

3.2

Proof and Perpendicular Lines

What you should learn

GOAL 1 Write different types of proofs.

GOAL 2 Prove results about perpendicular lines.

Why you should learn it

▼ Make conclusions from things you see in **real life**, such as the reflection in Ex. 28.



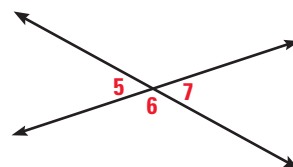
GOAL 1 COMPARING TYPES OF PROOFS

There is more than one way to write a proof. The two-column proof below is from Lesson 2.6. It can also be written as a paragraph proof or as a *flow* proof. A **flow proof** uses arrows to show the flow of the logical argument. Each reason in a flow proof is written below the statement it justifies.

EXAMPLE 1 Comparing Types of Proof

GIVEN ► $\angle 5$ and $\angle 6$ are a linear pair.
 $\angle 6$ and $\angle 7$ are a linear pair.

PROVE ► $\angle 5 \cong \angle 7$



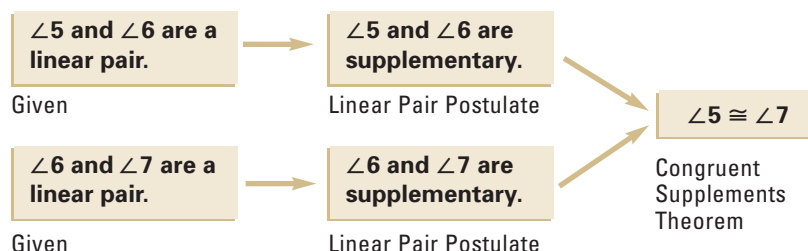
Method 1 Two-column Proof

Statements	Reasons
1. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	1. Given
2. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	2. Linear Pair Postulate
3. $\angle 5 \cong \angle 7$	3. Congruent Supplements Theorem

Method 2 Paragraph Proof

Because $\angle 5$ and $\angle 6$ are a linear pair, the Linear Pair Postulate says that $\angle 5$ and $\angle 6$ are supplementary. The same reasoning shows that $\angle 6$ and $\angle 7$ are supplementary. Because $\angle 5$ and $\angle 7$ are both supplementary to $\angle 6$, the Congruent Supplements Theorem says that $\angle 5 \cong \angle 7$.

Method 3 Flow Proof

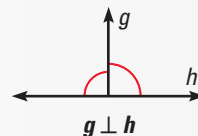


GOAL 2 PROVING RESULTS ABOUT PERPENDICULAR LINES

THEOREMS

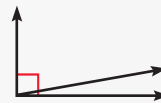
THEOREM 3.1

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.



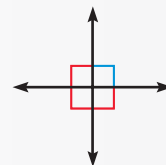
THEOREM 3.2

If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.



THEOREM 3.3

If two lines are perpendicular, then they intersect to form four right angles.



You will prove Theorem 3.2 and Theorem 3.3 in Exercises 17–19.

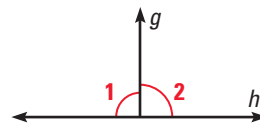


EXAMPLE 2 Proof of Theorem 3.1

Write a proof of Theorem 3.1.

SOLUTION

GIVEN $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 2$ are a linear pair.



PROVE $g \perp h$

Plan for Proof Use $m\angle 1 + m\angle 2 = 180^\circ$ and $m\angle 1 = m\angle 2$ to show $m\angle 1 = 90^\circ$.

$\angle 1$ and $\angle 2$ are a linear pair.

Given

$\angle 1$ and $\angle 2$ are supplementary.

Linear Pair Postulate

$m\angle 1 + m\angle 2 = 180^\circ$

Def. of supplementary $\angle$ s

$\angle 1 \cong \angle 2$

Given

$m\angle 1 = m\angle 2$

Def. of $\cong$ angles

$m\angle 1 + m\angle 1 = 180^\circ$

Substitution prop. of equality

$2 \cdot (m\angle 1) = 180^\circ$

Distributive prop.

$m\angle 1 = 90^\circ$

Div. prop. of equality

$\angle 1$ is a right $\angle$.

