

3.3

Parallel Lines and Transversals

What you should learn

GOAL 1 Prove and use results about parallel lines and transversals.

GOAL 2 Use properties of parallel lines to solve real-life problems, such as estimating Earth's circumference in **Example 5**.

Why you should learn it

▼ Properties of parallel lines help you understand how rainbows are formed, as in **Ex. 30**.



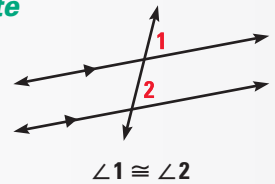
GOAL 1 PROPERTIES OF PARALLEL LINES

In the activity on page 142, you may have discovered the following results.

POSTULATE

POSTULATE 15 *Corresponding Angles Postulate*

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

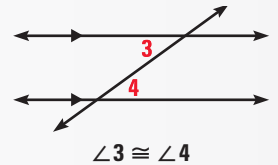


You are asked to prove Theorems 3.5, 3.6, and 3.7 in Exercises 27–29.

THEOREMS ABOUT PARALLEL LINES

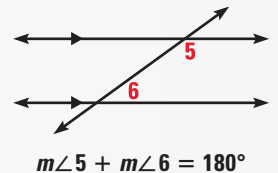
THEOREM 3.4 *Alternate Interior Angles*

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.



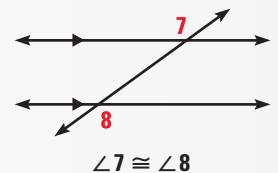
THEOREM 3.5 *Consecutive Interior Angles*

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.



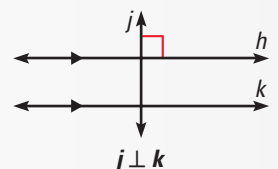
THEOREM 3.6 *Alternate Exterior Angles*

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.



THEOREM 3.7 *Perpendicular Transversal*

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.



STUDENT HELP

Study Tip

When you prove a theorem, the hypotheses of the theorem becomes the GIVEN, and the conclusion is what you must PROVE.

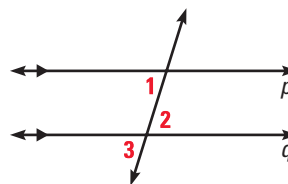
EXAMPLE 1 *Proving the Alternate Interior Angles Theorem*

Prove the Alternate Interior Angles Theorem.

SOLUTION

GIVEN ▶ $p \parallel q$

PROVE ▶ $\angle 1 \cong \angle 2$

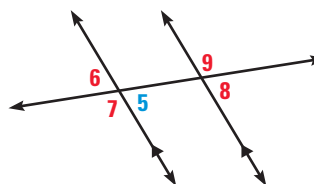


Statements	Reasons
1. $p \parallel q$	1. Given
2. $\angle 1 \cong \angle 3$	2. Corresponding Angles Postulate
3. $\angle 3 \cong \angle 2$	3. Vertical Angles Theorem
4. $\angle 1 \cong \angle 2$	4. Transitive Property of Congruence

EXAMPLE 2 *Using Properties of Parallel Lines*

Given that $m\angle 5 = 65^\circ$, find each measure. Tell which postulate or theorem you use.

- a. $m\angle 6$
- b. $m\angle 7$
- c. $m\angle 8$
- d. $m\angle 9$



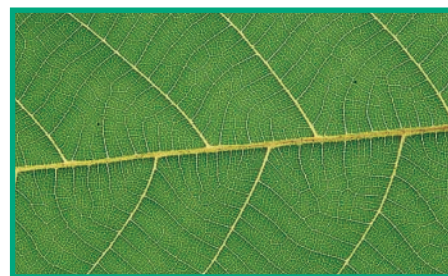
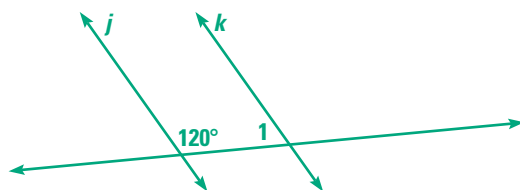
SOLUTION

- a. $m\angle 6 = m\angle 5 = 65^\circ$ Vertical Angles Theorem
- b. $m\angle 7 = 180^\circ - m\angle 5 = 115^\circ$ Linear Pair Postulate
- c. $m\angle 8 = m\angle 5 = 65^\circ$ Corresponding Angles Postulate
- d. $m\angle 9 = m\angle 7 = 115^\circ$ Alternate Exterior Angles Theorem

FOCUS ON CAREERS

EXAMPLE 3 *Classifying Leaves*

BOTANY Some plants are classified by the arrangement of the veins in their leaves. In the diagram of the leaf, $j \parallel k$. What is $m\angle 1$?



