3.5 Using Properties of Parallel Lines

What you should learn

GOAL 1 Use properties of parallel lines in **real-life** situations, such as building a CD rack in **Example 3**.

GOAL 2 Construct parallel lines using straightedge and compass.

Why you should learn it

▼ To understand how light bends when it passes through glass or water, as in **Ex. 42**.

REAL LIFE

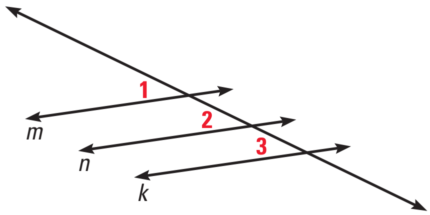
GOAL 1 USING PARALLEL LINES IN REAL LIFE

When a team of rowers competes, each rower keeps his or her oars parallel to the adjacent rower's oars. If any two *adjacent* oars on the same side of the boat are parallel, does this imply that *any two* oars on that side are parallel? This question is examined below.

Example 1 justifies Theorem 3.11, and you will prove Theorem 3.12 in Exercise 38.

EXAMPLE 1 *Proving Two Lines are Parallel*

Lines m, n, and k represent three of the oars above. $m \parallel n$ and $n \parallel k$. Prove that $m \parallel k$.



SOLUTION

GIVEN ▶ $m \parallel n,\ n \parallel k$

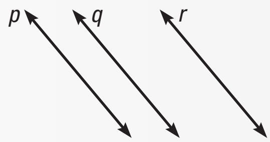
PROVE ▶ $m \parallel k$

Statements	Reasons
1. $m \parallel n$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Corresponding Angles Postulate
3. $n \parallel k$	**3.** Given
4. $\angle 2 \cong \angle 3$	**4.** Corresponding Angles Postulate
5. $\angle 1 \cong \angle 3$	**5.** Transitive Property of Congruence
6. $m \parallel k$	**6.** Corresponding Angles Converse

THEOREMS ABOUT PARALLEL AND PERPENDICULAR LINES

THEOREM 3.11

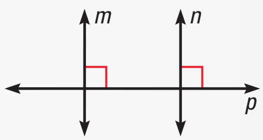
If two lines are parallel to the same line, then they are parallel to each other.



If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

THEOREM 3.12

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

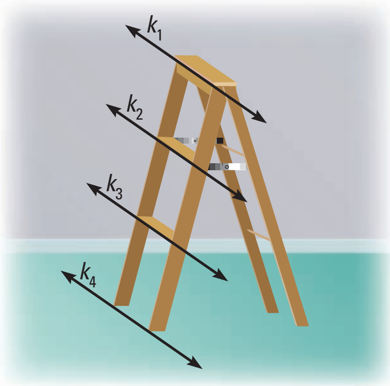


If $m \perp p$ and $n \perp p$, then $m \parallel n$.

Logical Reasoning

EXAMPLE 2 *Explaining Why Steps are Parallel*

In the diagram at the right, each step is parallel to the step immediately below it and the bottom step is parallel to the floor. Explain why the top step is parallel to the floor.



SOLUTION

You are given that $k_1 \parallel k_2$ and $k_2 \parallel k_3$.
By transitivity of parallel lines, $k_1 \parallel k_3$.
Since $k_1 \parallel k_3$ and $k_3 \parallel k_4$, it follows that $k_1 \parallel k_4$.
So, the top step is parallel to the floor.

EXAMPLE 3 *Building a CD Rack*

You are building a CD rack. You cut the sides, bottom, and top so that each corner is composed of two 45° angles. Prove that the top and bottom front edges of the CD rack are parallel.

