2.4 Reasoning with Properties from Algebra

What you should learn

GOAL 1 Use properties from algebra.

GOAL 2 Use properties of length and measure to justify segment and angle relationships, such as the angles at the turns of a racetrack, as in **Example 5** and **Ex. 28**.

Why you should learn it

▼ Using algebraic properties helps you when rewriting a formula, such as the formula for an athlete's target heart rate in **Example 3**.

REAL LIFE

GOAL 1 Using Properties from Algebra

Many properties from algebra concern the equality of real numbers. Several of these are summarized in the following list.

ALGEBRAIC PROPERTIES OF EQUALITY

Let a, b, and c be real numbers.

ADDITION PROPERTY	If $a = b$, then $a + c = b + c$.
SUBTRACTION PROPERTY	If $a = b$, then $a - c = b - c$.
MULTIPLICATION PROPERTY	If $a = b$, then $ac = bc$.
DIVISION PROPERTY	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
REFLEXIVE PROPERTY	For any real number a, $a = a$.
SYMMETRIC PROPERTY	If $a = b$, then $b = a$.
TRANSITIVE PROPERTY	If $a = b$ and $b = c$, then $a = c$.
SUBSTITUTION PROPERTY	If $a = b$, then a can be substituted for b in any equation or expression.

Properties of equality along with other properties from algebra, such as the *distributive property*,

$$a(b + c) = ab + ac$$

can be used to solve equations. For instance, you can use the subtraction property of equality to solve the equation $x + 3 = 7$. By subtracting 3 from each side of the equation, you obtain $x = 4$.

EXAMPLE 1 Writing Reasons

Solve $5x - 18 = 3x + 2$ and write a reason for each step.

Using Algebra

SOLUTION

$5x - 18 = 3x + 2$	Given
$2x - 18 = 2$	Subtraction property of equality
$2x = 20$	Addition property of equality
$x = 10$	Division property of equality

EXAMPLE 2 Writing Reasons

STUDENT HELP
INTERNET HOMEWORK HELP
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Solve $55z - 3(9z + 12) = -64$ and write a reason for each step.

SOLUTION

$55z - 3(9z + 12) = -64$	**Given**
$55z - 27z - 36 = -64$	**Distributive property**
$28z - 36 = -64$	**Simplify.**
$28z = -28$	**Addition property of equality**
$z = -1$	**Division property of equality**

EXAMPLE 3 Using Properties in Real Life

REAL LIFE **FITNESS** Before exercising, you should find your target heart rate. This is the rate at which you achieve an effective workout while not placing too much strain on your heart. Your target heart rate r (in beats per minute) can be determined from your age a (in years) using the equation $a = 220 - \frac{10}{7}r$.

a. Solve the formula for r and write a reason for each step.

b. Use the result to find the target heart rate for a 16 year old.

c. Find the target heart rate for the following ages: 20, 30, 40, 50, and 60. What happens to the target heart rate as a person gets older?

SOLUTION

a.

$a = 220 - \frac{10}{7}r$	**Given**
$a + \frac{10}{7}r = 220$	**Addition property of equality**
$\frac{10}{7}r = 220 - a$	**Subtraction property of equality**
$r = \frac{7}{10}(220 - a)$	**Multiplication property of equality**

b. Using $a = 16$, the target heart rate is:

$r = \frac{7}{10}(220 - a)$	**Given**
$r = \frac{7}{10}(220 - 16)$	**Substitute 16 for *a*.**
$r = 142.8$	**Simplify.**

▶ The target heart rate for a 16 year old is about 143 beats per minute.

c. From the table, the target heart rate appears to decrease as a person ages.

Age	20	30	40	50	60
Rate	140	133	126	119	112

GOAL 2 USING PROPERTIES OF LENGTH AND MEASURE

The algebraic properties of equality can be used in geometry.

CONCEPT SUMMARY — PROPERTIES OF EQUALITY

	SEGMENT LENGTH	ANGLE MEASURE
REFLEXIVE	For any segment AB, $AB = AB$.	For any angle A, $m\angle A = m\angle A$.
SYMMETRIC	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
TRANSITIVE	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Logical Reasoning

EXAMPLE 4 Using Properties of Length

In the diagram, $AB = CD$. The argument below shows that $AC = BD$.

$AB = CD$	Given
$AB + BC = BC + CD$	Addition property of equality
$AC = AB + BC$	Segment Addition Postulate
$BD = BC + CD$	Segment Addition Postulate
$AC = BD$	Substitution property of equality

EXAMPLE 5 Using Properties of Measure

AUTO RACING The Talladega Superspeedway racetrack in Alabama has four banked turns, which are described in the diagram at the left. Use the given information about the maximum banking angle of the four turns to find $m\angle 4$.

