2.3 Deductive Reasoning

What *you should learn*

GOAL 1 Use symbolic notation to represent logical statements.

GOAL 2 Form conclusions by applying the laws of logic to true statements, such as statements about a trip to Alabama in **Example 6**.

Why *you should learn it*

▼ The laws of logic help you with classification. For instance, the Law of Syllogism is used to determine true statements about birds in **Example 5**.

GOAL 1 USING SYMBOLIC NOTATION

In Lesson 2.1 you learned that a conditional statement has a hypothesis and a conclusion. Conditional statements can be written using symbolic notation, where p represents the hypothesis, q represents the conclusion, and $\rightarrow$ is read as "implies." Here are some examples.

$$\text{If } \underbrace{\textbf{the sun is out}}_{p}\text{, then } \underbrace{\textbf{the weather is good}}_{q}.$$

This conditional statement can be written symbolically as follows:

$$\text{If } p\text{, then } q \qquad \text{or} \qquad p \rightarrow q.$$

To form the converse of an "If p, then q" statement, simply switch p and q.

$$\text{If } \underbrace{\textbf{the weather is good}}_{q}\text{, then } \underbrace{\textbf{the sun is out}}_{p}.$$

The converse can be written symbolically as follows:

$$\text{If } q\text{, then } p \qquad \text{or} \qquad q \rightarrow p.$$

A biconditional statement can be written using symbolic notation as follows:

$$\text{If } p\text{, then } q \text{ and if } q\text{, then } p \qquad \text{or} \qquad p \leftrightarrow q.$$

Most often a biconditional statement is written in this form:

$$p \text{ if and only if } q.$$

EXAMPLE 1 *Using Symbolic Notation*

Let p be "the value of x is -5" and let q be "the absolute value of x is 5."

a. Write $p \rightarrow q$ in words.

b. Write $q \rightarrow p$ in words.

c. Decide whether the biconditional statement $p \leftrightarrow q$ is true.

SOLUTION

a. If the value of x is -5, then the absolute value of x is 5.

b. If the absolute value of x is 5, then the value of x is -5.

c. The conditional statement in part (a) is true, but its converse in part (b) is false. So, the biconditional statement $p \leftrightarrow q$ is false.

STUDENT HELP

Study Tip
The negation of a negative statement is a positive statement. This is shown at the right with the statement, "$\angle 3$ is not acute."

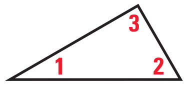
To write the inverse and contrapositive in symbolic notation, you need to be able to write the negation of a statement symbolically. The symbol for negation ($\sim$) is written before the letter. Here are some examples.

STATEMENT	SYMBOL	NEGATION	SYMBOL
$\angle 3$ measures 90°.	p	$\angle 3$ does not measure 90°.	$\sim p$
$\angle 3$ is not acute.	q	$\angle 3$ is acute.	$\sim q$

The inverse and contrapositive of $p \rightarrow q$ are as follows:

Inverse: $\sim p \rightarrow \sim q$

If $\angle 3$ does not measure 90°, then $\angle 3$ is acute.



Contrapositive: $\sim q \rightarrow \sim p$

If $\angle 3$ is acute, then $\angle 3$ does not measure 90°.

Notice that the inverse is false, but the contrapositive is true.

EXAMPLE 2 *Writing an Inverse and a Contrapositive*

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Let p be "it is raining" and let q be "the soccer game is canceled."

a. Write the contrapositive of $p \rightarrow q$.

b. Write the inverse of $p \rightarrow q$.

SOLUTION

a. Contrapositive: $\sim q \rightarrow \sim p$

If the soccer game is not canceled, then it is not raining.

b. Inverse: $\sim p \rightarrow \sim q$

If it is not raining, then the soccer game is not canceled.

. . . .

Recall from Lesson 2.1 that a conditional statement is equivalent to its contrapositive and that the converse and inverse are equivalent.

Equivalent Statements	Equivalent Statements
Conditional Statement $p \rightarrow q$ If the car will start, then the battery is charged.	**Converse** $q \rightarrow p$ If the battery is charged, then the car will start.
Contrapositive $\sim q \rightarrow \sim p$ If the battery is not charged, then the car will not start.	**Inverse** $\sim p \rightarrow \sim q$ If the car will not start, then the battery is not charged.

