2.5 Proving Statements about Segments

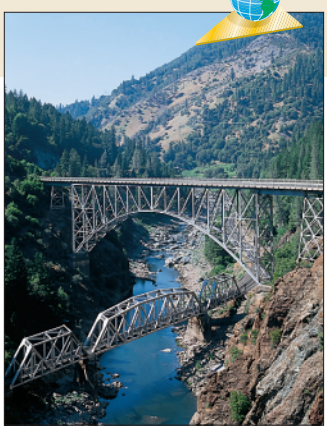
What you should learn

GOAL 1 Justify statements about congruent segments.

GOAL 2 Write reasons for steps in a proof.

Why you should learn it

▼ Properties of congruence allow you to justify segment relationships in **real life**, such as the segments in the trestle bridge shown and in **Exs. 3–5.**



GOAL 1 PROPERTIES OF CONGRUENT SEGMENTS

A true statement that follows as a result of other true statements is called a **theorem**. All theorems must be proved. You can prove a theorem using a *two-column proof*. A **two-column proof** has numbered statements and reasons that show the logical order of an argument.

THEOREM

THEOREM 2.1 ***Properties of Segment Congruence***

Segment congruence is reflexive, symmetric, and transitive. Here are some examples:

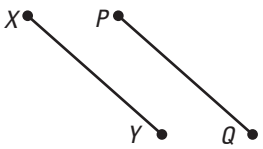
REFLEXIVE For any segment AB, $\overline{AB} \cong \overline{AB}$.

SYMMETRIC If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

TRANSITIVE If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

EXAMPLE 1 *Symmetric Property of Segment Congruence*

You can prove the Symmetric Property of Segment Congruence as follows.



GIVEN ▶ $\overline{PQ} \cong \overline{XY}$

PROVE ▶ $\overline{XY} \cong \overline{PQ}$

Statements	Reasons
1. $\overline{PQ} \cong \overline{XY}$	1. Given
2. $PQ = XY$	2. Definition of congruent segments
3. $XY = PQ$	3. Symmetric property of equality
4. $\overline{XY} \cong \overline{PQ}$	4. Definition of congruent segments

STUDENT HELP

Study Tip

When writing a reason for a step in a proof, you must use one of the following: given information, a definition, a property, a postulate, or a previously proven theorem.

You are asked to complete proofs for the Reflexive and Transitive Properties of Segment Congruence in Exercises 6 and 7.

. . . . . .

A proof can be written in paragraph form, called **paragraph proof**. Here is a paragraph proof for the Symmetric Property of Segment Congruence.

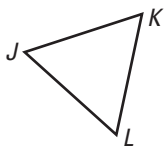
Paragraph Proof You are given that $\overline{PQ} \cong \overline{XY}$. By the definition of congruent segments, $PQ = XY$. By the symmetric property of equality, $XY = PQ$. Therefore, by the definition of congruent segments, it follows that $\overline{XY} \cong \overline{PQ}$.

GOAL 2 USING CONGRUENCE OF SEGMENTS

EXAMPLE 2 *Using Congruence*

Proof

Use the diagram and the given information to complete the missing steps and reasons in the proof.



GIVEN ▶ $LK = 5, JK = 5, \overline{JK} \cong \overline{JL}$

PROVE ▶ $\overline{LK} \cong \overline{JL}$

Statements	Reasons
1. **a.**	1. Given
2. **b.**	2. Given
3. $LK = JK$	3. Transitive property of equality
4. $\overline{LK} \cong \overline{JK}$	4. **c.**
5. $\overline{JK} \cong \overline{JL}$	5. Given
6. **d.**	6. Transitive Property of Congruence

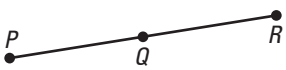
SOLUTION

a. $LK = 5$ **b.** $JK = 5$ **c.** Definition of congruent segments **d.** $\overline{LK} \cong \overline{JL}$

EXAMPLE 3 *Using Segment Relationships*

Proof

In the diagram, Q is the midpoint of $\overline{PR}$.



Show that PQ and QR are each equal to $\frac{1}{2}PR$.

SOLUTION

Decide what you know and what you need to prove. Then write the proof.

GIVEN ▶ Q is the midpoint of $\overline{PR}$.

PROVE ▶ $PQ = \frac{1}{2}PR$ and $QR = \frac{1}{2}PR$.

