2.1

Conditional Statements

Coastal Research Amphibious Buggy

GOAL 1 **RECOGNIZING CONDITIONAL STATEMENTS**

In this lesson you will study a type of logical statement called a conditional statement. A **conditional statement** has two parts, a *hypothesis* and a *conclusion*. When the statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**. Here is an example:

If **it is noon in Georgia**, then **it is 9 A.M. in California**.

Hypothesis　　　　　**Conclusion**

EXAMPLE 1 *Rewriting in If-Then Form*

Rewrite the conditional statement in *if-then form*.

a. Two points are collinear if they lie on the same line.

b. All sharks have a boneless skeleton.

c. A number divisible by 9 is also divisible by 3.

SOLUTION

a. If two points lie on the same line, then they are collinear.

b. If a fish is a shark, then it has a boneless skeleton.

c. If a number is divisible by 9, then it is divisible by 3.

. . . .

Conditional statements can be either true or false. To show that a conditional statement is true, you must present an argument that the conclusion follows for *all* cases that fulfill the hypothesis. To show that a conditional statement is false, describe a single counterexample that shows the statement is not always true.

EXAMPLE 2 *Writing a Counterexample*

Write a counterexample to show that the following conditional statement is false.

If $x^2 = 16$, then $x = 4$.

SOLUTION

As a counterexample, let $x = -4$. The hypothesis is true, because $(-4)^2 = 16$. However, the conclusion is false. This implies that the given conditional statement is false.

The **converse** of a conditional statement is formed by switching the hypothesis and conclusion. Here is an example.

> **Statement**: If you see lightning, then you hear thunder.

> **Converse**: If you hear thunder, then you see lightning.

EXAMPLE 3 *Writing the Converse of a Conditional Statement*

Write the converse of the following conditional statement.

> **Statement**: If two segments are congruent, then they have the same length.

SOLUTION

> **Converse**: If two segments have the same length, then they are congruent.

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A statement can be altered by **negation**, that is, by writing the negative of the statement. Here are some examples.

STATEMENT	NEGATION
$m\angle A = 30°$	$m\angle A \neq 30°$
$\angle A$ is acute.	$\angle A$ is not acute.

When you negate the hypothesis and conclusion of a conditional statement, you form the **inverse**. When you negate the hypothesis and conclusion of the converse of a conditional statement, you form the **contrapositive**.

Original	If $m\angle A = 30°$, then $\angle A$ is acute.
Inverse	If $m\angle A \neq 30°$, then $\angle A$ is not acute.
Converse	If $\angle A$ is acute, then $m\angle A = 30°$.
Contrapositive	If $\angle A$ is not acute, then $m\angle A \neq 30°$.

Both false Both true

When two statements are both true or both false, they are called **equivalent statements**. A conditional statement is equivalent to its contrapositive. Similarly, the inverse and converse of any conditional statement are equivalent. This is shown in the table above.

EXAMPLE 4 *Writing an Inverse, Converse, and Contrapositive*

Write the (**a**) inverse, (**b**) converse, and (**c**) contrapositive of the statement.

> If there is snow on the ground, then flowers are not in bloom.

SOLUTION

a. **Inverse**: If there is no snow on the ground, then flowers are in bloom.

b. **Converse**: If flowers are not in bloom, then there is snow on the ground.

c. **Contrapositive**: If flowers are in bloom, then there is no snow on the ground.

In Chapter 1, you studied four postulates.

Ruler Postulate	(Lesson 1.3, page 17)
Segment Addition Postulate	(Lesson 1.3, page 18)
Protractor Postulate	(Lesson 1.4, page 27)
Angle Addition Postulate	(Lesson 1.4, page 27)

Remember that postulates are assumed to be true—they form the foundation on which other statements (called *theorems*) are built.

POINT, LINE, AND PLANE POSTULATES

POSTULATE 5 Through any two points there exists exactly one line.

POSTULATE 6 A line contains at least two points.

POSTULATE 7 If two lines intersect, then their intersection is exactly one point.

POSTULATE 8 Through any three noncollinear points there exists exactly one plane.

POSTULATE 9 A plane contains at least three noncollinear points.

POSTULATE 10 If two points lie in a plane, then the line containing them lies in the plane.

**POSTULATE 11** If two planes intersect, then their intersection is a line.

EXAMPLE 5 *Identifying Postulates*

Logical Reasoning

Use the diagram at the right to give examples of Postulates 5 through 11.

