12.5 Volume of Pyramids and Cones

What you should learn

GOAL 1 Find the volume of pyramids and cones.

GOAL 2 Find the volume of pyramids and cones in **real life**, such as the nautical prism in **Example 4**.

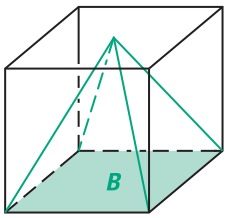
Why you should learn it

▼ Learning to find volumes of pyramids and cones is important in **real life**, such as in finding the volume of a volcano shown below and in **Ex. 34**.

Mount St. Helens

GOAL 1 FINDING VOLUMES OF PYRAMIDS AND CONES

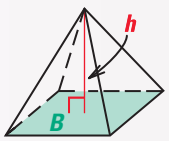
In Lesson 12.4, you learned that the volume of a prism is equal to Bh, where B is the area of the base and h is the height. From the figure at the right, it is clear that the volume of the pyramid with the same base area B and the same height h must be less than the volume of the prism. The volume of the pyramid is one third the volume of the prism.



THEOREMS

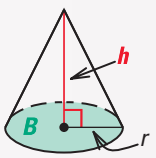
THEOREM 12.9 ***Volume of a Pyramid***

The volume V of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.



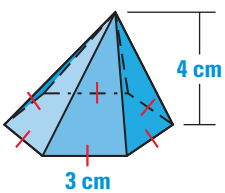
THEOREM 12.10 ***Volume of a Cone***

The volume V of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$, where B is the area of the base, h is the height, and r is the radius of the base.



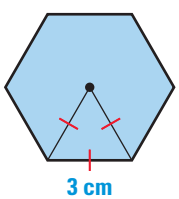
EXAMPLE 1 *Finding the Volume of a Pyramid*

Find the volume of the pyramid with the regular base.



SOLUTION

The base can be divided into six equilateral triangles. Using the formula for the area of an equilateral triangle, $\frac{1}{4}\sqrt{3} \cdot s^2$, the area of the base B can be found as follows:



$$6 \cdot \frac{1}{4}\sqrt{3} \cdot s^2 = 6 \cdot \frac{1}{4}\sqrt{3} \cdot 3^2 = \frac{27}{2}\sqrt{3}\ \text{cm}^2.$$

Use Theorem 12.9 to find the volume of the pyramid.

$$V = \frac{1}{3}Bh \qquad \text{Formula for volume of pyramid}$$

$$= \frac{1}{3}\left(\frac{27}{2}\sqrt{3}\right)(4) \qquad \text{Substitute.}$$

$$= 18\sqrt{3} \qquad \text{Simplify.}$$

▶ So, the volume of the pyramid is $18\sqrt{3}$, or about 31.2 cubic centimeters.

STUDENT HELP

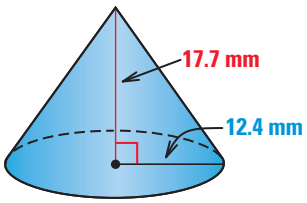
Study Tip

The formulas given in Theorems 12.9 and 12.10 apply to all pyramids and cones, whether right or oblique. This follows from Cavalieri's Principle, stated in Lesson 12.4.

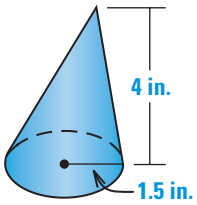
EXAMPLE 2 Finding the Volume of a Cone

Find the volume of each cone.

a. Right circular cone



b. Oblique circular cone



SOLUTION

a. Use the formula for the volume of a cone.

$$V = \frac{1}{3}Bh \qquad \text{Formula for volume of cone}$$

$$= \frac{1}{3}(\pi r^2)h \qquad \text{Base area equals } \pi r^2.$$

$$= \frac{1}{3}(\pi 12.4^2)(17.7) \qquad \text{Substitute.}$$

$$\approx 907.18\pi \qquad \text{Simplify.}$$

▶ So, the volume of the cone is about 907.18π, or 2850 cubic millimeters.