Proof

SOLUTION

GIVEN ▶ $m\angle 1 = 45°$, $m\angle 2 = 45°$
$m\angle 3 = 45°$, $m\angle 4 = 45°$

PROVE ▶ $\overline{BA} \parallel \overline{CD}$

Statement	Reason
$m\angle ABC = m\angle 1 + m\angle 2$	Angle Addition Postulate
$m\angle 1 = 45°$, $m\angle 2 = 45°$	Given
$m\angle ABC = 90°$	Substitution property
$\angle ABC$ is a right angle.	Def. of right angle
$\overline{BA} \perp \overline{BC}$	Def. of $\perp$ lines
$m\angle BCD = m\angle 3 + m\angle 4$	Angle Addition Postulate
$m\angle 3 = 45°$, $m\angle 4 = 45°$	Given
$m\angle BCD = 90°$	Substitution property
$\angle BCD$ is a right angle.	Def. of right angle
$\overline{BC} \perp \overline{CD}$	Def. of $\perp$ lines
$\overline{BA} \parallel \overline{CD}$	In a plane, 2 lines $\perp$ to the same line are $\parallel$.

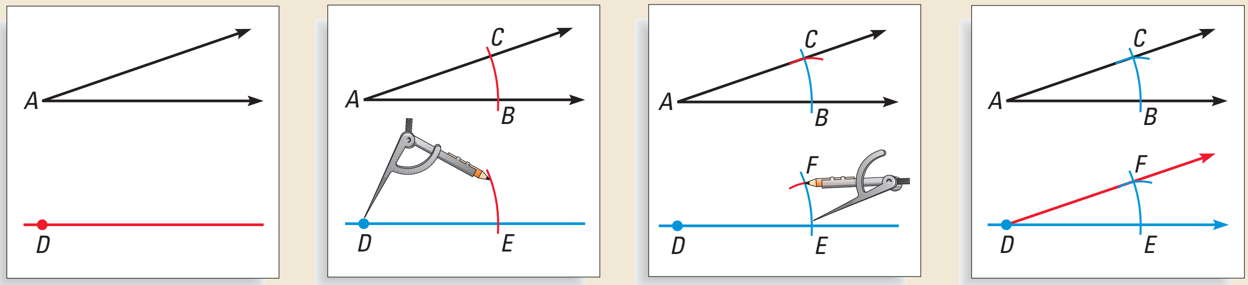
GOAL 2 CONSTRUCTING PARALLEL LINES

To construct parallel lines, you first need to know how to copy an angle.

ACTIVITY Construction

Copying an Angle

Use these steps to construct an angle that is congruent to a given $\angle A$.



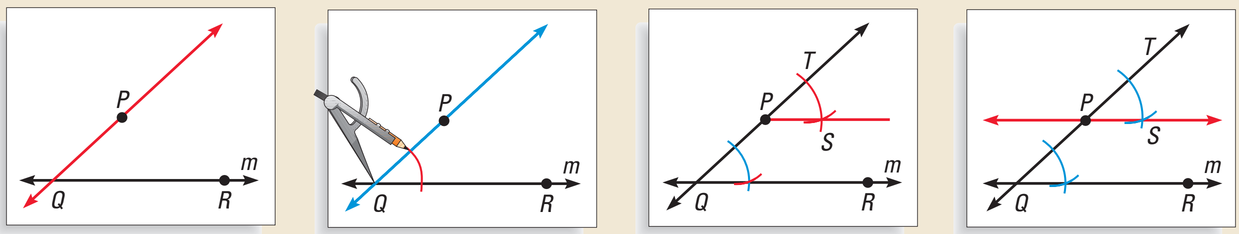
1. Draw a line. Label a point on the line *D*.
2. Draw an arc with center *A*. Label *B* and *C*. With the same radius, draw an arc with center *D*. Label *E*.
3. Draw an arc with radius *BC* and center *E*. Label the intersection *F*.
4. Draw $\overrightarrow{DF}$. $\angle EDF \cong \angle BAC$.

In Chapter 4, you will learn why the *Copying an Angle* construction works. You can use the *Copying an Angle* construction to construct two congruent corresponding angles. If you do, the sides of the angles will be parallel.

ACTIVITY Construction

Parallel Lines

Use these steps to construct a line that passes through a given point *P* and is parallel to a given line *m*.



