2.2 Definitions and Biconditional Statements

What you should learn

GOAL 1 Recognize and use definitions.

GOAL 2 Recognize and use biconditional statements.

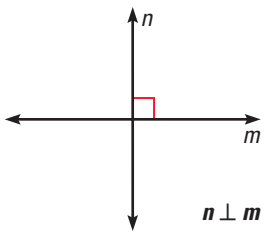
Why you should learn it

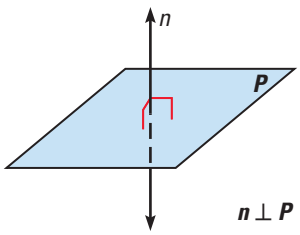
▼ You can use biconditional statements to help analyze geographic relations, such as whether three cities in Florida lie on the same line, as in **Ex. 50**.

GOAL 1 Recognizing and Using Definitions

In Lesson 1.2 you learned that a *definition* uses known words to describe a new word. Here are two examples.

Two lines are called **perpendicular lines** if they intersect to form a right angle. A **line perpendicular to a plane** is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it. The symbol $\perp$ is read as "is perpendicular to."

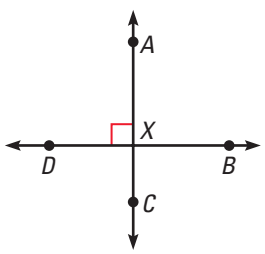




All definitions can be interpreted "forward" and "backward." For instance, the definition of perpendicular lines means (1) if two lines are perpendicular, then they intersect to form a right angle, *and* (2) if two lines intersect to form a right angle, then they are perpendicular.

EXAMPLE 1 Using Definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a. Points D, X, and B are collinear.

b. $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{DB}$.

c. $\angle AXB$ is adjacent to $\angle CXD$.

SOLUTION

a. This statement is true. Two or more points are *collinear* if they lie on the same line. The points D, X, and B all lie on line $\overleftrightarrow{DB}$ so they are collinear.

b. This statement is true. The right angle symbol in the diagram indicates that the lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{DB}$ intersect to form a right angle. So, the lines are perpendicular.

c. This statement is false. By definition, adjacent angles must share a common side. Because $\angle AXB$ and $\angle CXD$ do not share a common side, they are not adjacent.

STUDENT HELP

Study Tip

When a conditional statement contains the word "if," the hypothesis does not always follow the "if." This is shown in the "only-if" statement at the right.

GOAL 2 USING BICONDITIONAL STATEMENTS

Conditional statements are not always written in if-then form. Another common form of a conditional statement is *only-if* form. Here is an example.

It is Saturday, only if **I am working at the restaurant**.

Hypothesis **Conclusion**

You can rewrite this conditional statement in if-then form as follows:

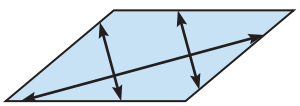
If **it is Saturday**, then **I am working at the restaurant**.

A **biconditional statement** is a statement that contains the phrase "if and only if." Writing a biconditional statement is equivalent to writing a conditional statement *and* its converse.

EXAMPLE 2 *Rewriting a Biconditional Statement*

The biconditional statement below can be rewritten as a conditional statement and its converse.

Three lines are coplanar if and only if they lie in the same plane.



Conditional statement: If three lines are coplanar, then they lie in the same plane.

Converse: If three lines lie in the same plane, then they are coplanar.

A biconditional statement can be either true or false. To be true, *both* the conditional statement and its converse must be true. This means that a true biconditional statement is true both "forward" and "backward." All definitions can be written as true biconditional statements.

EXAMPLE 3 *Analyzing a Biconditional Statement*

Using Algebra

Consider the following statement: $x = 3$ if and only if $x^2 = 9$.

a. Is this a biconditional statement?

b. Is the statement true?

SOLUTION

a. The statement is biconditional because it contains "if and only if."

b. The statement can be rewritten as the following statement and its converse.

Conditional statement: If $x = 3$, then $x^2 = 9$.