Statements	Reasons
1. Q is the midpoint of $\overline{PR}$.	1. Given
2. $PQ = QR$	2. Definition of midpoint
3. $PQ + QR = PR$	3. Segment Addition Postulate
4. $PQ + PQ = PR$	4. Substitution property of equality
5. $2 \cdot PQ = PR$	5. Distributive property
6. $PQ = \frac{1}{2}PR$	6. Division property of equality
7. $QR = \frac{1}{2}PR$	7. Substitution property of equality

STUDENT HELP

Study Tip

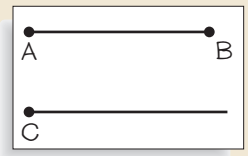
The distributive property can be used to simplify a sum, as in Step 5 of the proof. You can think of $PQ + PQ$ as follows: $1(PQ) + 1(PQ) = (1 + 1)(PQ) = 2 \cdot PQ$.

ACTIVITY

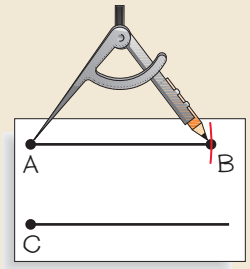
Construction

Copy a Segment

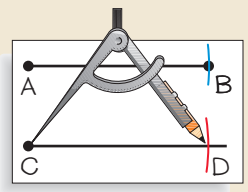
Use the following steps to construct a segment that is congruent to $\overline{AB}$.



1 Use a straightedge to draw a segment longer than $\overline{AB}$. Label the point C on the new segment.



2 Set your compass at the length of $\overline{AB}$.



3 Place the compass point at C and mark a second point, D, on the new segment. $\overline{CD}$ is congruent to $\overline{AB}$.

You will practice copying a segment in Exercises 12–15. It is an important construction because copying a segment is used in many constructions throughout this course.

GUIDED PRACTICE

Vocabulary Check ✓

1. An example of the Symmetric Property of Segment Congruence is "If $\overline{AB} \cong$ ___?___, then $\overline{CD} \cong$ ___?___."

Concept Check ✓

2. **ERROR ANALYSIS** In the diagram below, $\overline{CB} \cong \overline{SR}$ and $\overline{CB} \cong \overline{QR}$. Explain what is wrong with Michael's argument.

Because $\overline{CB} \cong \overline{SR}$ and $\overline{CB} \cong \overline{QR}$, then $\overline{CB} \cong \overline{AC}$ by the Transitive Property of Segment Congruence.

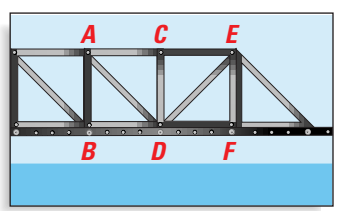
Skill Check ✓

BRIDGES **The diagram below shows a portion of a trestle bridge, where $\overline{BF} \perp \overline{CD}$ and D is the midpoint of $\overline{BF}$.**

3. Give a reason why $\overline{BD}$ and $\overline{FD}$ are congruent.

4. Are $\angle CDE$ and $\angle FDE$ complementary? Explain.

5. If $\overline{CE}$ and $\overline{BD}$ are congruent, explain why $\overline{CE}$ and $\overline{FD}$ are congruent.



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

PROVING THEOREM 2.1 **Copy and complete the proof for two of the cases of the Properties of Segment Congruence Theorem.**

6. Reflexive Property of Segment Congruence

GIVEN ▶ EF is a line segment

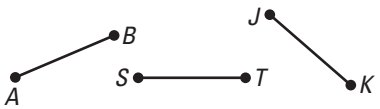
PROVE ▶ $\overline{EF} \cong \overline{EF}$

Statements	Reasons
1. $EF = EF$	1. ?
2. ?	2. Definition of congruent segments

7. Transitive Property of Segment Congruence

GIVEN ▶ $\overline{AB} \cong \overline{JK}$, $\overline{JK} \cong \overline{ST}$

PROVE ▶ $\overline{AB} \cong \overline{ST}$

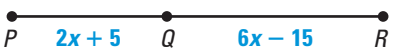


Statements	Reasons
1. $\overline{AB} \cong \overline{JK}$, $\overline{JK} \cong \overline{ST}$	1. ?
2. $AB = JK$, $JK = ST$	2. ?
3. $AB = ST$	3. ?
4. $\overline{AB} \cong \overline{ST}$	4. ?

