

Chapter Summary

WHAT did you learn?

Use properties of polyhedra. (12.1)

Find the surface area of prisms and cylinders. (12.2)

Find the surface area of pyramids and cones. (12.3)

Find the volume of prisms and cylinders. (12.4)

Find the volume of pyramids and cones. (12.5)

Find the surface area and volume of a sphere. (12.6)

Find the surface area and volume of similar solids. (12.7)

WHY did you learn it?

Classify crystals by their shape. (p. 725)

Determine the surface area of a wax cylinder record. (p. 733)

Find the area of each lateral face of a pyramid, such as the Pyramid Arena in Tennessee. (p. 735)

Find the volume of a fish tank, such as the tank at the New England Aquarium. (p. 748)

Find the volume of a volcano, such as Mount St. Helens. (p. 757)

Find the surface area of a planet, such as Earth. (p. 763)

Use the scale factor of a model car to determine dimensions on the actual car. (p. 770)

How does Chapter 12 fit into the BIGGER PICTURE of geometry?

Solids can be assigned three types of measure. For instance, the height and radius of a cylinder are one-dimensional measures. The surface area of a cylinder is a two-dimensional measure, and the volume of a cylinder is a three-dimensional measure. Assigning measures to plane regions and to solids is one of the primary goals of geometry. In fact, the word geometry means “Earth measure.”

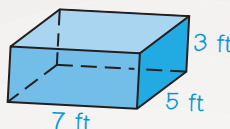
STUDY STRATEGY

How did generalizing formulas help you?

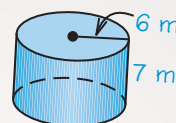
The list of similar concepts you made, following the **Study Strategy** on p. 718, may resemble this one.

Generalizing Formulas

The same concept is used to find the surface area of a prism and the surface area of a cylinder. For example, the surface areas can be found by adding twice the area of the base, $2B$, to the lateral area L .



$$\begin{aligned} S &= 2B + L \\ &= 2(l \cdot w) + Ph \\ &= 2(7 \cdot 5) + 24 \cdot 3 \\ &= 142 \text{ ft}^2 \end{aligned}$$



$$\begin{aligned} S &= 2B + L \\ &= 2(\pi r^2) + Ch \\ &= 2(\pi(6)^2) + (\pi \cdot 12)7 \\ &= 156\pi \text{ m}^2 \end{aligned}$$

Chapter Review

VOCABULARY

- polyhedron, p. 719
- face, p. 719
- edge, p. 719
- vertex, p. 719
- regular polyhedron, p. 720
- convex, p. 720
- cross section, p. 720
- Platonic solids, p. 721
- tetrahedron, p. 721
- octahedron, p. 721
- dodecahedron, p. 721
- icosahedron, p. 721
- prism, p. 728
- bases, p. 728
- lateral faces, p. 728
- right prism, p. 728
- oblique prism, p. 728
- surface area of a polyhedron, p. 728
- lateral area of a polyhedron, p. 728
- net, p. 729
- cylinder, p. 730
- right cylinder, p. 730
- lateral area of a cylinder, p. 730
- surface area of a cylinder, p. 730
- pyramid, p. 735
- regular pyramid, p. 735
- circular cone, p. 737
- lateral surface of a cone, p. 737
- right cone, p. 737
- volume of a solid, p. 743
- sphere, p. 759
- center of a sphere, p. 759
- radius of a sphere, p. 759
- chord of a sphere, p. 759
- diameter of a sphere, p. 759
- great circle, p. 760
- hemisphere, p. 760
- similar solids, p. 766

12.1

EXPLORING SOLIDS

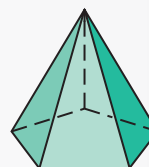
Examples on
pp. 719–722

EXAMPLE The solid at the right has 6 faces and 10 edges. The number of vertices can be found using Euler's Theorem.

$$F + V = E + 2$$

$$6 + V = 10 + 2$$

$$V = 6$$



Use Euler's Theorem to find the unknown number.

1. Faces: 32
Vertices: ?
Edges: 90

2. Faces: ?
Vertices: 6
Edges: 10

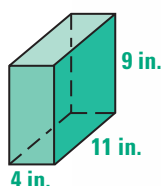
3. Faces: 5
Vertices: 5
Edges: ?