1. Draw points *Q* and *R* on *m*. Draw $\overleftrightarrow{PQ}$.
2. Draw an arc with the compass point at *Q* so that it crosses $\overrightarrow{QP}$ and $\overrightarrow{QR}$.
3. Copy $\angle PQR$ on $\overleftrightarrow{QP}$ as shown. Be sure the two angles are corresponding. Label the new angle $\angle TPS$ as shown.
4. Draw $\overleftrightarrow{PS}$. Because $\angle TPS$ and $\angle PQR$ are congruent corresponding angles, $\overleftrightarrow{PS} \parallel \overleftrightarrow{QR}$.

GUIDED PRACTICE

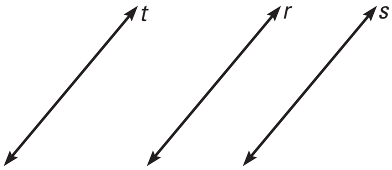
Concept Check ✓

Skill Check ✓

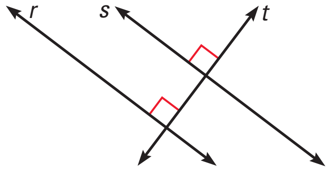
1. Name two ways, from this lesson, to prove that two lines are parallel.
if they are || to the same line, if they are ⊥ to the same line

State the theorem that you can use to prove that *r* is parallel to *s*.

2. GIVEN ▶ $r \parallel t$, $t \parallel s$

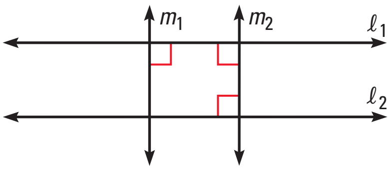


3. GIVEN ▶ $r \perp t$, $t \perp s$

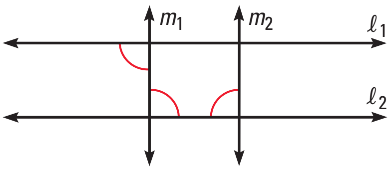


Determine which lines, if any, must be parallel. Explain your reasoning.

4.



5.



6. Draw any angle $\angle A$. Then construct $\angle B$ congruent to $\angle A$.

7. Given a line ℓ and a point P not on ℓ, describe how to construct a line through P parallel to ℓ.

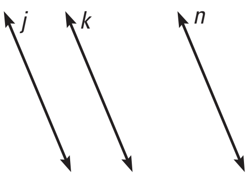
PRACTICE AND APPLICATIONS

STUDENT HELP

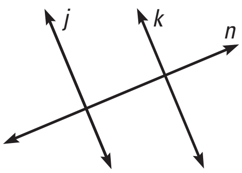
Extra Practice to help you master skills is on p. 808.

LOGICAL REASONING **State the postulate or theorem that allows you to conclude that $j \parallel k$.**

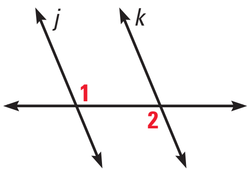
8. GIVEN ▶ $j \parallel n$, $k \parallel n$



9. GIVEN ▶ $j \perp n$, $k \perp n$

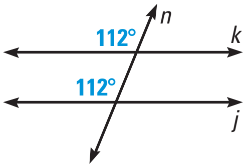


10. GIVEN ▶ $\angle 1 \cong \angle 2$



SHOWING LINES ARE PARALLEL **Explain how you would show that $k \parallel j$. State any theorems or postulates that you would use.**

11.



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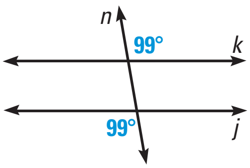

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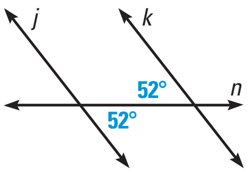


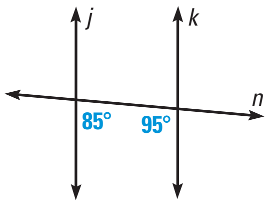
14. *Writing* Make a list of all the ways you know to prove that two lines are parallel.