USING ALGEBRA **Solve for the variable using the given information. Explain your steps.**

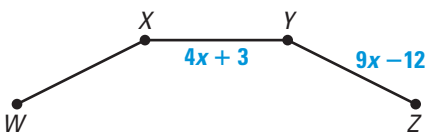
8. GIVEN ▶ $\overline{AB} \cong \overline{BC}$, $\overline{CD} \cong \overline{BC}$

9. GIVEN ▶ $PR = 46$

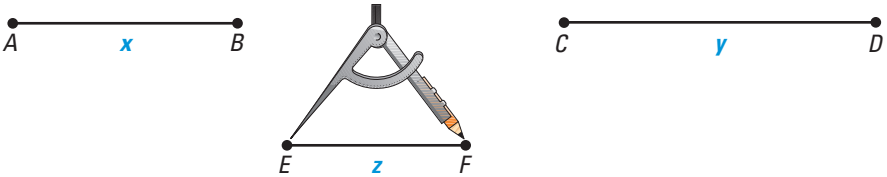


10. GIVEN ▶ $\overline{ST} \cong \overline{SR}$, $\overline{QR} \cong \overline{SR}$

11. GIVEN ▶ $\overline{XY} \cong \overline{WX}$, $\overline{YZ} \cong \overline{WX}$



CONSTRUCTION **In Exercises 12–15, use the segments, along with a straightedge and compass, to construct a segment with the given length.**



12. $x + y$ **13.** $y - z$ **14.** $3x - z$ **15.** $z + y - 2x$

STUDENT HELP

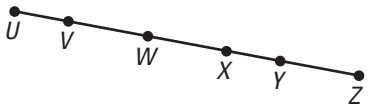
HOMEWORK HELP

Example 1: Exs. 6, 7
Example 2: Exs. 16–18
Example 3: Exs. 16–18

16. DEVELOPING PROOF Write a complete proof by rearranging the reasons listed on the pieces of paper.

GIVEN ▶ $\overline{UV} \cong \overline{XY}$, $\overline{VW} \cong \overline{WX}$, $\overline{WX} \cong \overline{YZ}$

PROVE ▶ $\overline{UW} \cong \overline{XZ}$

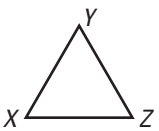


Statements	Reasons
1. $\overline{UV} \cong \overline{XY}$, $\overline{VW} \cong \overline{WX}$, $\overline{WX} \cong \overline{YZ}$	Transitive Property of Segment Congruence
2. $\overline{VW} \cong \overline{YZ}$	Addition property of equality
3. $UV = XY$, $VW = YZ$	Definition of congruent segments
4. $UV + VW = XY + YZ$	Given
5. $UV + VW = UW$, $XY + YZ = XZ$	Segment Addition Postulate
6. $UW = XZ$	Definition of congruent segments
7. $\overline{UW} \cong \overline{XZ}$	Substitution property of equality

TWO-COLUMN PROOF **Write a two-column proof.**

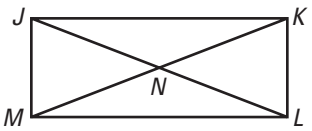
17. GIVEN ▶ $XY = 8$, $XZ = 8$, $\overline{XY} \cong \overline{ZY}$

PROVE ▶ $\overline{XZ} \cong \overline{ZY}$

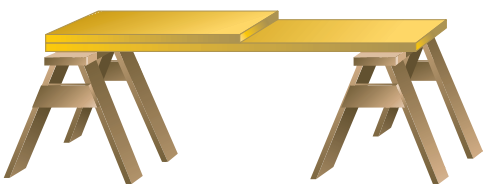


18. GIVEN ▶ $\overline{NK} \cong \overline{NL}$, $NK = 13$

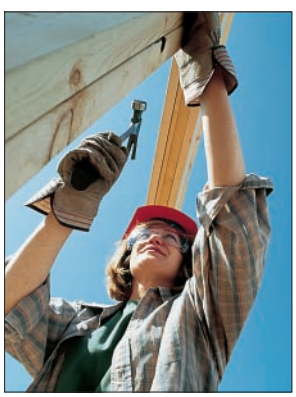
PROVE ▶ $NL = 13$



19. CARPENTRY You need to cut ten wood planks that are the same size. You measure and cut the first plank. You cut the second piece, using the first plank as a guide, as in the diagram below. The first plank is put aside and the second plank is used to cut a third plank. You follow this pattern for the rest of the planks. Is the last plank the same length as the first plank? Explain.



