

Challenge: Skills and Applications

For use with pages 96–101

1.
 - a. How is the product $4 \cdot 6$ related to 5^2 ?
 - b. How is the product $5 \cdot 7$ related to 6^2 ?
 - c. Make a conjecture about how the product of two positive integers n and $n + 2$ is related to the square of the integer between them.
 - d. Write a convincing argument to justify your conjecture.
2.
 - a. Find the value of $\frac{1}{1 \cdot 2}$, $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3}$, and $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4}$.
 - b. Conjecture the value of $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)}$.
 - c. Prove your conjecture. (Hint: $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$)
3.
 - a. Prove: The sum of any two consecutive positive integers is an odd number.
 - b. Prove: The sum of any three consecutive positive integers is a multiple of 3.
4.
 - a. Prove: The square of an even integer is always an even integer.
 - b. Prove: The square of an odd integer is always an odd integer.

In Exercises 5–8, decide whether the relationship is *reflexive*, *symmetric*, and/or *transitive*.

5. **Set:** cities

Relationship: “is at least as large as”

Example: Los Angeles is at least as large as Cincinnati.

6. **Set:** angles

Relationship: “is a supplement of”

Example: $\angle A$ is a supplement of $\angle B$.

7. **Set:** line segments

Relationship: “is the same length as”

Example: \overline{PQ} is the same length as \overline{RS} .

8. **Set:** triangles

Relationship: “has a larger perimeter than”

Example: $\triangle ABC$ has a larger perimeter than $\triangle DEF$.