Converse: If $x^2 = 9$, then $x = 3$.

▶ The first of these statements is true, but the second is false. So, the biconditional statement is false.

EXAMPLE 4 Writing a Biconditional Statement

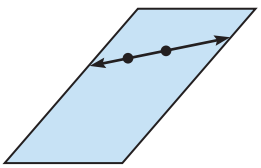
Each of the following statements is true. Write the converse of each statement and decide whether the converse is *true* or *false*. If the converse is true, combine it with the original statement to form a true biconditional statement. If the converse is false, state a counterexample.

a. If two points lie in a plane, then the line containing them lies in the plane.

b. If a number ends in 0, then the number is divisible by 5.

SOLUTION

a. **Converse:** If a line containing two points lies in a plane, then the points lie in the plane.



The converse is true, as shown in the diagram. So, it can be combined with the original statement to form the true biconditional statement written below.

Biconditional statement: Two points lie in a plane if and only if the line containing them lies in the plane.

b. **Converse:** If a number is divisible by 5, then the number ends in 0.

The converse is false. As a counterexample, consider the number 15. It is divisible by 5, but it does not end in 0, as shown at the right.

$10 \div 5 = 2$
$\blacktriangleright\ 15 \div 5 = 3$
$20 \div 5 = 4$

. . . . . .

Knowing how to use true biconditional statements is an important tool for reasoning in geometry. For instance, if you can write a true biconditional statement, then you can use the conditional statement or the converse to justify an argument.

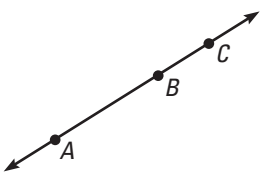
EXAMPLE 5 Writing a Postulate as a Biconditional

The second part of the Segment Addition Postulate is the converse of the first part. Combine the statements to form a true biconditional statement.

SOLUTION

The first part of the Segment Addition Postulate can be written as follows:

If B lies between points A and C, then $AB + BC = AC$.



The converse of this is as follows:

If $AB + BC = AC$, then B lies between A and C.

Combining these statements produces the following true biconditional statement:

Point B lies between points A and C if and only if $AB + BC = AC$.

STUDENT HELP

Study Tip

Unlike definitions, not all postulates can be written as true biconditional statements.

GUIDED PRACTICE

Vocabulary Check ✓

1. Describe in your own words what a *true biconditional statement* is.

Concept Check ✓

2. **ERROR ANALYSIS** What is wrong with Jared's argument below?

> The statements "I eat cereal only if it is morning" and "If I eat cereal, then it is morning" are not equivalent.

Skill Check ✓

Tell whether the statement is a biconditional.

3. I will work after school only if I have the time.
4. An angle is called a right angle if and only if it measures 90°.
5. Two segments are congruent if and only if they have the same length.

Rewrite the biconditional statement as a conditional statement and its converse.

6. The ceiling fan runs if and only if the light switch is on.
7. You scored a touchdown if and only if the football crossed the goal line.
8. The expression $3x + 4$ is equal to 10 if and only if x is 2.

WINDOWS **Decide whether the statement about the window shown is true. Explain your answer using the definitions you have learned.**

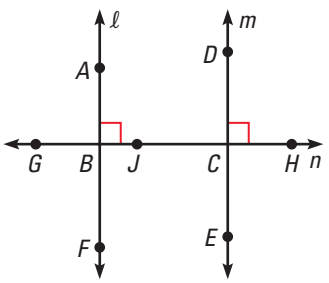
9. The points D, E, and F are collinear.
10. $m\angle CBA = 90°$
11. $\angle DBA$ and $\angle EBC$ are not complementary.
12. $\overline{DE} \perp \overline{AC}$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 805.