SOLUTION

- $m\angle 1 + 120^\circ = 180^\circ$ Consecutive Interior Angles Theorem
- $m\angle 1 = 60^\circ$ Subtract.



REAL LIFE **BOTANY** Botanists study plants and environmental issues such as conservation, weed control, and re-vegetation.

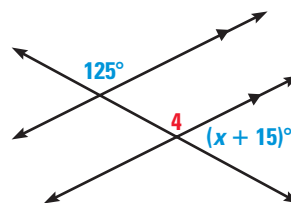
CAREER LINK
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GOAL 2 PROPERTIES OF SPECIAL PAIRS OF ANGLES



EXAMPLE 4 Using Properties of Parallel Lines

Use properties of parallel lines to find the value of x .



SOLUTION

$$m\angle 4 = 125^\circ \quad \text{Corresponding Angles Postulate}$$

$$m\angle 4 + (x + 15)^\circ = 180^\circ \quad \text{Linear Pair Postulate}$$

$$125^\circ + (x + 15)^\circ = 180^\circ \quad \text{Substitute.}$$

$$x = 40 \quad \text{Subtract.}$$

EXAMPLE 5 Estimating Earth's Circumference

HISTORY CONNECTION Eratosthenes was a Greek scholar. Over 2000 years ago, he estimated Earth's circumference by using the fact that the Sun's rays are parallel.

Eratosthenes chose a day when the Sun shone exactly down a vertical well in Syene at noon. On that day, he measured the angle the Sun's rays made with a vertical stick in Alexandria at noon. He discovered that

$$m\angle 2 \approx \frac{1}{50} \text{ of a circle.}$$

By using properties of parallel lines, he knew that $m\angle 1 = m\angle 2$. So he reasoned that

$$m\angle 1 \approx \frac{1}{50} \text{ of a circle.}$$

At the time, the distance from Syene to Alexandria was believed to be **575 miles**.

$$\frac{1}{50} \text{ of a circle} \approx \frac{575 \text{ miles}}{\text{Earth's circumference}}$$

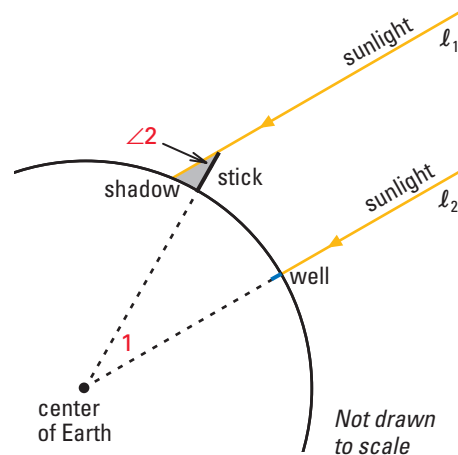
$$\text{Earth's circumference} \approx 50(575 \text{ miles}) \quad \leftarrow \text{Use cross product property.}$$

$$\approx 29,000 \text{ miles}$$

How did Eratosthenes know that $m\angle 1 = m\angle 2$?

SOLUTION

Because the Sun's rays are parallel, $\ell_1 \parallel \ell_2$. Angles 1 and 2 are alternate interior angles, so $\angle 1 \cong \angle 2$. By the definition of congruent angles, $m\angle 1 = m\angle 2$.



STUDENT HELP



APPLICATION LINK

Visit our Web site www.mcdougallittell.com for more information about Eratosthenes' estimate in Example 5.

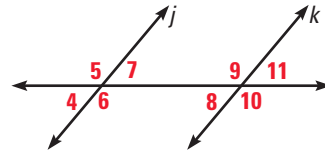
GUIDED PRACTICE

Vocabulary Check ✓

1. Sketch two parallel lines cut by a transversal. Label a pair of consecutive interior angles.

Concept Check ✓

2. In the figure at the right, $j \parallel k$. How many angle measures must be given in order to find the measure of every angle? Explain your reasoning.



Skill Check ✓

State the postulate or theorem that justifies the statement.