Def. of right angle

$g \perp h$

Def. of $\perp$ lines

STUDENT HELP

Study Tip

When you write a complicated proof, it may help to write a plan first. The plan will also help others to understand your proof.

You have now studied three types of proofs.

- 1. TWO-COLUMN PROOF** This is the most formal type of proof. It lists numbered statements in the left column and a reason for each statement in the right column.
- 2. PARAGRAPH PROOF** This type of proof describes the logical argument with sentences. It is more conversational than a two-column proof.
- 3. FLOW PROOF** This type of proof uses the same statements and reasons as a two-column proof, but the logical flow connecting the statements is indicated by arrows.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

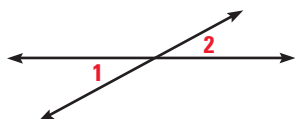
Skill Check ✓

1. Define *perpendicular lines*.

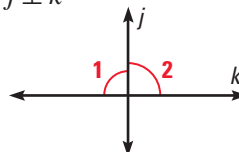
2. Which postulate or theorem guarantees that there is only one line that can be constructed perpendicular to a given line from a given point not on the line?

Write the postulate or theorem that justifies the statement about the diagram.

3. $\angle 1 \cong \angle 2$

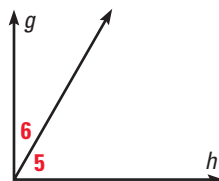


4. $j \perp k$

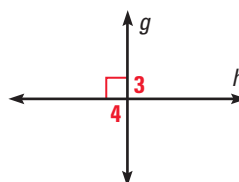


Write the postulate or theorem that justifies the statement, given that $g \perp h$.

5. $m\angle 5 + m\angle 6 = 90^\circ$

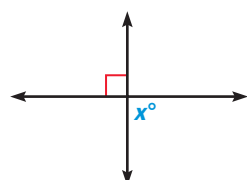


6. $\angle 3$ and $\angle 4$ are right angles.

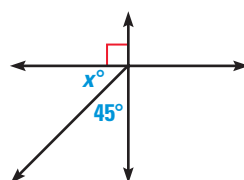


Find the value of x .

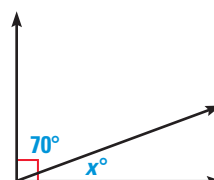
7.



8.



9.



10. **ERROR ANALYSIS** It is given that $\angle ABC \cong \angle CBD$. A student concludes that because $\angle ABC$ and $\angle CBD$ are congruent adjacent angles, $\overleftrightarrow{AB} \perp \overleftrightarrow{CB}$. What is wrong with this reasoning? Draw a diagram to support your answer.

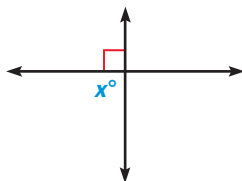
PRACTICE AND APPLICATIONS

STUDENT HELP

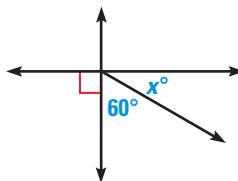
Extra Practice
to help you master
skills is on p. 807.

xy USING ALGEBRA Find the value of x .

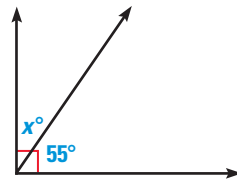
11.



12.

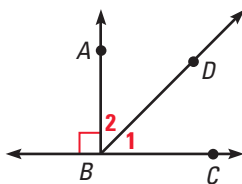


13.

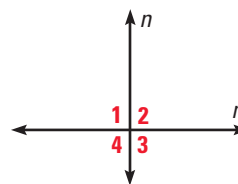


LOGICAL REASONING What can you conclude about the labeled angles?

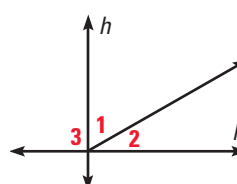
14. $\overline{AB} \perp \overline{CB}$



15. $n \perp m$



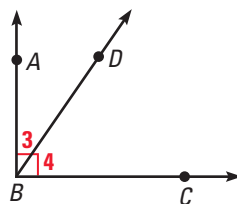
16. $h \perp k$



17. **DEVELOPING PARAGRAPH PROOF** Fill in the lettered blanks to complete the proof of Theorem 3.2.