PERPENDICULAR LINES **Use the diagram to determine whether the statement is *true* or *false*.**



13. Points A, F, and G are collinear.
14. $\angle DCJ$ and $\angle DCH$ are supplementary.
15. $\overline{DC}$ is perpendicular to line ℓ.
16. $\overline{FB}$ is perpendicular to line n.
17. $\angle FBJ$ and $\angle JBA$ are complementary.
18. Line m bisects $\angle JCH$.
19. $\angle ABJ$ and $\angle DCH$ are supplementary.

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 13–19
Example 2: Exs. 20–23
Example 3: Exs. 28–31
Example 4: Exs. 32–37
Example 5: Exs. 44–46

BICONDITIONAL STATEMENTS **Rewrite the biconditional statement as a conditional statement and its converse.**

20. Two angles are congruent if and only if they have the same measure.

21. A ray bisects an angle if and only if it divides the angle into two congruent angles.

22. Two lines are perpendicular if and only if they intersect to form right angles.

23. A point is a midpoint of a segment if and only if it divides the segment into two congruent segments.

FINDING COUNTEREXAMPLES **Give a counterexample that demonstrates that the converse of the statement is false.**

24. If an angle measures 94°, then it is obtuse.

25. If two angles measure 42° and 48°, then they are complementary.

26. If Terry lives in Tampa, then she lives in Florida.

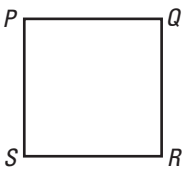
27. If a polygon is a square, then it has four sides.

ANALYZING BICONDITIONAL STATEMENTS **Determine whether the biconditional statement about the diagram is *true* or *false*. If false, provide a counterexample.**

28. $\overline{SR}$ is perpendicular to $\overline{QR}$ if and only if $\angle SRQ$ measures 90°.

29. PQ and PS are equal if and only if PQ and PS are both 8 centimeters.

30. $\angle PQR$ and $\angle QRS$ are supplementary if and only if $m\angle PQR = m\angle QRS = 90°$.



31. $\angle PSR$ measures 90° if and only if $\angle PSR$ is a right angle.

FOCUS ON APPLICATIONS

REAL LIFE **SNOW LEOPARDS** The pale coat of the snow leopard, as mentioned in Ex. 37, allows the animal to blend in with the snow 3960 meters (13,000 feet) high in the mountains of Central Asia.

REWRITING STATEMENTS **Rewrite the true statement in if-then form and write the converse. If the converse is true, combine it with the if-then statement to form a true biconditional statement. If the converse is false, provide a counterexample.**

32. Adjacent angles share a common side.

33. Two circles have the same circumference if they have the same diameter.

34. The perimeter of a triangle is the sum of the lengths of its sides.

35. All leopards have spots.

36. Panthers live in the forest.

37. A leopard is a snow leopard if the leopard has pale gray fur.

USING ALGEBRA **Determine whether the statement can be combined with its converse to form a true biconditional.**

38. If $3u + 2 = u + 12$, then $u = 5$.

39. If $v = 1$, then $9v - 4v = 2v + 3v$.

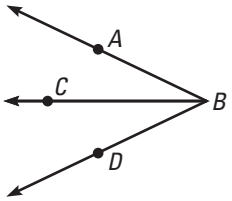
40. If $w^2 - 10 = w + 2$, then $w = 4$.

41. If $x^3 - 27 = 0$, then $x = 3$.

42. If $y = -3$, then $y^2 = 9$.

43. If $z = 3$, then $7 + 18z = 5z + 7 + 13z$.

44. REWRITING A POSTULATE Write the converse of the Angle Addition Postulate and decide whether the converse is *true* or *false*. If true, write the postulate as a true biconditional. If false, provide a counterexample.



Angle Addition Postulate: If C is in the interior of $\angle ABD$, then $m\angle ABC + m\angle CBD = m\angle ABD$.

45. *Writing* Give an example of a true biconditional statement.

46. MUSICAL GROUPS The table shows four different groups, along with the number of instrumentalists in each group. Write your own definitions of the musical groups and verify that they are true biconditional statements by writing each definition "forward" and "backward." The first one is started for you.

Sample: A musical group is a *piano trio* if and only if it contains exactly one pianist, one violinist, and one cellist.

