CHAPTER 2

Chapter Summary

WHAT did you learn?	WHY did you learn it?
Recognize and analyze conditional statements, and write their inverses, converses, and contrapositives. (2.1)	Use postulates about points, lines, and planes to analyze real-life objects, such as a research buggy. (p. 77)
Recognize and use definitions and biconditional statements. (2.2)	Rewrite postulates in a form suitable for solving a particular problem, such as analyzing geographic relations. (p. 84)
Use symbolic notation to represent logical statements. (2.3)	Decide whether a logical statement is valid. (p. 89)
Use the laws of logic to write a logical argument. (2.3)	Write true statements about birds using a list of facts about birds. (p. 90)
Use properties from algebra. (2.4)	Use properties from algebra to solve equations, such as an athlete's target heart rate. (p. 97)
Use properties of length and measure. (2.4)	Find the measure of the angle of a banked turn at the Talladega Superspeedway. (p. 98)
Use the properties of segment congruence to prove statements about segments. (2.5)	Prove statements about segments in real life, such as the segments in a trestle bridge. (p. 104)
Use the properties of angle congruence to prove properties about special pairs of angles. (2.6)	Decide which angles are congruent when constructing a picture frame. (p. 115)

How does it fit into the BIGGER PICTURE of geometry?

In this chapter, you were introduced to the formal side of geometry. You learned that the structure of geometry consists of undefined terms, defined terms, postulates, and theorems. You were also introduced to the need for proofs, and the form of a proof. In later chapters, you will study other ways to write proofs. The goal of writing a proof will remain the same—to convince a person about the truth of a statement.

STUDY STRATEGY

How was previewing each lesson helpful?

Some of the notes you made while previewing a lesson, following the **Study Strategy** on page 70, may resemble these.

Previewing Lesson 2.1

- Counterexamples were used in Lesson 1.1 to show that a statement was not always true. Lesson 2.1 seems to use counterexamples for the same reason.
- Postulate was defined in Chapter 1, and four postulates were presented. Lesson 2.1 presents seven more postulates.

CHAPTER 2

Chapter Review

VOCABULARY

- conditional statement, p. 71
- if-then form, p. 71
- hypothesis, p. 71
- conclusion, p. 71
- converse, p. 72
- negation, p. 72
- inverse, p. 72
- contrapositive, p. 72
- equivalent statement, p. 72
- perpendicular lines, p. 79
- line perpendicular to a plane, p. 79
- biconditional statement, p. 80
- logical argument, p. 89
- Law of Detachment, p. 89
- Law of Syllogism, p. 90
- theorem, p. 102
- two-column proof, p. 102
- paragraph proof, p. 102

2.1 CONDITIONAL STATEMENTS

Examples on pp. 71–74

EXAMPLES

If-then form	If a person is 2 meters tall, then he or she is 6.56 feet tall.
Inverse	If a person is not 2 meters tall, then he or she is not 6.56 feet tall.
Converse	If a person is 6.56 feet tall, then he or she is 2 meters tall.
Contrapositive	If a person is not 6.56 feet tall, then he or she is not 2 meters tall.

Write the statement in if-then form. Determine the hypothesis and conclusion, and write the inverse, converse, and contrapositive.

1. We are dismissed early if there is a teacher's meeting.

2. I prepare dinner on Wednesday nights.

Fill in the blank. Then draw a sketch that illustrates your answer.

3. Through any three noncollinear points there exists __?__ plane.

4. A line contains at least __?__ points.

2.2 DEFINITIONS AND BICONDITIONAL STATEMENTS

Examples on pp. 79–81

EXAMPLE The statement "If a number ends in 0, then the number is divisible by 10," and its converse "If a number is divisible by 10, then the number ends in 0," are both true. This means that the statement can be written as the true biconditional statement, "A number is divisible by 10 if and only if it ends in 0."

Can the statement be written as a true biconditional statement?

5. If $x = 5$, then $x^2 = 25$.

6. A rectangle is a square if it has four congruent sides.

2.3 Deductive Reasoning

Examples on pp. 87–90

EXAMPLES Using symbolic notation, let p be "it is summer" and let q be "school is closed."

Statement	$p \to q$	If it is summer, then school is closed.
Inverse	$\sim p \to \sim q$	If it is not summer, then school is not closed.
Converse	$q \to p$	If the school is closed, then it is summer.
Contrapositive	$\sim q \to \sim p$	If school is not closed, then it is not summer.

Write the symbolic statement in words using *p* and *q* given below.

p: $\angle A$ is a right angle. q: The measure of $\angle A$ is 90°.

7. $q \to p$ **8.** $\sim q \to \sim p$ **9.** $\sim p$ **10.** $\sim p \to \sim q$

Use the Law of Syllogism to write the statement that follows from the pair of true statements.

11. If there is a nice breeze, then the mast is up.
If the mast is up, then we will sail to Dunkirk.

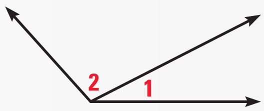
12. If Chess Club meets today, then it is Thursday.
If it is Thursday, then the garbage needs to be taken out.

2.4 Reasoning with Properties from Algebra

Examples on pp. 96–98

EXAMPLE In the diagram, $m\angle 1 + m\angle 2 = 132°$ and $m\angle 2 = 105°$. The argument shows that $m\angle 1 = 27°$.

$m\angle 1 + m\angle 2 = 132°$	Given
$m\angle 2 = 105°$	Given
$m\angle 1 + 105° = 132°$	Substitution property of equality
$m\angle 1 = 27°$	Subtraction property of equality



Match the statement with the property.