STUDENT HELP

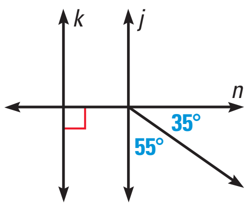
HOMEWORK HELP
Example 1: Exs. 8–24
Example 2: Exs. 8–24
Example 3: Exs. 8–24

SHOWING LINES ARE PARALLEL Explain how you would show that $k \parallel j$.

15.



16.



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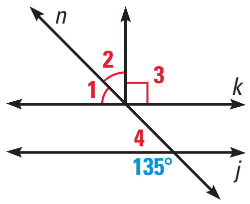
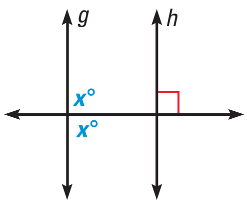


USING ALGEBRA Explain how you would show that $g \parallel h$.

18.



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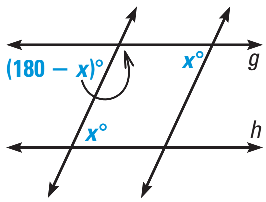

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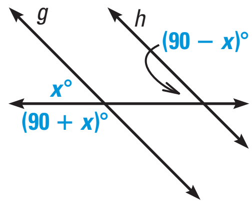
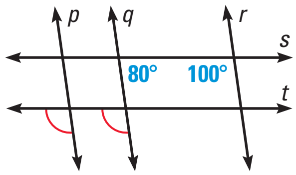


NAMING PARALLEL LINES Determine which lines, if any, must be parallel. Explain your reasoning.

21.



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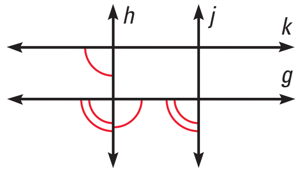

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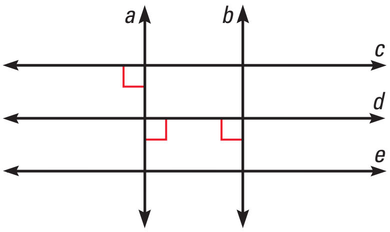

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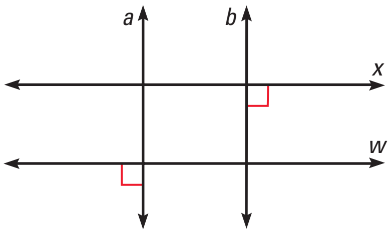


CONSTRUCTIONS Use a straightedge to draw an angle that fits the description. Then use the *Copying an Angle* construction on page 159 to copy the angle.

25. An acute angle

26. An obtuse angle

27. CONSTRUCTING PARALLEL LINES Draw a horizontal line and construct a line parallel to it through a point above the line.

28. CONSTRUCTING PARALLEL LINES Draw a diagonal line and construct a line parallel to it through a point to the right of the line.

29. JUSTIFYING A CONSTRUCTION Explain why the lines in Exercise 28 are parallel. Use a postulate or theorem from Lesson 3.4 to support your answer.

STUDENT HELP

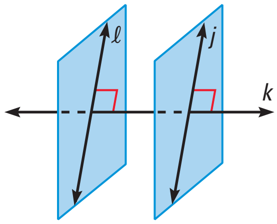
INTERNET

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with constructions in Exs. 25–29.

30. **FOOTBALL FIELD** The white lines along the long edges of a football field are called *sidelines*. *Yard lines* are perpendicular to the sidelines and cross the field every five yards. Explain why you can conclude that the yard lines are parallel.

31. **HANGING WALLPAPER** When you hang wallpaper, you use a tool called a *plumb line* to make sure one edge of the first strip of wallpaper is vertical. If the edges of each strip of wallpaper are parallel and there are no gaps between the strips, how do you know that the rest of the strips of wallpaper will be parallel to the first?