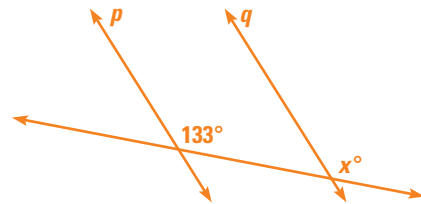
3. $\angle 2 \cong \angle 7$

4. $\angle 4 \cong \angle 5$

5. $m\angle 3 + m\angle 5 = 180^\circ$

6. $\angle 2 \cong \angle 6$

7. In the diagram of the feather below, lines p and q are parallel. What is the value of x ?

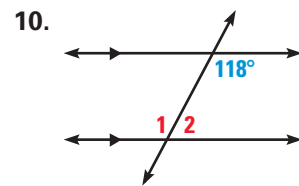
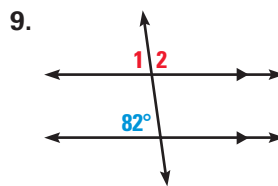
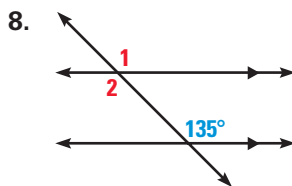


PRACTICE AND APPLICATIONS

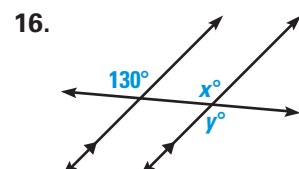
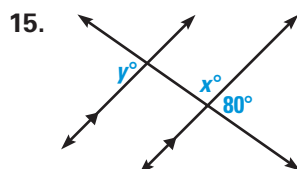
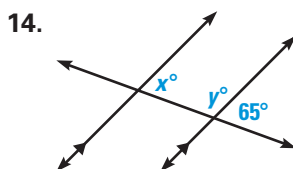
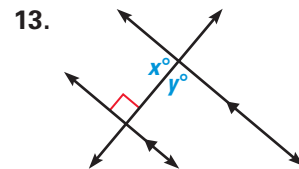
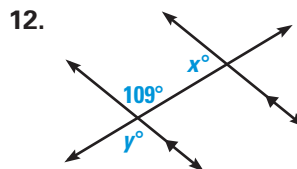
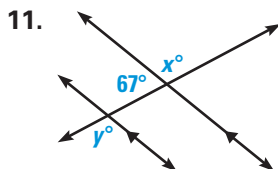
STUDENT HELP

Extra Practice to help you master skills is on p. 808.

USING PARALLEL LINES Find $m\angle 1$ and $m\angle 2$. Explain your reasoning.



USING PARALLEL LINES Find the values of x and y . Explain your reasoning.



STUDENT HELP

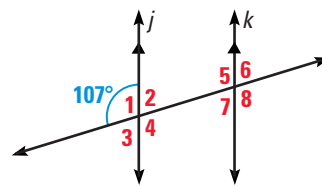
HOMEWORK HELP

- Example 1: Exs. 27–29
- Example 2: Exs. 8–17
- Example 3: Exs. 8–17
- Example 4: Exs. 18–26
- Example 5: Ex. 30

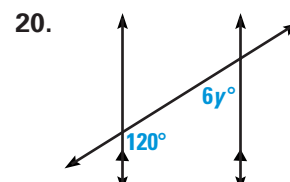
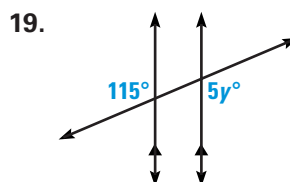
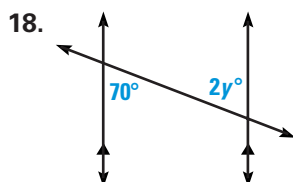
17. USING PROPERTIES OF PARALLEL LINES

Use the given information to find the measures of the other seven angles in the figure at the right.

