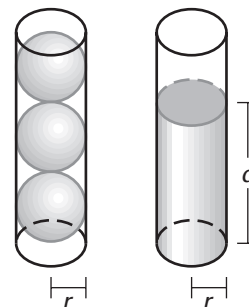# Challenge: Skills and Applications

For use with pages 759–765

1. Suppose that three spherical balls of wax of radius r are packed into a cylinder, also of radius r, as shown. If the wax is melted down, to what depth will the can be filled? Give your answer in terms of r.

2. If a hole of radius r is drilled through the center of a sphere of radius R, we refer to the remaining portion of the sphere as a *bead* with *inner radius r* and *outer radius R*. The *height* of the bead is h, as shown. It can be shown (using calculus) that the volume of a bead depends only on its height. Since the volume is the same whatever the value of r, you can find the answer to parts (a) and (b) by thinking of the special case where $r = 0$ and the bead is actually a sphere.
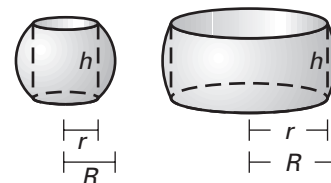
   a. What is the volume of a bead with height 12 cm?

   b. Give a formula for the volume V of a bead in terms of the height h.

3. Suppose a spherical bowl of radius R is filled with liquid to a depth d, where $d < R$. Complete the following steps to find a formula for the volume of the liquid. Let r be the radius of the circle that forms the top surface of the liquid.

   a. Explain why the height of the bead with outer radius R and inner radius r is $2R - 2d$. Then use your formula from Exercise 2(b) to find the volume of the bead in terms of R and d.

   b. Refer to the cylinder shown at the right. First find the length of the unknown leg of the triangle in terms of R and d. Next use the Pythagorean Theorem to find r in terms of R and d. Then find the volume of the cylinder whose radius is r and whose height is $2R - 2d$.

   c. If the volumes you found in parts (a) and (b) are subtracted from $\frac{4}{3}\pi R^3$, the result is *twice* the volume of the liquid.
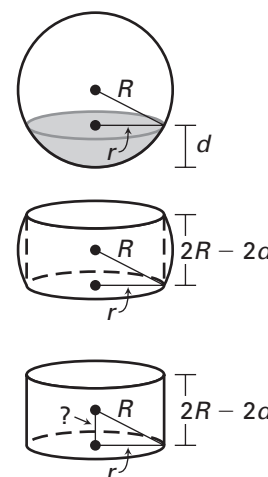
   Use this fact to show that the volume of the liquid is $V = \dfrac{\pi d^2}{3}(3R - d)$.

   (*Note:* Although we assumed that $d < R$ to derive this formula, the formula is actually valid for $0 \le d \le 2R$.)

4. Use the formula from Exercise 3(c) to find the volume of the liquid for each value of d. Make a sketch of the bowl with the given amount of liquid and tell whether the volume makes sense.

   a. $d = 0$ **b.** $d = R$ **c.** $d = 2R$

5. If $d = \dfrac{R}{2}$, the volume of the liquid is what fraction of the volume of the sphere?

   Does your result seem reasonable? Explain.