32. **ERROR ANALYSIS** It is given that $j \perp k$ and $k \perp \ell$. A student reasons that lines j and ℓ must be parallel. What is wrong with this reasoning? Sketch a counterexample to support your answer.



CATEGORIZING **Tell whether the statement is *sometimes*, *always*, or *never* true.**

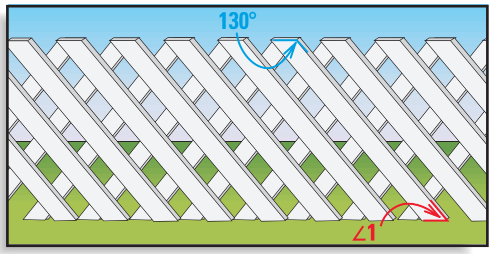
33. Two lines that are parallel to the same line are parallel to each other.

34. *In a plane*, two lines that are perpendicular to the same line are parallel to each other.

35. Two *noncoplanar* lines that are perpendicular to the same line are parallel to each other.

36. Through a point not on a line you can construct a parallel line.

37. **LATTICEWORK** You are making a lattice fence out of pieces of wood called slats. You want the top of each slat to be parallel to the bottom. At what angle should you cut $\angle 1$?

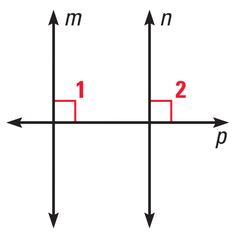


38. **PROVING THEOREM 3.12** Rearrange the statements to write a flow proof of Theorem 3.12. Remember to include a reason for each statement.

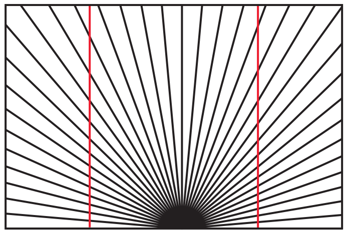
GIVEN ▶ $m \perp p$, $n \perp p$

PROVE ▶ $m \parallel n$

$\angle 1 \cong \angle 2$	$n \perp p$	$\angle 1$ is a right $\angle$.
$m \parallel n$	$m \perp p$	$\angle 2$ is a right $\angle$.



39. OPTICAL ILLUSION The radiating lines make it hard to tell if the red lines are straight. Explain how you can answer the question using only a straightedge and a protractor.



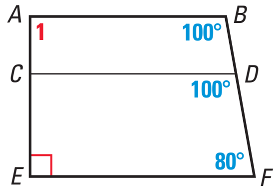
a. Are the red lines straight?

b. Are the red lines parallel?

40. CONSTRUCTING WITH PERPENDICULARS Draw a horizontal line ℓ and a point P not on ℓ. Construct a line m through P perpendicular to ℓ. Draw a point Q not on m or ℓ. Construct a line n through Q perpendicular to m. What postulate or theorem guarantees that the lines ℓ and n are parallel?

Test Preparation

41. MULTI-STEP PROBLEM Use the information given in the diagram at the right.



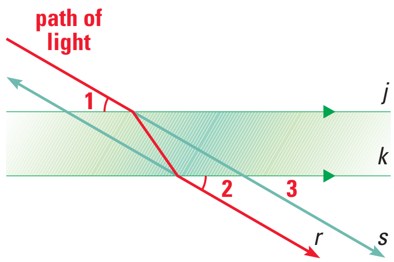
a. Explain why $\overline{AB} \parallel \overline{CD}$.

b. Explain why $\overline{CD} \parallel \overline{EF}$.

c. *Writing* What is $m\angle 1$? How do you know?

★ Challenge

42. SCIENCE CONNECTION When light enters glass, the light bends. When it leaves glass, it bends again. If both sides of a pane of glass are parallel, light leaves the pane at the same angle at which it entered. Prove that the path of the exiting light is parallel to the path of the entering light.