12.2

SURFACE AREA OF PRISMS AND CYLINDERS

Examples on
pp. 728–731

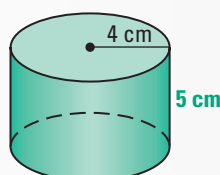
EXAMPLES The surface area of a right prism and a right cylinder are shown.



$$S = 2B + Ph$$

$$= 2(44) + 30(9)$$

$$= 358 \text{ in.}^2$$

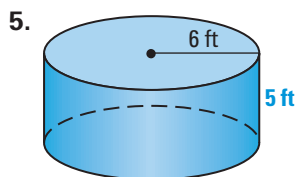
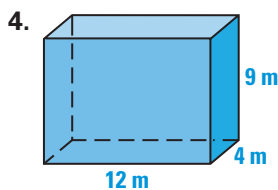


$$S = 2\pi r^2 + 2\pi rh$$

$$= 2\pi(4^2) + 2\pi(4)(5)$$

$$\approx 226.2 \text{ cm}^2$$

Find the surface area of the right prism or right cylinder. Round your result to two decimal places.

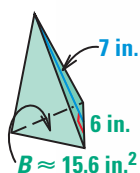


12.3

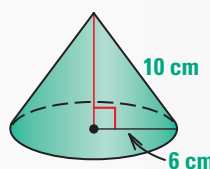
SURFACE AREA OF PYRAMIDS AND CONES

Examples on
pp. 735–737

EXAMPLES The surface area of a regular pyramid and a right cone are shown.

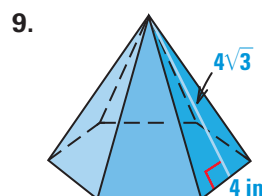
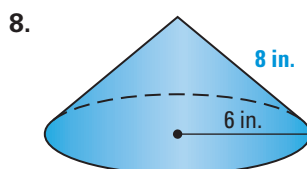
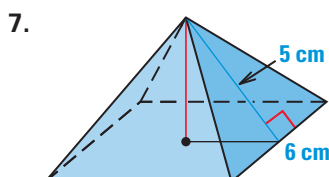


$$\begin{aligned} S &= B + \frac{1}{2}P\ell \\ &\approx 15.6 + \frac{1}{2}(18)(7) \\ &\approx 78.6 \text{ in.}^2 \end{aligned}$$



$$\begin{aligned} S &= \pi r^2 + \pi r\ell \\ &= \pi(6)^2 + \pi(6)(10) \\ &\approx 301.6 \text{ cm}^2 \end{aligned}$$

Find the surface area of the regular pyramid or right cone. Round your result to two decimal places.

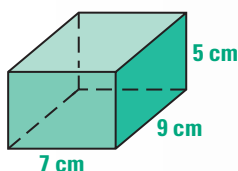


12.4

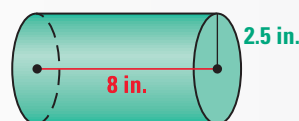
VOLUME OF PRISMS AND CYLINDERS

Examples on
pp. 743–745

EXAMPLES The volume of a rectangular prism and a right cylinder are shown.



$$V = Bh = (7 \cdot 9)(5) = 315 \text{ cm}^3$$



$$V = \pi r^2 h = \pi(2.5^2)(8) \approx 157.1 \text{ in.}^3$$

Find the volume of the described solid.

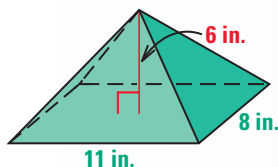
10. A side of a cube measures 8 centimeters.
11. A right prism has a height of 37.2 meters and regular hexagonal bases, each with a base edge of 21 meters.
12. A right cylinder has a radius of 3.5 inches and a height of 8 inches.

12.5

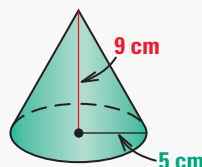
VOLUME OF PYRAMIDS AND CONES

Examples on
pp. 752–754

EXAMPLES The volume of a right pyramid and a right cone are shown.



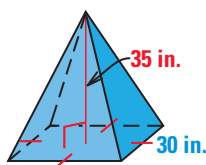
$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(11 \cdot 8)(6) \\ &= 176 \text{ in.}^3 \end{aligned}$$



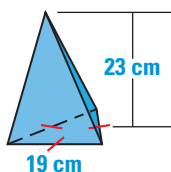
$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(5^2)(9) \\ &\approx 235.6 \text{ cm}^3 \end{aligned}$$

Find the volume of the pyramid or cone.

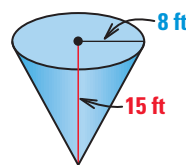
13.



14.



15.