$m\angle 1 + m\angle 2 = 66°$

$m\angle 1 + m\angle 2 + m\angle 3 = 99°$

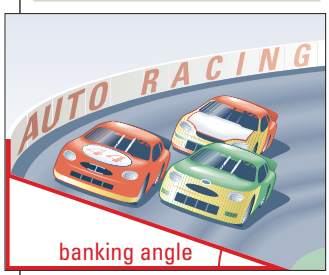
$m\angle 3 = m\angle 1$

$m\angle 1 = m\angle 4$

SOLUTION

$m\angle 1 + m\angle 2 = 66°$	Given
$m\angle 1 + m\angle 2 + m\angle 3 = 99°$	Given
$66° + m\angle 3 = 99°$	Substitution property of equality
$m\angle 3 = 33°$	Subtraction property of equality
$m\angle 3 = m\angle 1$, $m\angle 1 = m\angle 4$	Given
$m\angle 3 = m\angle 4$	Transitive property of equality
$m\angle 4 = 33°$	Substitution property of equality

FOCUS ON APPLICATIONS



REAL LIFE **AUTO RACING** Banked turns help the cars travel around the track at high speeds. The angles provide an inward force that helps keep the cars from flying off the track.

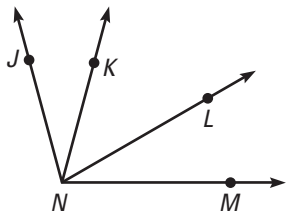
GUIDED PRACTICE

Vocabulary Check ✓

1. Name the property that makes the following statement true: "If $m\angle 3 = m\angle 5$, then $m\angle 5 = m\angle 3$."

Concept Check ✓

Use the diagram at the right.



2. Explain how the addition property of equality supports this statement: "If $m\angle JNK = m\angle LNM$, then $m\angle JNL = m\angle KNM$."

3. Explain how the subtraction property of equality supports this statement: "If $m\angle JNL = m\angle KNM$, then $m\angle JNK = m\angle LNM$."

Skill Check ✓

In Exercises 4–8, match the conditional statement with the property of equality.

4. If $JK = PQ$ and $PQ = ST$, then $JK = ST$.

5. If $m\angle S = 30°$, then $5° + m\angle S = 35°$.

6. If $ST = 2$ and $SU = ST + 3$, then $SU = 5$.

7. If $m\angle K = 45°$, then $3(m\angle K) = 135°$.

8. If $m\angle P = m\angle Q$, then $m\angle Q = m\angle P$.

A. Addition property

B. Substitution property

C. Transitive property

D. Symmetric property

E. Multiplication property

9. **WIND-CHILL FACTOR** If the wind is blowing at 20 miles per hour, you can find the wind-chill temperature W (in degrees Fahrenheit) by using the equation $W = 1.42T - 38.5$, where T is the actual temperature (in degrees Fahrenheit). Solve this equation for T and write a reason for each step. What is the actual temperature if the wind chill temperature is -24.3°F and the wind is blowing at 20 miles per hour?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

COMPLETING STATEMENTS **In Exercises 10–14, use the property to complete the statement.**

10. Symmetric property of equality: If $m\angle A = m\angle B$, then ___?___.

11. Transitive property of equality: If $BC = CD$ and $CD = EF$, then ___?___.

12. Substitution property of equality: If $LK + JM = 12$ and $LK = 2$, then ___?___.

13. Subtraction property of equality: If $PQ + ST = RS + ST$, then ___?___.

14. Division property of equality: If $3(m\angle A) = 90°$, then $m\angle A =$ ___?___.

15. Copy and complete the argument below, giving a reason for each step.

$2(3x + 1) = 5x + 14$	**Given**
$6x + 2 = 5x + 14$	___?___
$x + 2 = 14$	___?___
$x = 12$	___?___

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 10–23
Example 2: Exs. 15–23
Example 3: Exs. 29–31
Example 4: Exs. 24–27
Example 5: Ex. 28

SOLVING EQUATIONS **In Exercises 16–23, solve the equation and state a reason for each step.**

16. $p - 1 = 6$

17. $q + 9 = 13$

18. $2r - 7 = 9$

19. $7s + 20 = 4s - 13$

20. $3(2t + 9) = 30$

21. $-2(-w + 3) = 15$

22. $26u + 4(12u - 5) = 128$

23. $3(4v - 1) - 8v = 17$

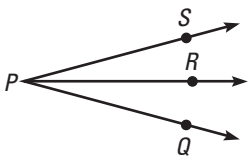
24. **LOGICAL REASONING** In the diagram, $m\angle RPQ = m\angle RPS$. Verify each step in the argument that shows $m\angle SPQ = 2(m\angle RPQ)$.

$m\angle RPQ = m\angle RPS$

$m\angle SPQ = m\angle RPQ + m\angle RPS$

$m\angle SPQ = m\angle RPQ + m\angle RPQ$

$m\angle SPQ = 2(m\angle RPQ)$



25. **LOGICAL REASONING** In the diagram, $m\angle ABF = m\angle BCG$ and $m\angle ABF = 90°$. Verify each step in the argument that shows $\overleftrightarrow{GK} \perp \overleftrightarrow{AD}$.