In the table above the conditional statement and its contrapositive are true. The converse and inverse are false. (Just because a car won't start does not imply that its battery is dead.)

GOAL 2 USING THE LAWS OF LOGIC

Deductive reasoning uses facts, definitions, and accepted properties in a logical order to write a **logical argument**. This differs from *inductive reasoning*, in which previous examples and patterns are used to form a conjecture.

EXAMPLE 3 *Using Inductive and Deductive Reasoning*

The following examples show how inductive and deductive reasoning differ.

a. Andrea knows that Robin is a sophomore and Todd is a junior. All the other juniors that Andrea knows are older than Robin. Therefore, Andrea reasons *inductively* that Todd is older than Robin based on past observations.

b. Andrea knows that Todd is older than Chan. She also knows that Chan is older than Robin. Andrea reasons *deductively* that Todd is older than Robin based on accepted statements.

There are two *laws of deductive reasoning*. The first is the **Law of Detachment**, shown below. The **Law of Syllogism** follows on the next page.

LAW OF DETACHMENT

If $p \rightarrow q$ is a true conditional statement and p is true, then q is true.

EXAMPLE 4 *Using the Law of Detachment*

State whether the argument is valid.

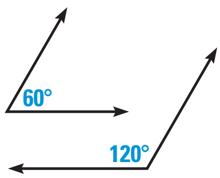
a. Jamal knows that if he misses the practice the day before a game, then he will not be a starting player in the game. Jamal misses practice on Tuesday so he concludes that he will not be able to start in the game on Wednesday.

b. If two angles form a linear pair, then they are supplementary; $\angle A$ and $\angle B$ are supplementary. So, $\angle A$ and $\angle B$ form a linear pair.

SOLUTION

a. This logical argument is a valid use of the Law of Detachment. It is given that both a statement $(p \rightarrow q)$ and its hypothesis (p) are true. So, it is valid for Jamal to conclude that the conclusion (q) is true.

b. This logical argument is not a valid use of the Law of Detachment. Given that a statement $(p \rightarrow q)$ and its conclusion (q) are true does not mean the hypothesis (p) is true. The argument implies that all supplementary angles form a linear pair.



The diagram shows that this is not a valid conclusion.

FOCUS ON CAREERS

ZOOLOGY Zoologists use facts about a bird's appearance to classify it. Ornithology is the branch of zoology that studies birds.

CAREER LINK
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LAW OF SYLLOGISM

If $p \rightarrow q$ and $q \rightarrow r$ are true conditional statements, then $p \rightarrow r$ is true.

EXAMPLE 5 *Using the Law of Syllogism*

ZOOLOGY Write some conditional statements that can be made from the following true statements using the Law of Syllogism.

1. If a bird is the fastest bird on land, then it is the largest of all birds.
2. If a bird is the largest of all birds, then it is an ostrich.
3. If a bird is a bee hummingbird, then it is the smallest of all birds.
4. If a bird is the largest of all birds, then it is flightless.
5. If a bird is the smallest bird, then it has a nest the size of a walnut half-shell.

SOLUTION

Here are the conditional statements that use the Law of Syllogism.

a. If a bird is the fastest bird on land, then it is an ostrich. (Use 1 and 2.)

b. If a bird is a bee hummingbird, then it has a nest the size of a walnut half-shell. (Use 3 and 5.)

c. If a bird is the fastest bird on land, then it is flightless. (Use 1 and 4.)

EXAMPLE 6 *Using the Laws of Deductive Reasoning*

Logical Reasoning

Over the summer, Mike visited Alabama. Given the following true statements, can you conclude that Mike visited the Civil Rights Memorial?

If Mike visits Alabama, then he will spend a day in Montgomery.

If Mike spends a day in Montgomery, then he will visit the Civil Rights Memorial.

Civil Rights Memorial in Montgomery, Alabama

SOLUTION

Let p, q, and r represent the following.

p: Mike visits Alabama.

q: Mike spends a day in Montgomery.

r: Mike visits the Civil Rights Memorial.

Because $p \rightarrow q$ is true and $q \rightarrow r$ is true, you can apply the Law of Syllogism to conclude that $p \rightarrow r$ is true.