FOCUS ON CAREERS



REAL LIFE CARPENTRY
For many projects, carpenters need boards that are all the same length. For instance, equally-sized boards in the house frame above insure stability.

INTERNET CAREER LINK
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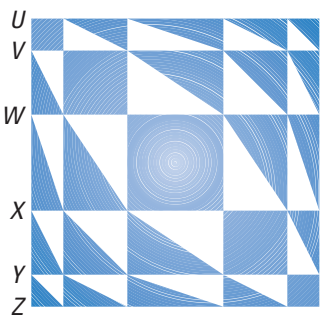
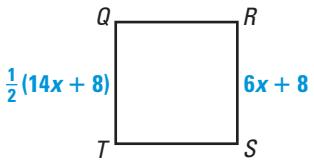
20. OPTICAL ILLUSION To create the illusion, a special grid was used. In the grid, corresponding row heights are the same measure. For instance, $\overline{UV}$ and $\overline{ZY}$ are congruent. You decide to make this design yourself. You draw the grid, but you need to make sure that the row heights are the same. You measure $\overline{UV}$, $\overline{UW}$, $\overline{ZY}$, and $\overline{ZX}$. You find that $\overline{UV} \cong \overline{ZY}$ and $\overline{UW} \cong \overline{ZX}$. Write an argument that allows you to conclude that $\overline{VW} \cong \overline{YX}$.



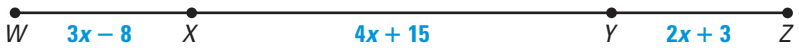
Test Preparation

21. MULTIPLE CHOICE In $QRST$, $\overline{QT} \cong \overline{TS}$ and $\overline{RS} \cong \overline{TS}$. What is x?



(A) 1 (B) 4 (C) 12 (D) 16 (E) 32

22. MULTIPLE CHOICE In the figure shown below, $\overline{WX} \cong \overline{YZ}$. What is the length of $\overline{XZ}$?



(A) 25 (B) 34 (C) 59 (D) 60 (E) 84

Challenge

REPRESENTING SEGMENT LENGTHS **In Exercises 23–26, suppose point *T* is the midpoint of $\overline{RS}$ and point *W* is the midpoint of $\overline{RT}$. If $\overline{XY} \cong \overline{RT}$ and $\overline{TS}$ has a length of *z*, write the length of the segment in terms of *z*.**

23. $\overline{RT}$ **24.** $\overline{XY}$ **25.** $\overline{RW}$ **26.** $\overline{WT}$

EXTRA CHALLENGE
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27. CRITICAL THINKING Suppose M is the midpoint of $\overline{AB}$, P is the midpoint of $\overline{AM}$, and Q is the midpoint of $\overline{PM}$. If a and b are the coordinates of points A and B on a number line, find the coordinates of P and Q in terms of a and b.

MIXED REVIEW

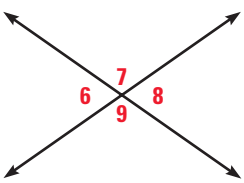
FINDING COUNTEREXAMPLES **Find a counterexample that shows the statement is false.** (Review 1.1)

28. For every number n, $2^n > n + 1$.

29. The sum of an even number and an odd number is always even.

30. If a number is divisible by 5, then it is divisible by 10.

FINDING ANGLE MEASURES **In Exercises 31–34, use the diagram to find the angle measure.** (Review 1.6 for 2.6)



31. If $m\angle 6 = 64°$, then $m\angle 7 =$ ___?___.

32. If $m\angle 8 = 70°$, then $m\angle 6 =$ ___?___.

33. If $m\angle 9 = 115°$, then $m\angle 8 =$ ___?___.

34. If $m\angle 7 = 108°$, then $m\angle 8 =$ ___?___.

35. Write the contrapositive of the conditional statement, "If Matthew wins this wrestling match, then he will win first place." (Review 2.1)

36. Is the converse of a true conditional statement always true? Explain. (Review 2.1)

USING SYMBOLIC NOTATION **Let *p* be "the car is in the garage" and let *q* be "Mark is home." Write the statement in words and symbols.** (Review 2.3)

37. The conditional statement $p \rightarrow q$ **38.** The converse of $p \rightarrow q$

39. The inverse of $p \rightarrow q$ **40.** The contrapositive of $p \rightarrow q$