13. If $m\angle S = 45°$, then $m\angle S + 45° = 90°$.

14. If $UV = VW$, then $VW = UV$.

15. If $AE = EG$ and $EG = JK$, then $AE = JK$.

16. If $m\angle K = 9°$, then $3(m\angle K) = 27°$.

A. Symmetric property of equality

B. Multiplication property of equality

C. Addition property of equality

D. Transitive property of equality

Solve the equation and state a reason for each step.

17. $5(3y + 2) = 25$ **18.** $8t - 4 = 5t + 8$ **19.** $23 + 11d - 2c = 12 - 2c$

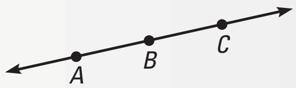
2.5 PROVING STATEMENTS ABOUT SEGMENTS

Examples on pp. 102–104

EXAMPLE A proof that shows $AC = 2 \cdot BC$ is shown below.

GIVEN ▶ $AB = BC$

PROVE ▶ $AC = 2 \cdot BC$

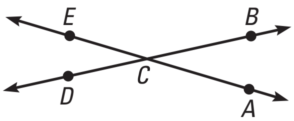


Statements	Reasons
1. $AB = BC$	**1.** Given
2. $AC = AB + BC$	**2.** Segment Addition Postulate
3. $AC = BC + BC$	**3.** Substitution property of equality
4. $AC = 2 \cdot BC$	**4.** Distributive property

20. Write a two-column proof.

GIVEN ▶ $\overline{AE} \cong \overline{BD}$, $\overline{CD} \cong \overline{CE}$

PROVE ▶ $\overline{AC} \cong \overline{BC}$



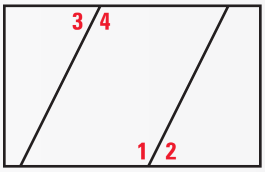
2.6 PROVING STATEMENTS ABOUT ANGLES

Examples on pp. 109–112

EXAMPLE A proof that shows $\angle 2 \cong \angle 3$ is shown below.

GIVEN ▶ $\angle 1$ and $\angle 2$ form a linear pair,
$\angle 3$ and $\angle 4$ form a linear pair,
$\angle 1 \cong \angle 4$

PROVE ▶ $\angle 2 \cong \angle 3$

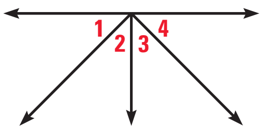


Statements	Reasons
1. $\angle 1$ and $\angle 2$ form a linear pair, $\angle 3$ and $\angle 4$ form a linear pair, $\angle 1 \cong \angle 4$	**1.** Given
2. $\angle 1$ and $\angle 2$ are supplementary, $\angle 3$ and $\angle 4$ are supplementary	**2.** Linear Pair Postulate
3. $\angle 2 \cong \angle 3$	**3.** Congruent Supplements Theorem

21. Write a two-column proof using the given information.

GIVEN ▶ $\angle 1$ and $\angle 2$ are complementary,
$\angle 3$ and $\angle 4$ are complementary,
$\angle 1 \cong \angle 3$

PROVE ▶ $\angle 2 \cong \angle 4$



CHAPTER 2

Chapter Test

State the postulate that shows that the statement is false.

1. Plane R contains only two points A and B.

2. Plane M and plane N are two distinct planes that intersect at exactly two distinct points.

3. Any three noncollinear points define at least three distinct planes.

4. Points A and B are two distinct points in plane Q. Line $\overleftrightarrow{AB}$ does not intersect plane Q.

Find a counterexample that demonstrates that the converse of the statement is false.

5. If an angle measures 34°, then the angle is acute.

6. If the lengths of two segments are each 17 feet, then the segments are congruent.

7. If two angles measure 32° and 148°, then they are supplementary.

8. If you chose number 13, then you chose a prime number.

State what conclusions can be made if $x = 5$ and the given statement is true.

9. If $x > x - 2$, then $y = 14x$.

10. If $-x < 2x < 11$, then $x = y - 12$.

11. If $|x| > -x$, then $y = -x$.

12. If $y = 4x$, then $z = 2x + y$.

In Exercises 13–16, name the property used to make the conclusion.

13. If $13 = x$, then $x = 13$.

14. If $x = 3$, then $5x = 15$.

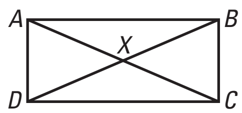
15. If $x = y$ and $y = 4$, then $x = 4$.

16. If $x + 3 = 17$, then $x = 14$.

17. **PROOF** Write a two-column proof.

GIVEN ▶ $\overline{AX} \cong \overline{DX}$, $\overline{XB} \cong \overline{XC}$

PROVE ▶ $\overline{AC} \cong \overline{BD}$



18. **PLUMBING** A plumber is replacing a small section of a leaky pipe. To find the length of new pipe that he will need, he first measures the leaky section of the old pipe with a steel tape measure, and then uses this measure to find the same length of new pipe. What property of segment congruence does this process illustrate? Use the wording of the property to explain how it is illustrated.

19. **PACKAGING** A tool and die company produces a part that is to be packed in triangular boxes. To maximize space and minimize cost, the boxes need to be designed to fit together in shipping cartons. If $\angle 1$ and $\angle 2$ have to be complementary, $\angle 3$ and $\angle 4$ have to be complementary, and $m\angle 2 = m\angle 3$, describe the relationship between $\angle 1$ and $\angle 4$.