GIVEN $\overrightarrow{BA} \perp \overrightarrow{BC}$

PROVE $\angle 3$ and $\angle 4$ are complementary.



Because $\overrightarrow{BA} \perp \overrightarrow{BC}$, $\angle ABC$ is a a. and $m\angle ABC =$ b.

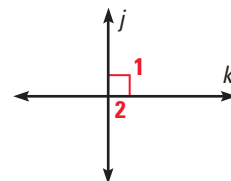
According to the c. Postulate, $m\angle 3 + m\angle 4 = m\angle ABC$. So, by the substitution property of equality, d. + e. = f.

By definition, $\angle 3$ and $\angle 4$ are complementary.

18. **DEVELOPING FLOW PROOF** Fill in the lettered blanks to complete the proof of part of Theorem 3.3. Because the lines are perpendicular, they intersect to form a right angle. Call that $\angle 1$.

GIVEN $j \perp k$, $\angle 1$ and $\angle 2$ are a linear pair.

PROVE $\angle 2$ is a right angle.



$\angle 1$ and $\angle 2$ are a linear pair.

Given

a.

Linear Pair Postulate

$m\angle 1 + m\angle 2 = 180^\circ$

b.

$j \perp k$

Given

$\angle 1$ is a right $\angle$.

Def. of $\perp$ lines

$m\angle 1 = 90^\circ$

c.

$90^\circ + m\angle 2 = 180^\circ$

d.

Subtr. prop. of equality

f.

$\angle 2$ is a right $\angle$.

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 17–23

Example 2: Exs. 11–19,
24, 25

STUDENT HELP



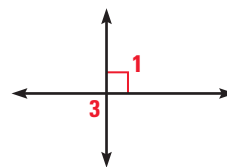
HOMEWORK HELP

Visit our Web site
www.mcdougallittell.com
for help with writing
proofs in Exs. 17–24.

19. **DEVELOPING TWO-COLUMN PROOF** Fill in the blanks to complete the proof of part of Theorem 3.3.

GIVEN $\angle 1$ is a right angle.

PROVE $\angle 3$ is a right angle.



Statements	Reasons
1. $\angle 1$ and $\angle 3$ are vertical angles.	1. Definition of vertical angles
2. $\underline{\hspace{1cm}}$	2. Vertical Angles Theorem
3. $m\angle 1 = m\angle 3$	3. $\underline{\hspace{1cm}}$
4. $\angle 1$ is a right angle.	4. $\underline{\hspace{1cm}}$
5. $\underline{\hspace{1cm}}$	5. Definition of right angle
6. $\underline{\hspace{1cm}}$	6. Substitution prop. of equality
7. $\underline{\hspace{1cm}}$	7. Definition of right angle

- DEVELOPING PROOF** In Exercises 20–23, use the following information.
Dan is trying to figure out how to prove that $\angle 5 \cong \angle 6$ below. First he wrote everything that he knew about the diagram, as shown below in blue.

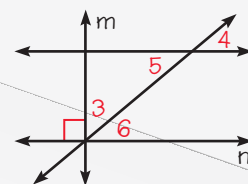
GIVEN $m \perp n$, $\angle 3$ and $\angle 4$ are complementary.

PROVE $\angle 5 \cong \angle 6$

$m \perp n \rightarrow \angle 3$ and $\angle 6$ are complementary.

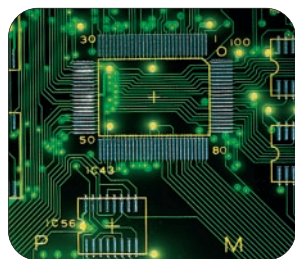
$\angle 3$ and $\angle 4$ are complementary.

$\angle 4$ and $\angle 5$ are vertical angles. $\rightarrow \angle 4 \cong \angle 5$



$\angle 4 \cong \angle 6 \rightarrow \angle 5 \cong \angle 6$

FOCUS ON APPLICATIONS



CIRCUIT BOARDS

The lines on circuit boards are made of metal and carry electricity. The lines must not touch each other or the electricity will flow to the wrong place, creating a *short circuit*.