12.6

SURFACE AREA AND VOLUME OF SPHERES

Examples on
pp. 759–761

EXAMPLES The surface area and volume of the sphere are shown.

$$\begin{aligned} S &= 4\pi r^2 = 4\pi(7^2) \approx 615.8 \text{ in.}^2 \\ V &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7^3) \approx 1436.8 \text{ in.}^3 \end{aligned}$$



16. Find the surface area and volume of a sphere with a radius of 14 meters.

17. Find the surface area and volume of a sphere with a radius of 0.5 inch.

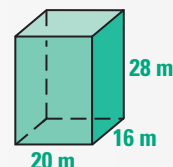
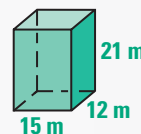
12.7

SIMILAR SOLIDS

Examples on
pp. 766–768

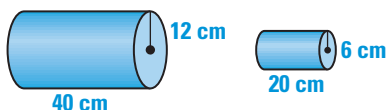
EXAMPLES The ratios of the corresponding linear measurements of the two right prisms are equal, so the solids are similar with a scale factor of 3:4.

$$\text{lengths: } \frac{15}{20} = \frac{3}{4} \quad \text{widths: } \frac{12}{16} = \frac{3}{4} \quad \text{heights: } \frac{21}{28} = \frac{3}{4}$$

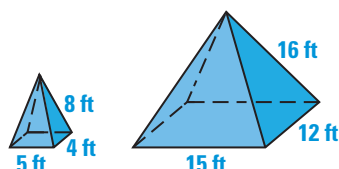


Decide whether the solids are similar. If so, find their scale factor.

18.

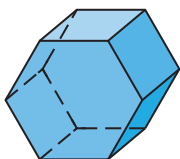


19.

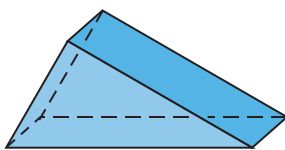


Determine the number of faces, vertices, and edges of the solids.

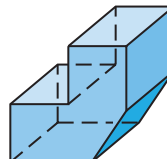
1.



2.



3.



xy USING ALGEBRA Sketch the solid described and find its missing measurement. (B is the base area, P is the base perimeter, h is the height, S is the surface area, r is the radius, and ℓ is the slant height.)

4. Right rectangular prism: $B = 44 \text{ m}^2$, $P = 30 \text{ m}$, $h = 7 \text{ m}$, $S = \underline{\hspace{1cm}}$

5. Right cylinder: $r = 8.6 \text{ in.}$, $h = \underline{\hspace{1cm}}$, $S = 784\pi \text{ in.}^2$

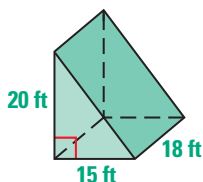
6. Regular pyramid: $B = 100 \text{ ft}^2$, $P = 40 \text{ ft}$, $\ell = \underline{\hspace{1cm}}$, $S = 340 \text{ ft}^2$

7. Right cone: $r = 12 \text{ yd}$, $\ell = 17 \text{ yd}$, $S = \underline{\hspace{1cm}}$

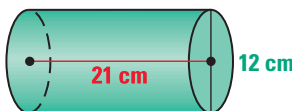
8. Sphere: $r = 34 \text{ cm}$, $S = \underline{\hspace{1cm}}$

In Exercises 9–11, find the volume of the right solid.

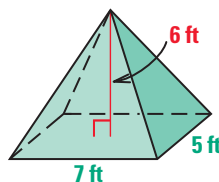
9.



10.



11.

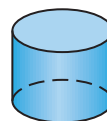


12. Draw a net for each solid in Exercises 9–11. Label the dimensions of the net.

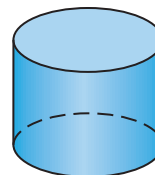
13. The scale factor of two spheres is 1:5. The radius of the smaller sphere is 3 centimeters. What is the volume of the larger sphere?

14. Describe the possible intersections of a plane and a sphere.

15. What is the scale factor of the two cylinders at the right?



$$V = 8\pi \text{ m}^3$$



$$V = 27\pi \text{ m}^3$$

16. **CANNED GOODS** Find the volume and surface area of a prism with a height of 6 inches and a 4 inch by 4 inch square base. Compare the results with the volume and surface area of a cylinder with a height of 7.64 inches and a diameter of 4 inches.

SILOS Suppose you are building a silo. The shape of your silo is a right prism with a regular 15-gon for a base, as shown. The height of your silo is 59 feet.

17. What is the area of the floor of your silo?

18. Find the lateral area and volume of your silo.

19. What are the lateral area and volume of a larger silo that is in a 1:1.25 ratio with yours?