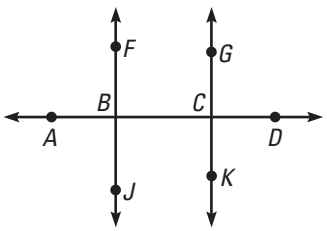
$m\angle ABF = 90°$

$m\angle ABF = m\angle BCG$

$m\angle BCG = 90°$

$\angle BCG$ is a right angle.

$\overleftrightarrow{GK} \perp \overleftrightarrow{AD}$



DEVELOPING ARGUMENTS **In Exercises 26 and 27, give an argument for the statement, including a reason for each step.**

26. If $\angle 1$ and $\angle 2$ are right angles, then they are supplementary.

27. If B lies between A and C and $AB = 3$ and $BC = 8$, then $AC = 11$.

FOCUS ON PEOPLE

BILL ELLIOTT holds the qualifying record at Daytona International Speedway with a speed of 210.364 miles per hour.

28. **AUTO RACING** Some facts about the maximum banking angles of Daytona International Speedway at corners 1, 2, 3, and 4 are at the right. Find $m\angle 3$. Explain your steps. (Banked corners are described on page 98.)

$m\angle 1 + m\angle 3 + m\angle 4 = 93°$

$m\angle 2 + m\angle 4 = 62°$

$m\angle 2 = m\angle 3$

$m\angle 1 = m\angle 2$

PAY RAISES **In Exercises 29–31, suppose you receive a raise at work. You can calculate your percent increase by using the pay raise formula $c(r + 1) = n$, where c is your current wage (in dollars per hour), r is your percent increase (as a decimal), and n is your new wage (in dollars per hour).**

29. Solve the formula for r and write a reason for each step.

30. Use the result from Exercise 29 to find your percent increase if your current wage is \$10.00 and your new wage will be \$10.80.

31. Suppose Donald gets a 6% pay raise and his new wage is \$12.72. Find Donald's old wage. Explain the steps you used to find your answer.

Test Preparation

32. MULTI-STEP PROBLEM State a reason that makes the statement true.

a. If $4(x - 5 + 2x) = 0.5(12x - 16)$, then $4x - 20 + 8x = 6x - 8$.

b. If $4x - 20 + 8x = 6x - 8$, then $12x - 20 = 6x - 8$.

c. If $12x - 20 = 6x - 8$, then $6x - 20 = -8$.

d. If $6x - 20 = -8$, then $6x = 12$.

e. If $6x = 12$, then $x = 2$.

f. *Writing* Use parts (a) through (e) to provide an argument for "If $4(x - 5 + 2x) = 0.5(12x - 16)$, then $x = 2$."

★ Challenge

DETERMINING PROPERTIES **Decide whether the relationship is *reflexive*, *symmetric*, or *transitive*. When the relationship does not have any of these properties, give a counterexample.**

33. **Set:** students in a geometry class
Relationship: "earned the same grade as"
Example: Jim earned the same grade as Mario.

34. **Set:** letters of the alphabet
Relationship: "comes after"
Example: H comes after G.

EXTRA CHALLENGE
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MIXED REVIEW

USING THE DISTANCE FORMULA **Find the distance between the two points. Round your result to two decimal places.** (Review 1.3 for 2.5)

35. $A(4, 5), B(-3, -2)$ **36.** $E(-7, 6), F(2, 0)$ **37.** $J(1, 1), K(-1, 11)$

38. $P(8, -4), Q(1, -4)$ **39.** $S(9, -1), T(2, -6)$ **40.** $V(7, 10), W(1, 5)$

DETERMINING ENDPOINTS **In Exercises 41–44, you are given an endpoint and the midpoint of a line segment. Find the coordinates of the other endpoint. Each midpoint is denoted by *M*(*x*, *y*).** (Review 1.5 for 2.5)

41. $B(5, 7)$, $M(-1, 0)$ **42.** $C(-4, -5)$, $M(3, -6)$ **43.** $F(0, 9)$, $M(6, -2)$ **44.** $Q(-1, 14)$, $M(2, 7)$

45. Given that $m\angle A = 48°$, what are the measures of a complement and a supplement of $\angle A$? (Review 1.6)

ANALYZING STATEMENTS **Use the diagram shown at the right to determine whether the statement is *true* or *false*.** (Review 2.2)

46. Points G, L, and J are collinear.

47. $\overline{BC} \perp \overline{FG}$

48. $\angle ECB \cong \angle ACD$

49. $\angle JHL$ and $\angle JHF$ are complementary.

50. $\overleftrightarrow{AK} \perp \overleftrightarrow{BD}$