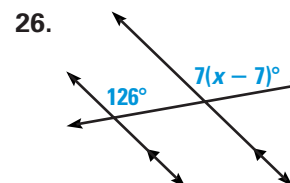
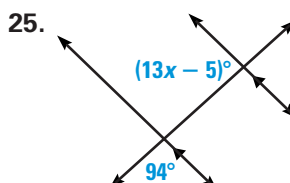
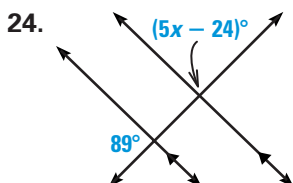
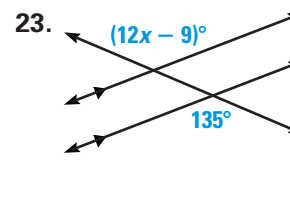
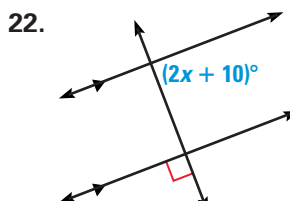
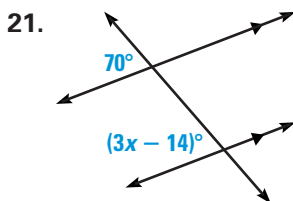
GIVEN $\triangleright j \parallel k, m\angle 1 = 107^\circ$



18. USING ALGEBRA Find the value of y .



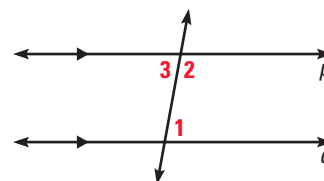
21. USING ALGEBRA Find the value of x .




27. DEVELOPING PROOF Complete the proof of the Consecutive Interior Angles Theorem.

GIVEN $\triangleright p \parallel q$

PROVE $\triangleright \angle 1$ and $\angle 2$ are supplementary.



STUDENT HELP
 **HOMEWORK HELP**
 Visit our Web site
www.mcdougallittell.com
 for help with proving
 theorems in Exs. 27–29.

Statements	Reasons
1. <u> ?</u>	1. Given
2. $\angle 1 \cong \angle 3$	2. <u> ?</u>
3. <u> ?</u>	3. Definition of congruent angles
4. <u> ?</u>	4. Definition of linear pair
5. $m\angle 3 + m\angle 2 = 180^\circ$	5. <u> ?</u>
6. <u> ?</u>	6. Substitution prop. of equality
7. $\angle 1$ and $\angle 2$ are supplementary.	7. <u> ?</u>

STUDENT HELP

Study Tip

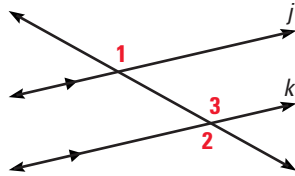
When you prove a theorem you may use any previous theorem, but you may not use the one you're proving.

PROVING THEOREMS 3.6 AND 3.7 In Exercises 28 and 29, complete the proof.

28. To prove the Alternate Exterior Angles Theorem, first show that $\angle 1 \cong \angle 3$. Then show that $\angle 3 \cong \angle 2$. Finally, show that $\angle 1 \cong \angle 2$.

GIVEN $\triangleright j \parallel k$

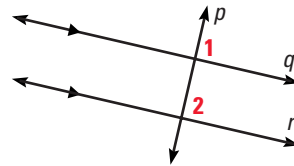
PROVE $\triangleright \angle 1 \cong \angle 2$



29. To prove the Perpendicular Transversal Theorem, show that $\angle 1$ is a right angle, $\angle 1 \cong \angle 2$, $\angle 2$ is a right angle, and finally that $p \perp r$.

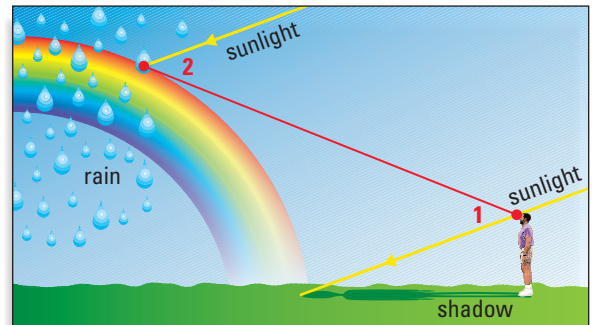
GIVEN $\triangleright p \perp q, q \parallel r$

PROVE $\triangleright p \perp r$



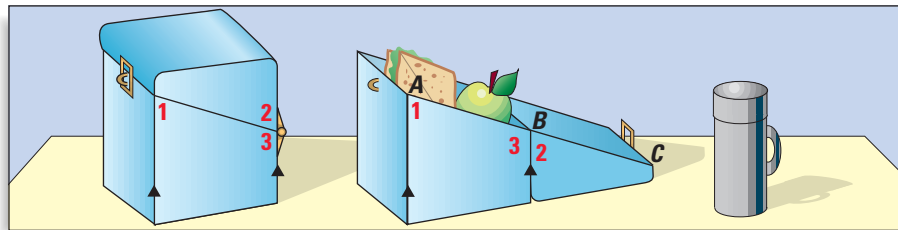
30. FORMING RAINBOWS

When sunlight enters a drop of rain, different colors leave the drop at different angles. That's what makes a rainbow. For red light, $m\angle 2 = 42^\circ$. What is $m\angle 1$? How do you know?



