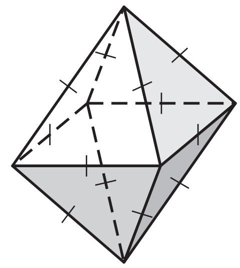
LESSON 12.5

NAME ______________________ DATE __________

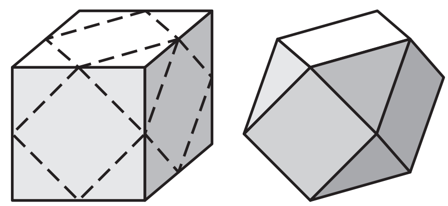
Challenge: Skills and Applications

For use with pages 752–758

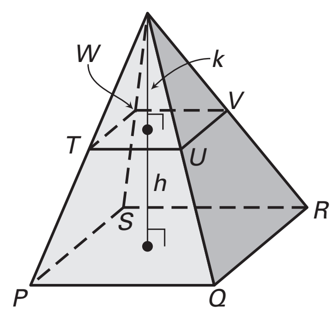
1. The figure shown is a regular octahedron. Each face is an equilateral triangle with a side length of $3\sqrt{2}$ in. Find the volume of the regular octahedron.



2. A *cuboctahedron* has 6 square faces and 8 equilateral triangle faces. It can be made by slicing off the corners of a cube, as shown. If each edge of a cuboctahedron has length $3\sqrt{2}$ cm, find the volume of the cuboctahedron. (*Hint:* Find the volume of the original cube, and subtract the volume of the corners that are removed.)



3. The *frustum* of a pyramid or cone is obtained by slicing off the top portion of the pyramid or cone, as shown, where the cut is parallel to the base of the pyramid or cone. Let B be the area of the base ($PQRS$), and let C be the area of the cut surface ($TUVW$). Let h be the height of the frustum, and let k be the height of the small pyramid (or cone).



a. Show that the volume V of the frustum is given by $V = \frac{1}{3}Bh + \frac{1}{3}(B - C)k.$

b. The cut surface is similar to the base of the original pyramid or cone, and the side lengths are in the ratio $k : h + k$. Use areas of similar figures to show that $\frac{h}{k} + 1 = \sqrt{\frac{B}{C}}.$

c. Show that $\frac{k}{h} = \frac{C + \sqrt{BC}}{B - C}.$

d. Show that $V = \frac{h}{3}\left(B + C + \sqrt{BC}\right).$

4. In the diagram for Exercise 3, suppose the area of $PQRS$ is 50 ft^2 and the area of $TUVW$ is 8 ft^2. Let $h = 3$ ft.

a. Use your result from Exercise 3 to find the volume of the frustum.

b. Find k. Then subtract the volume of the small pyramid from the volume of the large pyramid to verify your answer to part (a).