SOLUTION

a. Postulate 5: There is exactly one line (line *n*) that passes through the points *A* and *B*.

b. Postulate 6: Line *n* contains at least two points. For instance, line *n* contains the points *A* and *B*.

c. Postulate 7: Lines *m* and *n* intersect at point *A*.

d. Postulate 8: Plane *P* passes through the noncollinear points *A, B,* and *C*.

e. Postulate 9: Plane *P* contains at least three noncollinear points, *A, B,* and *C*.

f. Postulate 10: Points *A* and *B* lie in plane *P.* So, line *n*, which contains points *A* and *B*, also lies in plane *P.*

g. Postulate 11: Planes *P* and *Q* intersect. So, they intersect in a line, labeled in the diagram as line *m*.
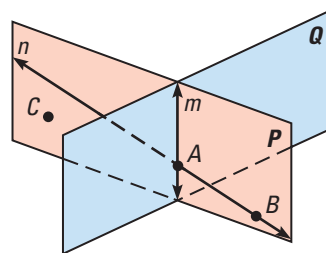
EXAMPLE 6 **Rewriting a Postulate**

a. Rewrite Postulate 5 in if-then form.

b. Write the inverse, converse, and contrapositive of Postulate 5.

SOLUTION

a. Postulate 5 can be rewritten in if-then form as follows:

> If two points are distinct, then there is exactly one line that passes through them.

b. **Inverse:** If two points are not distinct, then it is not true that there is exactly one line that passes through them.

Converse: If exactly one line passes through two points, then the two points are distinct.

Contrapositive: If it is not true that exactly one line passes through two points, then the two points are not distinct.

**EXAMPLE 7** **Using Postulates and Counterexamples**

Logical Reasoning

Decide whether the statement is *true* or *false*. If it is false, give a counterexample.

a. A line can be in more than one plane.

b. Four noncollinear points are always coplanar.

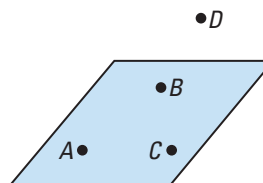**c.** Two nonintersecting lines can be noncoplanar.

SOLUTION

a. In the diagram at the right, line *k* is in plane *S* and line *k* is in plane *T*.

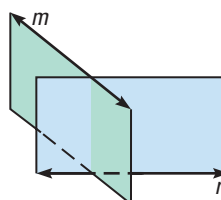So, it is *true* that a line can be in more than one plane.
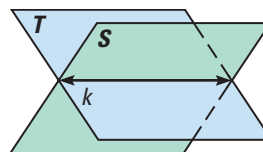
b. Consider the points *A*, *B*, *C*, and *D* at the right. The points *A*, *B*, and *C* lie in a plane, but there is no plane that contains all four points.

So, as shown in the counterexample at the right, it is *false* that four noncollinear points are always coplanar.

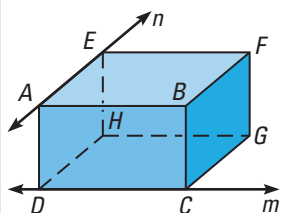**c.** In the diagram at the right, line *m* and line *n* are nonintersecting and are also noncoplanar.

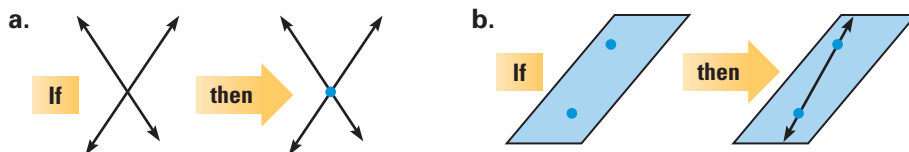So, it is *true* that two nonintersecting lines can be noncoplanar.

GUIDED PRACTICE

Vocabulary Check ✓

1. The ___?___ of a conditional statement is found by switching the hypothesis and conclusion.

Concept Check ✓

2. State the postulate described in each diagram.

a.

b.

Skill Check ✓

3. Write the hypothesis and conclusion of the statement, "If the dew point equals the air temperature, then it will rain."

In Exercises 4 and 5, write the statement in if-then form.

4. When threatened, the African ball python protects itself by coiling into a ball with its head in the middle.

5. The measure of a right angle is 90°.

6. Write the inverse, converse, and contrapositive of the conditional statement, "If a cactus is of the *cereus* variety, then its flowers open at night."

Decide whether the statement is *true* or *false*. Make a sketch to help you decide.

7. Through three noncollinear points there exists exactly one line.

8. If a line and a plane intersect, and the line does not lie in the plane, then their intersection is a point.

PRACTICE AND APPLICATIONS

STUDENT HELP

► **Extra Practice**
to help you master skills is on p. 805.