b. Use the formula for the volume of a cone.

$$V = \frac{1}{3}Bh \qquad \text{Formula for volume of cone}$$

$$= \frac{1}{3}(\pi r^2)h \qquad \text{Base area equals } \pi r^2.$$

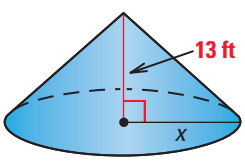
$$= \frac{1}{3}(\pi 1.5^2)(4) \qquad \text{Substitute.}$$

$$= 3\pi \qquad \text{Simplify.}$$

▶ So, the volume of the cone is 3π, or about 9.42 cubic inches.

EXAMPLE 3 Using the Volume of a Cone

Use the given measurements to solve for x.



Volume = 2614 ft^3

SOLUTION

$$V = \frac{1}{3}\pi r^2 h \qquad \text{Formula for volume}$$

$$2614 = \frac{1}{3}(\pi x^2)(13) \qquad \text{Substitute.}$$

$$7842 = 13\pi x^2 \qquad \text{Multiply each side by 3.}$$

$$192 \approx x^2 \qquad \text{Divide each side by } 13\pi.$$

$$13.86 \approx x \qquad \text{Find positive square root.}$$

▶ So, the radius of the cone is about 13.86 feet.

STUDENT HELP

Study Tip

To eliminate the fraction in an equation, you can multiply each side by the reciprocal of the fraction. This was done in Example 3.

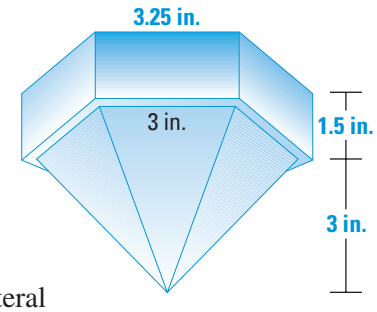
FOCUS ON APPLICATIONS

REAL LIFE **NAUTICAL PRISMS** Before electricity, nautical prisms were placed in the decks of sailing ships. By placing the hexagonal face flush with the deck, the prisms would draw light to the lower regions of the ship.

GOAL 2 USING VOLUME IN REAL-LIFE PROBLEMS

EXAMPLE 4 *Finding the Volume of a Solid*

NAUTICAL PRISMS A nautical prism is a solid piece of glass, as shown. Find its volume.



SOLUTION

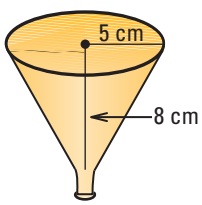
To find the volume of the entire solid, add the volumes of the prism and the pyramid. The bases of the prism and the pyramid are regular hexagons made up of six equilateral triangles. To find the area of each base, B, multiply the area of one of the equilateral triangles by 6, or $6\left(\frac{\sqrt{3}}{4}s^2\right)$, where s is the base edge.

Volume of prism	$= 6\left(\frac{\sqrt{3}}{4}s^2\right)h$	**Formula for volume of prism**
	$= 6\left(\frac{\sqrt{3}}{4}(3.25)^2\right)(1.5)$	**Substitute.**
	≈ 41.16	**Use a calculator.**
Volume of pyramid	$= \frac{1}{3} \cdot 6\left(\frac{\sqrt{3}}{4}s^2\right)h$	**Formula for volume of pyramid**
	$= \frac{1}{3} \cdot 6\left(\frac{\sqrt{3}}{4} \cdot 3^2\right)(3)$	**Substitute.**
	≈ 23.38	**Use a calculator.**

▶ The volume of the nautical prism is **41.16** + **23.38** or 64.54 cubic inches.