If Mike visits Alabama, then he will visit the Civil Rights Memorial.

▶ You are told that Mike visited Alabama, which means p is true. Using the Law of Detachment, you can conclude that he visited the Civil Rights Memorial.

GUIDED PRACTICE

Vocabulary Check ✓

1. If the statements $p \rightarrow q$ and $q \rightarrow r$ are true, then the statement $p \rightarrow r$ is true by the Law of __?__. If the statement $p \rightarrow q$ is true and p is true, then q is true by the Law of __?__.

Concept Check ✓

2. State whether the following argument uses inductive or deductive reasoning: "If it is Friday, then Kendra's family has pizza for dinner. Today is Friday, therefore, Kendra's family will have pizza for dinner."

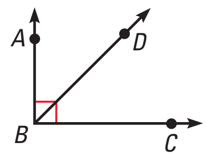
Skill Check ✓

3. Given the notation for a conditional statement is $p \rightarrow q$, what statement is represented by $q \rightarrow p$?

4. A conditional statement is defined in symbolic notation as $p \rightarrow q$. Use symbolic notation to write the inverse of $p \rightarrow q$.

5. Write the contrapositive of the following statement: "If you don't enjoy scary movies, then you wouldn't have liked this one."

6. If a ray bisects a right angle, then the congruent angles formed are complementary. In the diagram, $\angle ABC$ is a right angle. Are $\angle ABD$ and $\angle CBD$ complementary? Explain your reasoning.



7. If $f \rightarrow g$ and $g \rightarrow h$ are true statements, and f is true, does it follow that h is true? Explain.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 805.

WRITING STATEMENTS **Using p and q below, write the symbolic statement in words.**

p: Points X, Y, and Z are collinear.

q: Points X, Y, and Z lie on the same line.

8. $q \rightarrow p$ **9.** $\sim q$ **10.** $\sim p$

11. $\sim p \rightarrow \sim q$ **12.** $p \longleftrightarrow q$ **13.** $\sim q \rightarrow \sim p$

WRITING INVERSE AND CONTRAPOSITIVE **Given that the statement is of the form $p \rightarrow q$, write p and q. Then write the inverse and the contrapositive of $p \rightarrow q$ both symbolically and in words.**

14. If Jed gets a C on the exam, then he will get an A for the quarter.

15. If Alberto finds a summer job, then he will buy a car.

16. If the fuse has blown, then the light will not go on.

17. If the car is running, then the key is in the ignition.

18. If you dial 911, then there is an emergency.

19. If Gina walks to the store, then she will buy a newspaper.

20. If it is not raining, then Petra will ride her bike to school.

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 8–13
Example 2: Exs. 14–20
Example 3: Exs. 21, 22
Example 4: Exs. 23–25
Example 5: Exs. 30–48
Example 6: Exs. 30–48

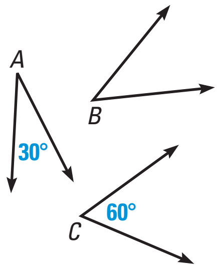
LOGICAL REASONING **Decide whether *inductive* or *deductive* reasoning is used to reach the conclusion. Explain your reasoning.**

21. For the past three Wednesdays the cafeteria has served macaroni and cheese for lunch. Dana concludes that the cafeteria will serve macaroni and cheese for lunch this Wednesday.

22. If you live in Nevada and are between the ages of 16 and 18, then you must take driver's education to get your license. Marcus lives in Nevada, is 16 years old, and has his driver's license. Therefore, Marcus took driver's education.

USING THE LAW OF DETACHMENT **State whether the argument is valid. Explain your reasoning.**

23. If the sum of the measures of two angles is 90°, then the two angles are complementary. Because $m\angle A + m\angle C = 90°$, $\angle A$ and $\angle C$ are complementary.



24. If two adjacent angles form a right angle, then the two angles are complementary. Because $\angle A$ and $\angle C$ are complementary, $\angle A$ and $\angle C$ are adjacent.

25. If $\angle A$ and $\angle C$ are acute angles, then any angle whose measure is between the measures of $\angle A$ and $\angle C$ is also acute. In the diagram above it is shown that $m\angle A \le m\angle B \le m\angle C$, so $\angle B$ must be acute.