Test Preparation

31. MULTI-STEP PROBLEM You are designing a lunch box like the one below.



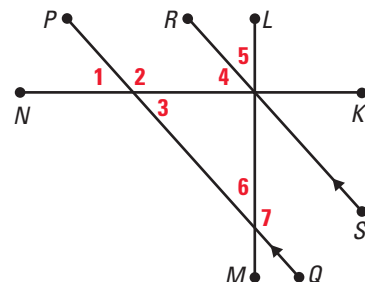
- The measure of $\angle 1$ is 70° . What is the measure of $\angle 2$? What is the measure of $\angle 3$?
- Writing* Explain why $\angle ABC$ is a straight angle.

★ Challenge

32. USING PROPERTIES OF PARALLEL LINES

Use the given information to find the measures of the other labeled angles in the figure. For each angle, tell which postulate or theorem you used.

GIVEN $\triangleright \overline{PQ} \parallel \overline{RS},$
 $\overline{LM} \perp \overline{NK},$
 $m\angle 1 = 48^\circ$



EXTRA CHALLENGE

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MIXED REVIEW

ANGLE MEASURES $\angle 1$ and $\angle 2$ are supplementary. Find $m\angle 2$. (Review 1.6)

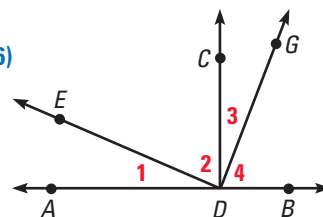
33. $m\angle 1 = 50^\circ$ 34. $m\angle 1 = 73^\circ$ 35. $m\angle 1 = 101^\circ$
 36. $m\angle 1 = 107^\circ$ 37. $m\angle 1 = 111^\circ$ 38. $m\angle 1 = 118^\circ$

CONVERSES Write the converse of the statement. (Review 2.1 for 3.4)

39. If the measure of an angle is 19° , then the angle is acute.
 40. I will go to the park if you go with me.
 41. I will go fishing if I do not have to work.

FINDING ANGLES Complete the statement, given that $\overrightarrow{DE} \perp \overrightarrow{DG}$ and $\overrightarrow{AB} \perp \overrightarrow{DC}$. (Review 2.6)

42. If $m\angle 1 = 23^\circ$, then $m\angle 2 = \underline{\quad? \quad}$.
 43. If $m\angle 4 = 69^\circ$, then $m\angle 3 = \underline{\quad? \quad}$.
 44. If $m\angle 2 = 70^\circ$, then $m\angle 4 = \underline{\quad? \quad}$.

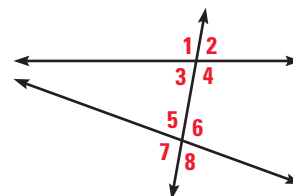


QUIZ 1

Self-Test for Lessons 3.1–3.3

Complete the statement. (Lesson 3.1)

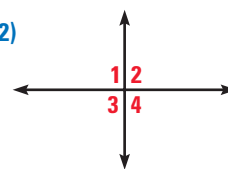
1. $\angle 2$ and $\underline{\quad? \quad}$ are corresponding angles.
 2. $\angle 3$ and $\underline{\quad? \quad}$ are consecutive interior angles.
 3. $\angle 3$ and $\underline{\quad? \quad}$ are alternate interior angles.
 4. $\angle 2$ and $\underline{\quad? \quad}$ are alternate exterior angles.



5. **PROOF** Write a plan for a proof. (Lesson 3.2)

GIVEN $\angle 1 \cong \angle 2$

PROVE $\angle 3$ and $\angle 4$ are right angles.



Find the value of x . (Lesson 3.3)

6. 7. 8.

9. **FLAG OF PUERTO RICO** Sketch the flag of Puerto Rico shown at the right. Given that $m\angle 3 = 55^\circ$, determine the measure of $\angle 1$. Justify each step in your argument. (Lesson 3.3)