EXAMPLE 5 *Using the Volume of a Cone*

REAL LIFE **AUTOMOBILES** If oil is being poured into the funnel at a rate of 147 milliliters per second and flows out of the funnel at a rate of 42 milliliters per second, estimate the time it will take for the funnel to overflow. $(1 \text{ mL} = 1 \text{ cm}^3)$



SOLUTION

First, find the approximate volume of the funnel.

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5^2)(8) \approx 209 \text{ cm}^3 = 209 \text{ mL}$$

The rate of accumulation of oil in the funnel is $147 - 42 = 105$ mL/s. To find the time it will take for the oil to fill the funnel, divide the volume of the funnel by the rate of accumulation of oil in the funnel as follows:

$$209 \text{ mL} \div \frac{105 \text{ mL}}{1 \text{ s}} = 209 \text{ mL} \times \frac{1 \text{ s}}{105 \text{ mL}} \approx 2 \text{ s}$$

▶ The funnel will overflow after about 2 seconds.

GUIDED PRACTICE

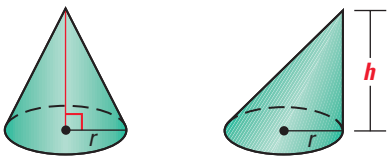
Vocabulary Check ✓

1. The volume of a cone with radius r and height h is $\frac{1}{3}$ the volume of a ___?___ with radius r and height h.

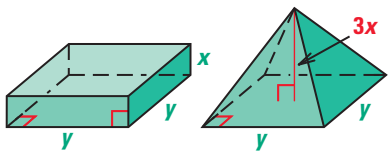
Concept Check ✓

Do the two solids have the same volume? Explain your answer.

2.



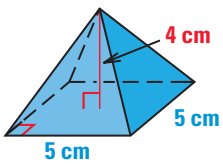
3.



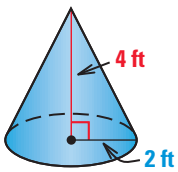
Skill Check ✓

In Exercises 4–6, find (a) the area of the base of the solid and (b) the volume of the solid.

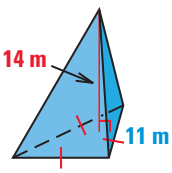
4.



5.



6.



7. CRITICAL THINKING You are given the radius and the slant height of a right cone. Explain how you can find the height of the cone.

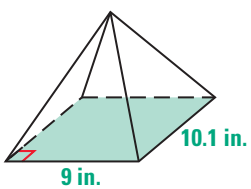
PRACTICE AND APPLICATIONS

STUDENT HELP

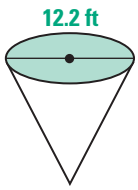
Extra Practice to help you master skills is on p. 826.

FINDING BASE AREAS **Find the area of the base of the solid.**

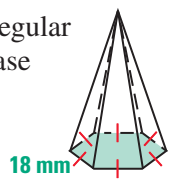
8.



9.

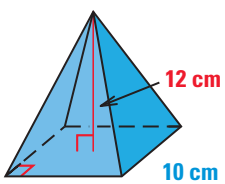


10. Regular base

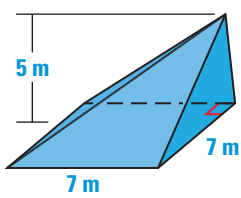


VOLUME OF A PYRAMID **Find the volume of the pyramid. Each pyramid has a regular polygon for a base.**

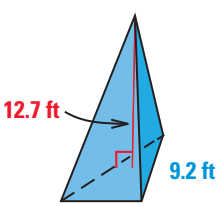
11.



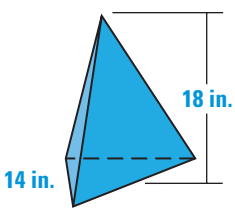
12.



13.



14.



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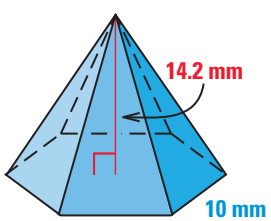
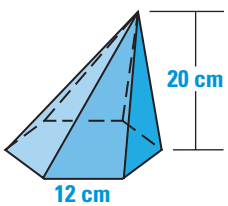


16.