USING ALGEBRA **State whether any conclusions can be made using the true statement, given that $x = 3$.**

26. If $x > 2x - 10$, then $x = y$.

27. If $2x + 3 < 4x < 5x$, then $y \le x$.

28. If $4x \ge 12$, then $y = 6x$.

29. If $x + 3 = 10$, then $y = x$.

MAKING CONCLUSIONS **Use the Law of Syllogism to write the statement that follows from the pair of true statements.**

30. If the sun is shining, then it is a beautiful day.

If it is a beautiful day, then we will have a picnic.

31. If the stereo is on, then the volume is loud.

If the volume is loud, then the neighbors will complain.

32. If Ginger goes to the movies, then Marta will go to the movies.

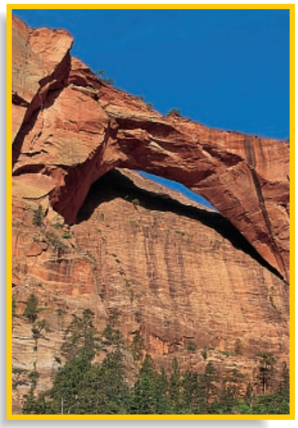
If Yumi goes to the movies, then Ginger will go to the movies.

USING DEDUCTIVE REASONING **Select the word that makes the concluding statement true.**

33. The Oak Terrace apartment building does not allow dogs. Serena lives at Oak Terrace. So, Serena (must, may, may not) keep a dog.

34. The Kolob Arch is the world's widest natural arch. The world's widest arch is in Zion National Park. So, the Kolob Arch (is, may be, is not) in Zion.

35. Zion National Park is in Utah. Jeremy spent a week in Utah. So, Jeremy (must have, may have, never) visited Zion National Park.



The Kolob Arch mentioned in Ex. 34, spans 310 feet.

USING THE LAWS OF LOGIC **In Exercises 36–42, use the diagram to give a reason for each true statement. In the diagram, $m\angle 2 = 115°$, $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$.**

36. p_1: $m\angle 2 = 115°$

37. $p_1 \rightarrow p_2$: If $m\angle 2 = 115°$, then $m\angle 1 = 65°$.

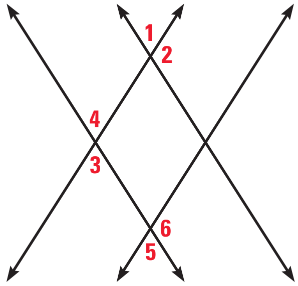
38. $p_2 \rightarrow p_3$: If $m\angle 1 = 65°$, then $m\angle 4 = 65°$.

39. $p_3 \rightarrow p_4$: If $m\angle 4 = 65°$, then $m\angle 3 = 65°$.

40. $p_4 \rightarrow p_5$: If $m\angle 3 = 65°$, then $m\angle 5 = 65°$.

41. $p_5 \rightarrow p_6$: If $m\angle 5 = 65°$, then $m\angle 6 = 115°$.

42. $p_1 \rightarrow p_6$: If $m\angle 2 = 115°$, then $m\angle 6 = 115°$.



43. *Writing* Describe a time in your life when you use deductive reasoning.

44. **CRITICAL THINKING** Describe an instance where inductive reasoning can lead to an incorrect conclusion.

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 45–48.

LOGICAL REASONING **In Exercises 45–48, use the true statements to determine whether the conclusion is *true* or *false*. Explain your reasoning.**

- If Diego goes shopping, then he will buy a pretzel.
- If the mall is open, then Angela and Diego will go shopping.
- If Angela goes shopping, then she will buy a pizza.
- The mall is open.

45. Diego bought a pretzel.

46. Angela and Diego went shopping.

47. Angela bought a pretzel.

48. Diego had some of Angela's pizza.

49. **ROBOTICS** Because robots can withstand higher temperatures than humans, a fire-fighting robot is under development. Write the following statements about the robot in order. Then use the Law of Syllogism to complete the statement, "If there is a fire, then __?__."

A. If the robot sets off a fire alarm, then it concludes there is a fire.

B. If the robot senses high levels of smoke and heat, then it sets off a fire alarm.

C. If the robot locates the fire, then the robot extinguishes the fire.

D. If there is a fire, then the robot senses high levels of smoke and heat.

E. If the robot concludes there is a fire, then it locates the fire.

50. **DOGS** Use the true statements to form other conditional statements.