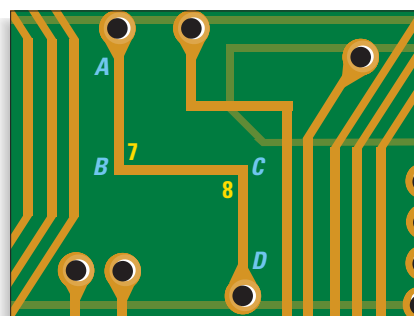
20. Write a justification for each statement Dan wrote in blue.
21. After writing all he knew, Dan wrote what he was supposed to prove in red. He also wrote $\angle 4 \cong \angle 6$ because he knew that if $\angle 4 \cong \angle 6$ and $\angle 4 \cong \angle 5$, then $\angle 5 \cong \angle 6$. Write a justification for this step.
22. How can you use Dan's blue statements to prove that $\angle 4 \cong \angle 6$?
23. Copy and complete Dan's flow proof.

24. **CIRCUIT BOARDS** The diagram shows part of a circuit board. Write any type of proof.

GIVEN $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$

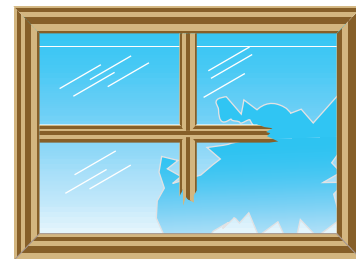
PROVE $\angle 7 \cong \angle 8$

Plan for Proof Show that $\angle 7$ and $\angle 8$ are both right angles.



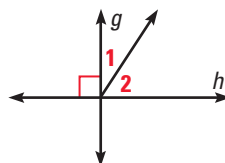
Test Preparation

25. **WINDOW REPAIR** Cathy is fixing a window frame. She fit two strips of wood together to make the crosspieces. For the glass panes to fit, each angle of the crosspieces must be a right angle. Must Cathy measure all four angles to be sure they are all right angles? Explain.



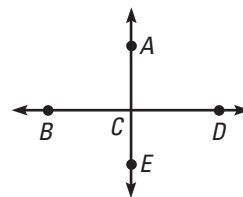
26. **MULTIPLE CHOICE** Which of the following is true if $g \perp h$?

- (A) $m\angle 1 + m\angle 2 > 180^\circ$
 (B) $m\angle 1 + m\angle 2 < 180^\circ$
 (C) $m\angle 1 + m\angle 2 = 180^\circ$
 (D) Cannot be determined



27. **MULTIPLE CHOICE** Which of the following must be true if $m\angle ACD = 90^\circ$?

- I. $\angle BCE$ is a right angle.
 II. $\overleftrightarrow{AE} \perp \overleftrightarrow{BD}$
 III. $\angle BCA$ and $\angle BCE$ are complementary.
 (A) I only (B) I and II only (C) III only
 (D) I, II, and III (E) None of these



★ Challenge

28. **REFLECTIONS** Ann has a full-length mirror resting against the wall of her room. Ann notices that the floor and its reflection do not form a straight angle. She concludes that the mirror is not perpendicular to the floor. Explain her reasoning.



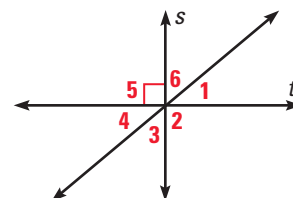
EXTRA CHALLENGE

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MIXED REVIEW

ANGLE MEASURES Complete the statement given that $s \perp t$. (Review 2.6 for 3.3)

29. If $m\angle 1 = 38^\circ$, then $m\angle 4 = \underline{\hspace{1cm}}$.
 30. $m\angle 2 = \underline{\hspace{1cm}}$
 31. If $m\angle 6 = 51^\circ$, then $m\angle 1 = \underline{\hspace{1cm}}$.
 32. If $m\angle 3 = 42^\circ$, then $m\angle 1 = \underline{\hspace{1cm}}$.



ANGLES List all pairs of angles that fit the description. (Review 3.1)

33. Corresponding angles
 34. Alternate interior angles
 35. Alternate exterior angles
 36. Consecutive interior angles