REWRITING STATEMENTS **Rewrite the conditional statement in if-then form.**

9. An object weighs one ton if it weighs 2000 pounds.

10. An object weighs 16 ounces if it weighs one pound.

11. Three points are collinear if they lie on the same line.

12. Blue trunkfish live in the waters of a coral reef.

13. Hagfish live in salt water.

STUDENT HELP

► **HOMEWORK HELP**
Example 1: Exs. 9–13
Example 2: Exs. 14–17
Example 3: Exs. 18–21
Example 4: Exs. 46–52
Example 5: Exs. 25–34
Example 6: Exs. 22–24
Example 7: Exs. 35–38

ANALYZING STATEMENTS **Decide whether the statement is *true* or *false*. If false, provide a counterexample.**

14. A point may lie in more than one plane.

15. If x^4 equals 81, then x must equal 3.

16. If it is snowing, then the temperature is below freezing.

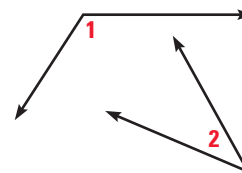**17.** If four points are collinear, then they are coplanar.

2.1 *Conditional Statements* **75**

WRITING CONVERSES Write the converse of the statement.

18. If ∠1 measures 123°, then ∠1 is obtuse.

19. If ∠2 measures 38°, then ∠2 is acute.

20. I will go to the mall if it is not raining.

21. I will go to the movies if it is raining.

REWRITING POSTULATES Rewrite the postulate in if-then form. Then write the inverse, converse, and contrapositive of the conditional statement.

22. A line contains at least two points.

23. Through any three noncollinear points there exists exactly one plane.

24. A plane contains at least three noncollinear points.

ILLUSTRATING POSTULATES Fill in the blank. Then draw a sketch that helps illustrate your answer.

25. If two lines intersect, then their intersection is ___?___ point(s).

26. Through any ___?___ points there exists exactly one line.

27. If two points lie in a plane, then the ___?___ containing them lies in the plane.

28. If two planes intersect, then their intersection is ___?___.

LINKING POSTULATES Use the diagram to state the postulate(s) that verifies the truth of the statement.
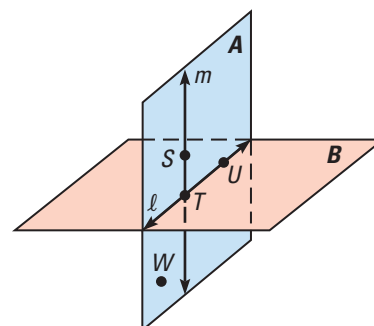
29. The points U and T lie on line ℓ.

30. Line ℓ contains points U and T.

31. The points W, S, and T lie in plane A.

32. The points S and T lie in plane A. Therefore, line m lies in plane A.

33. The planes A and B intersect in line ℓ.

34. Lines m and ℓ intersect at point T.

USING POSTULATES In Exercises 35–38, state the postulate that shows that the statement is false.
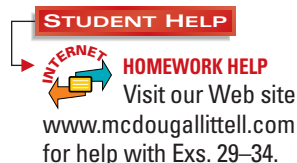
35. A line contains only one point.

36. Two planes intersect in exactly one point.

37. Three points, A, B, and C, are noncollinear, and two planes, M and N, each contain points A, B, and C.

38. Two points, P and Q, are collinear and two different lines, $\overleftrightarrow{RS}$ and $\overleftrightarrow{XY}$, each pass through points P and Q.

39. *Writing* Give an example of a true conditional statement with a true converse.

POINTS AND LINES IN SPACE Think of the intersection of the ceiling and the front wall of your classroom as line *k*. Think of the center of the floor as point *A* and the center of the ceiling as point *B*.

40. Is there more than one line that contains both points *A* and *B*?

41. Is there more than one plane that contains both points *A* and *B*?

42. Is there a plane that contains line *k* and point *A*?

43. Is there a plane that contains points *A*, *B*, and a point on the front wall?

USING ALGEBRA Find the inverse, converse, and contrapositive of the statement.

44. If $x = y$, then $5x = 5y$.

45. $6x - 6 = x + 14$ if $x = 4$.

QUOTES OF WISDOM Rewrite the statement in if-then form. Then (a) determine the hypothesis and conclusion, and (b) find the inverse of the conditional statement.