A. If a dog is a gazehound, then it hunts by sight.

B. If a hound bays (makes long barks while hunting), then it is a scent hound.

C. If a dog is a foxhound, then it does not hunt primarily by sight.

D. If a dog is a coonhound, then it bays when it hunts.

E. If a dog is a greyhound, then it is a gazehound.

Test Preparation

51. MULTI-STEP PROBLEM Let p be "Jana wins the contest" and q be "Jana gets two free tickets to the concert."

a. Write $p \rightarrow q$ in words.

b. Write the converse of $p \rightarrow q$, both in words and symbols.

c. Write the contrapositive of $p \rightarrow q$, both in words and symbols.

d. Suppose Jana gets two free tickets to the concert but does not win the contest. Is this a counterexample to the converse or to the contrapositive?

e. What do you need to know about the conditional statement from part (a) so the Law of Detachment can be used to conclude that Jana gets two free tickets to the concert?

f. *Writing* Use the statement in part (a) to write a second statement that uses the Law of Syllogism to reach a valid conclusion.

★ **Challenge**

CONTRAPOSITIVES **Use the true statements to answer the questions.**

- If a creature is a fly, then it has six legs.
- If a creature has six legs, then it is an insect.

52. Use symbolic notation to describe the statements.

53. Use the statements and the Law of Syllogism to write a conditional statement, both in words and symbols.

54. Write the contrapositive of each statement, both in words and symbols.

55. Using the contrapositives and the Law of Syllogism, write a conditional statement. Is the statement true? Does the Law of Syllogism work for contrapositives?

EXTRA CHALLENGE
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MIXED REVIEW

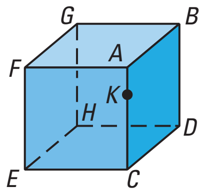
NAMING POINTS **Use the diagram to name a point.** (Review 1.2)

56. A third point collinear with A and C

57. A fourth point coplanar with A, C, and E

58. A point coplanar with A and B, but not coplanar with A, B, and C

59. A point coplanar with A and C, but not coplanar with E and F



FINDING ANGLE MEASURES **Find $m\angle ABD$ given that $\angle ABC$ and $\angle CBD$ are adjacent angles.** (Review 1.4 for 2.4)

60. $m\angle ABC = 20°$, $m\angle CBD = 10°$

61. $m\angle CBD = 13°$, $m\angle ABC = 28°$

62. $m\angle ABC = 3y + 1$, $m\angle CBD = 12 - y$

63. $m\angle CBD = 11 + 2f - g$, $m\angle ABC = 5g - 4 + f$

QUIZ 1

Self-Test for Lessons 2.1–2.3

Write the true statement in if-then form and write its converse. Determine whether the statement and its converse can be combined to form a true biconditional statement. (Lesson 2.1 and Lesson 2.2)

1. If today is June 4, then tomorrow is June 5.
2. A century is a period of 100 years.
3. Two circles are congruent if they have the same diameter.

LOGICAL REASONING **Use the true statements to answer the questions.** (Lesson 2.3)

- If John drives into the fence, then John's father will be angry.
- If John backs the car out, then John will drive into the fence.
- John backs the car out.

4. Does John drive into the fence?
5. Is John's father angry?

MATH & History — History of Recreational Logic Puzzles

APPLICATION LINK www.mcdougallittell.com

THEN

IN THE 1600S, puzzles involving "formal" logic first became popular in Europe. However, logic has been a part of games such as mancala and chess for thousands of years.

NOW

TODAY, logic games and puzzles are a popular pastime throughout the world. Lewis Carroll, author of *Alice in Wonderland*, was also a mathematician who wrote books on logic. The following problem is based on notes he wrote in his diary in the 1890s.

A says B lies; B says C lies; C says A and B lie.
Who is telling the truth? Who is lying?

Complete the exercises to solve the problem.

1. If A is telling the truth, then B is lying. What can you conclude about C's statement?
2. Assume A is telling the truth. Explain how this leads to a contradiction.
3. Who is telling the truth? Who is lying? How do you know? (*Hint*: For C to be lying, only one other person (A or B) must be telling the truth.)