GIVEN ▶ $\angle 1 \cong \angle 2, j \parallel k$

PROVE ▶ $r \parallel s$

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MIXED REVIEW

USING THE DISTANCE FORMULA Find the distance between the two points. (Review 1.3 for 3.6)

43. $A(0, -6), B(14, 0)$ **44.** $A(-3, -8), B(2, -1)$ **45.** $A(0, -7), B(6, 3)$

46. $A(-9, -5), B(-1, 11)$ **47.** $A(5, -7), B(-11, 6)$ **48.** $A(4, 4), B(-3, -3)$

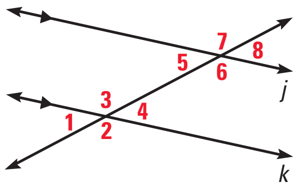
FINDING COUNTEREXAMPLES Give a counterexample that demonstrates that the converse of the statement is false. (Review 2.2)

49. If an angle measures 42°, then it is acute.

50. If two angles measure 150° and 30°, then they are supplementary.

51. If a polygon is a rectangle, then it contains four right angles.

52. USING PROPERTIES OF PARALLEL LINES Use the given information to find the measures of the other seven angles in the figure shown at the right. (Review 3.3)

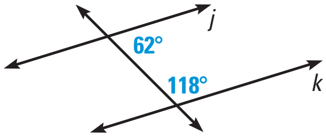


GIVEN ▶ $j \parallel k, m\angle 1 = 33°$

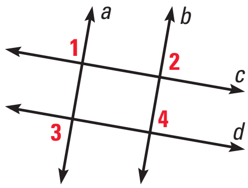
QUIZ 2

Self-Test for Lessons 3.4 and 3.5

1. In the diagram shown at the right, determine whether you can prove that lines j and k are parallel. If you can, state the postulate or theorem that you would use. **(Lesson 3.4)**



Use the given information and the diagram to determine which lines must be parallel. **(Lesson 3.5)**

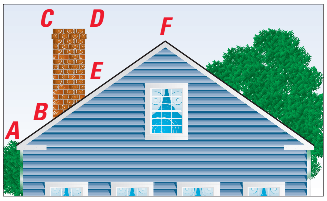


2. $\angle 1$ and $\angle 2$ are right angles.

3. $\angle 4 \cong \angle 3$

4. $\angle 2 \cong \angle 3$, $\angle 3 \cong \angle 4$.

5. **FIREPLACE CHIMNEY** In the illustration at the right, $\angle ABC$ and $\angle DEF$ are supplementary. Explain how you know that the left and right edges of the chimney are parallel. **(Lesson 3.4)**



MATH & History — Measuring Earth's Circumference

INTERNET APPLICATION LINK www.mcdougallittell.com

THEN **AROUND 230 B.C.,** the Greek scholar Eratosthenes estimated Earth's circumference. In the late 15th century, Christopher Columbus used a smaller estimate to convince the king and queen of Spain that his proposed voyage to India would take only 30 days.

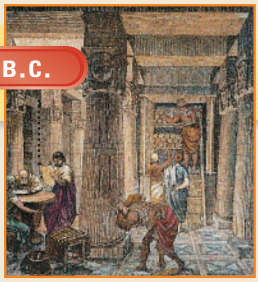
NOW **TODAY,** satellites and other tools are used to determine Earth's circumference with great accuracy.

1. The actual distance from Syene to Alexandria is about 500 miles. Use this value and the information on page 145 to estimate Earth's circumference. How close is your value to the modern day measurement in the table at the right?

Measuring Earth's Circumference	
Circumference estimated by Eratosthenes (230 B.C.)	About 29,000 mi
Circumference assumed by Columbus (about 1492)	About 17,600 mi
Modern day measurement	24,902 mi

Eratosthenes becomes the head of the library in Alexandria.

235 B.C.



1492

A replica of one of the ships used by Christopher Columbus.

1999

Photograph of Earth from space.