46. "If you tell the truth, you don't have to remember anything." — Mark Twain

47. "One can never consent to creep when one feels the impulse to soar."
— Helen Keller

48. "Freedom is not worth having if it does not include the freedom to make mistakes." — Mahatma Ghandi

49. "Early to bed and early to rise, makes a man healthy, wealthy, and wise."
— Benjamin Franklin

ADVERTISING In Exercises 50–52, use the following advertising slogan: "You want a great selection of used cars? Come and see Bargain Bob's Used Cars!"

50. Write the slogan in if-then form. What are the hypothesis and conclusion of the conditional statement?

51. Write the inverse, converse, and contrapositive of the conditional statement.

52. *Writing* Find a real-life advertisement or slogan similar to the one given. Then repeat Exercises 50 and 51 using the advertisement or slogan.

53. **TECHNOLOGY** Use geometry software to draw a segment with endpoints *A* and *C*. Draw a third point *B* not on $\overline{AC}$. Measure $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$. Move *B* closer to $\overline{AC}$ and observe the measures of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.

54. **RESEARCH BUGGY** The diagram at the right shows the 35 foot tall Coastal Research Amphibious Buggy, also known as CRAB. This vehicle moves along the ocean floor collecting data that are used to make an accurate map of the ocean floor. Using the postulates you have learned, make a conjecture about why the CRAB was built with three legs instead of four.
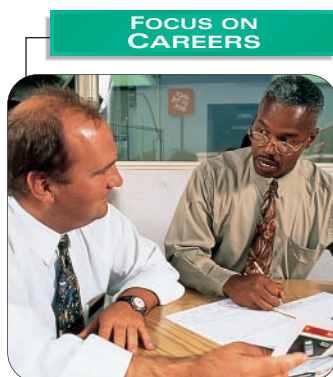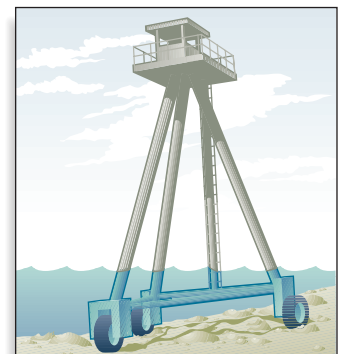
55. MULTIPLE CHOICE Use the conditional statement "If the measure of an angle is 44°, then the angle is acute" to decide which of the following are true.

   I. The statement is true.

   II. The converse of the statement is true.

   III. The contrapositive of the statement is true.

   Ⓐ I only   Ⓑ II only   Ⓒ I and II   Ⓓ I and III   Ⓔ I, II, and III

56. MULTIPLE CHOICE Which one of the following statements is *not* true?

   Ⓐ If $x = 2$, then $x^2 = 4$.

   Ⓑ If $x = -2$, then $x^2 = 4$.

   Ⓒ If $x^3 = -8$, then $x = -2$.

   Ⓓ If $x^2 = 4$, then $x = 2$.

   Ⓔ If $x = -2$, then $x^3 = -8$.

★ **Challenge**

MAKING A CONJECTURE Sketch a line *k* and a point *P* not on line *k*. Make a conjecture about how many planes can be drawn through line *k* and point *P*, and then answer the following questions.

57. Which postulate allows you to state that there are two points, R and S, on line k?

58. Which postulate allows you to conclude that exactly one plane X can be drawn to contain points P, R, and S?

59. Which postulate guarantees that line k is contained in plane X?

60. Was your conjecture correct?

MIXED REVIEW

DRAWING ANGLES Plot the points in a coordinate plane. Then classify ∠*ABC*. **(Review 1.4 for 2.2)**

61. $A(0, 7)$, $B(2, 2)$, $C(6, -1)$ **62.** $A(-1, 0)$, $B(-6, 4)$, $C(-6, -1)$

63. $A(1, 3)$, $B(1, -5)$, $C(-5, -5)$ **64.** $A(-3, -1)$, $B(2, 5)$, $C(3, -2)$

FINDING THE MIDPOINT Find the coordinates of the midpoint of the segment joining the two points. **(Review 1.5)**

65. $A(-2, 8)$, $B(4, -12)$ **66.** $A(8, 8)$, $B(-6, 1)$

67. $A(-7, -4)$, $B(4, 7)$ **68.** $A(0, -9)$, $B(-8, 5)$

69. $A(1, 4)$, $B(11, -6)$ **70.** $A(-10, -10)$, $B(2, 12)$

FINDING PERIMETER AND AREA Find the area and perimeter (or circumference) of the figure described. (Use $\pi \approx 3.14$ when necessary.) **(Review 1.7 for 2.2)**

71. circle, radius $= 6$ m   **72.** square, side $= 11$ cm

73. square, side $= 38.75$ mm   **74.** circle, diameter $= 23$ ft