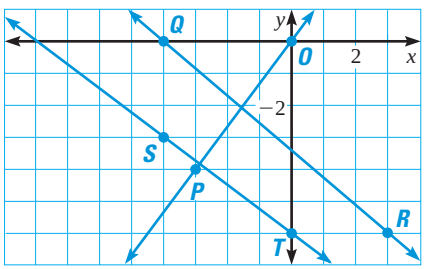
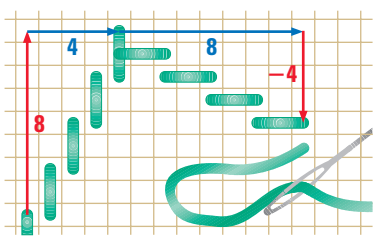


37. **NEEDLEPOINT** To check whether two stitched lines make a right angle, you can count the squares. For example, the lines at the right are perpendicular because one goes up 8 as it goes over 4, and the other goes over 8 as it goes down 4. Why does this mean the lines are perpendicular?



WRITING EQUATIONS **Line j is perpendicular to the line with the given equation and line j passes through P. Write an equation of line j.**

38. $y = \frac{1}{2}x - 1$, $P(0, 3)$

39. $y = \frac{5}{3}x + 2$, $P(5, 1)$

40. $y = -4x - 3$, $P(-2, 2)$

41. $3y + 4x = 12$, $P(-3, -4)$

FOCUS ON PEOPLE

REAL LIFE **HELAMAN FERGUSON'S** stone drill is suspended by six cables. The computer uses the lengths of the cables to calculate the coordinates of the drill tip.

WRITING EQUATIONS **The line with the given equation is perpendicular to line *j* at point *R*. Write an equation of line *j*.**

42. $y = -\frac{3}{4}x + 6, R(8, 0)$

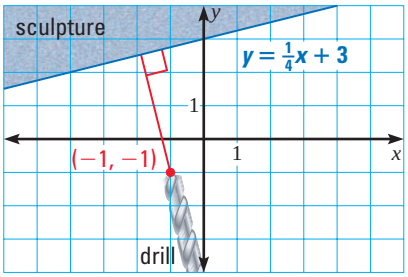
43. $y = \frac{1}{7}x - 11, R(7, -10)$

44. $y = 3x + 5, R(-3, -4)$

45. $y = -\frac{2}{5}x - 3, R(5, -5)$

46. **SCULPTURE** Helaman Ferguson designs sculptures on a computer. The computer is connected to his stone drill and tells how far he should drill at any given point. The distance from the drill tip to the desired surface of the sculpture is calculated along a line perpendicular to the sculpture.

Suppose the drill tip is at $(-1, -1)$ and the equation $y = \frac{1}{4}x + 3$ represents the surface of the sculpture. Write an equation of the line that passes through the drill tip and is perpendicular to the sculpture.



LINE RELATIONSHIPS **Decide whether the lines with the given equations are *parallel, perpendicular,* or *neither*.**

47. $y = -2x - 1$ $y = -2x - 3$

48. $y = -\frac{1}{2}x + 3$ $y = -\frac{1}{2}x + 5$

49. $y = -3x + 1$ $y = \frac{1}{3}x + 1$

50. $y = 4x + 10$ $y = -2x + 5$

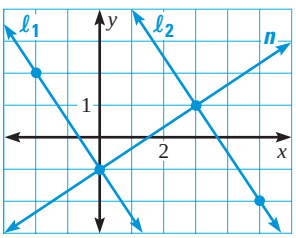
Test Preparation

51. **MULTI-STEP PROBLEM** Use the diagram at the right.

a. Is $\ell_1 \parallel \ell_2$? How do you know?

b. Is $\ell_2 \perp n$? How do you know?

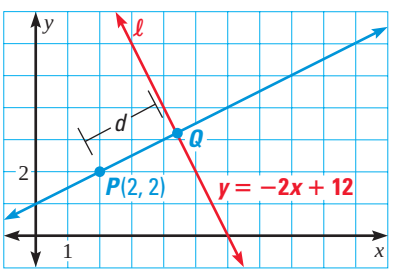
c. *Writing* Describe two ways to prove that $\ell_1 \perp n$.



Challenge

DISTANCE TO A LINE **In Exercises 52–54, use the following information.**

The distance from a point to a line is defined to be the length of the perpendicular segment from the point to the line. In the diagram at the right, the distance d between point P and line ℓ is given by QP.



52. Find an equation of $\overleftrightarrow{QP}$.

53. Solve a system of equations to find the coordinates of point Q, the intersection of the two lines.

54. Use the Distance Formula to find QP.

EXTRA CHALLENGE
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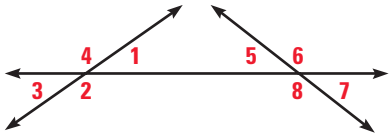
MIXED REVIEW

ANGLE MEASURES **Use the diagram to complete the statement.**
(Review 2.6 for 4.1)



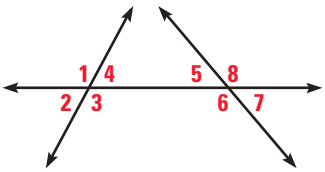
55. If $m\angle 5 = 38°$, then $m\angle 8 =$?.

56. If $m\angle 3 = 36°$, then $m\angle 4 =$?.

57. If $\angle 8 \cong \angle 4$ and $m\angle 2 = 145°$, then $m\angle 7 =$?.

58. If $m\angle 1 = 38°$ and $\angle 3 \cong \angle 5$, then $m\angle 6 =$?.

IDENTIFYING ANGLES **Use the diagram to complete the statement.**
(Review 3.1 for 4.1)



59. $\angle 3$ and ? are consecutive interior angles.

60. $\angle 1$ and ? are alternate exterior angles.

61. $\angle 4$ and ? are alternate interior angles.

62. $\angle 1$ and ? are corresponding angles.

63. *Writing* Describe the three types of proofs you have learned so far.
(Review 3.2)

QUIZ 3

Self-Test for Lessons 3.6 and 3.7

Find the slope of $\overleftrightarrow{AB}$. **(Lesson 3.6)**

1. $A(1, 2), B(5, 8)$

2. $A(2, -3), B(-1, 5)$

Write an equation of line j_2 that passes through point P and is parallel to line j_1. **(Lesson 3.6)**

3. line j_1: $y = 3x - 2$
$P(0, 2)$

4. line j_1: $y = \frac{1}{2}x + 1$
$P(2, -4)$

Decide whether k_1 and k_2 are perpendicular. **(Lesson 3.7)**

5. line k_1: $y = 2x - 1$
line k_2: $y = -\frac{1}{2}x + 2$

6. line k_1: $y - 3x = -2$
line k_2: $3y - x = 12$

7. **ANGLE OF REPOSE** When a granular substance is poured into a pile, the slope of the pile depends only on the substance. For example, when barley is poured into piles, every pile has the same slope. A pile of barley that is 5 feet tall would be about 10 feet wide. What is the slope of a pile of barley? **(Lesson 3.6)**