Musical group	Pianist	Violinist	Cellist	Violist
Piano trio	1	1	1	—
String quartet	—	2	1	1
String quintet	—	2	1	2
Piano quintet	1	2	1	1

TECHNOLOGY In Exercises 47–49, use geometry software to complete the statement.

47. If the sides of a square are doubled, then the area is ___?___.

48. If the sides of a square are doubled, then the perimeter is ___?___.

49. Decide whether the statements in Exercises 47 and 48 can be written as true biconditionals. If not, provide a counterexample.

50. AIR DISTANCES The air distance between Jacksonville, Florida, and Merritt Island, Florida, is 148 miles and the air distance between Merritt Island and Fort Pierce, Florida, is 70 miles. Given that the air distance between Jacksonville and Fort Pierce is 218 miles, does Merritt Island fall on the line connecting Jacksonville and Fort Pierce?

FOCUS ON APPLICATIONS

REAL LIFE **WINDS AT SEA** Along with wind speed, sailors need to know the direction of the wind. Flags, also known as telltales, help sailors determine wind direction.

INTERNET **APPLICATION LINK** www.mcdougallittell.com

WINDS AT SEA Use the portion of the Beaufort wind scale table shown to determine whether the biconditional statement is *true* or *false*. If false, provide a counterexample.

51. A storm is a hurricane if and only if the winds of the storm measure 64 knots or greater.

52. Winds at sea are classified as a strong gale if and only if the winds measure 34–40 knots.

53. Winds are classified as 10 on the Beaufort scale if and only if the winds measure 41–55 knots.

Beaufort Wind Scale for Open Sea		
Number	Knots	Description
8	34–40	gale winds
9	41–47	strong gale
10	48–55	storm
11	56–63	violent storm
12	64+	hurricane

Test Preparation

54. MULTIPLE CHOICE Which one of the following statements cannot be written as a true biconditional statement?

(A) Any angle that measures between 90° and 180° is obtuse.

(B) $2x - 5 = x + 1$ only if $x = 6$.

(C) Any angle that measures between 0° and 90° is acute.

(D) If two angles measure 110° and 70°, then they are supplementary.

(E) If the sum of the measures of two angles equals 180°, then they are supplementary.

55. MULTIPLE CHOICE Which of the following statements about the conditional statement "If two lines intersect to form a right angle, then they are perpendicular" is true?

I. The converse is true.
II. The statement can be written as a true biconditional.
III. The statement is false.

(A) I only (B) I and II only (C) II and III only
(D) III only (E) I, II, and III

Challenge

WRITING STATEMENTS **In Exercises 56 and 57, determine (a) whether the contrapositive of the true statement is *true* or *false* and (b) whether the true statement can be written as a true biconditional.**

56. If I am in Des Moines, then I am in the capital of Iowa.

57. If two angles measure 10° and 80°, then they are complementary.

58. LOGICAL REASONING You are given that the contrapositive of a statement is true. Will that help you determine whether the statement can be written as a true biconditional? Explain. (*Hint:* Use your results from Exercises 56 and 57.)

EXTRA CHALLENGE
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MIXED REVIEW

STUDYING ANGLES **Find the measures of a complement and a supplement of the angle.** (Review 1.6 for 2.3)

59. 87° **60.** 73° **61.** 14° **62.** 29°

FINDING PERIMETER AND AREA **Find the area and perimeter, or circumference of the figure described. (Use $\pi \approx 3.14$ when necessary.)** (Review 1.7 for 2.3)

63. rectangle: $w = 3$ ft, $l = 12$ ft

64. rectangle: $w = 7$ cm, $l = 10$ cm

65. circle: $r = 8$ in.

66. square: $s = 6$ m

CONDITIONAL STATEMENTS **Write the converse of the statement.** (Review 2.1 for 2.3)

67. If the sides of a rectangle are all congruent, then the rectangle is a square.

68. If $8x + 1 = 3x + 16$, then $x = 3$.