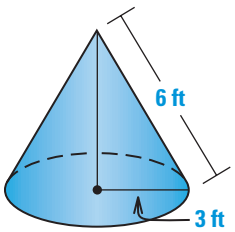
STUDENT HELP

HOMEWORK HELP

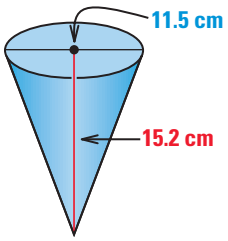
Example 1: Exs. 11–16
Example 2: Exs. 17–19
Example 3: Exs. 20–22
Example 4: Exs. 23–28
Example 5: Ex. 29

VOLUME OF A CONE **Find the volume of the cone. Round your result to two decimal places.**

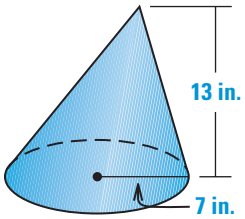
17.



18.

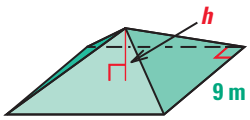


19.

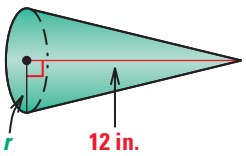


USING ALGEBRA **Solve for the variable using the given information.**

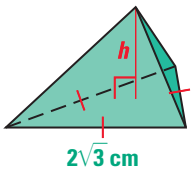
20. Volume $= 270\ m^3$



21. Volume $= 100\pi\ in.^3$



22. Volume $= 5\sqrt{3}\ cm^3$



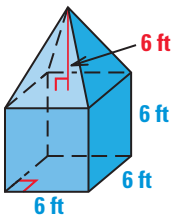
STUDENT HELP

INTERNET HOMEWORK HELP

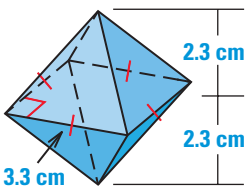
Visit our Web site www.mcdougallittell.com for help with Exs. 23–25.

COMPOSITE SOLIDS **Find the volume of the solid. The prisms, pyramids, and cones are right. Round the result to two decimal places.**

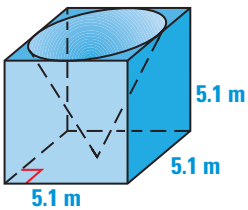
23.



24.



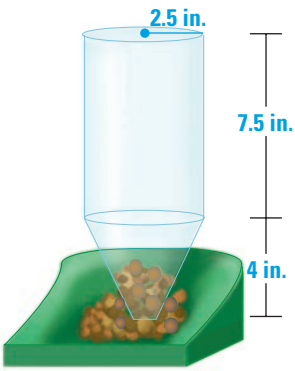
25.



AUTOMATIC FEEDER **In Exercises 26 and 27, use the diagram of the automatic pet feeder.** $(1\ cup = 14.4\ in.^3)$

26. Calculate the amount of food that can be placed in the feeder.

27. If a cat eats half of a cup of food, twice per day, will the feeder hold enough food for three days?



28. **ANCIENT CONSTRUCTION** Early civilizations in the Andes Mountains in Peru used cone-shaped adobe bricks to build homes. Find the volume of an adobe brick with a diameter of 8.3 centimeters and a slant height of 10.1 centimeters. Then calculate the amount of space 27 of these bricks would occupy in a mud mortar wall.

29. **SCIENCE CONNECTION** During a chemistry lab, you use a funnel to pour a solvent into a flask. The radius of the funnel is 5 centimeters and its height is 10 centimeters. If the solvent is being poured into the funnel at a rate of 80 milliliters per second and the solvent flows out of the funnel at a rate of 65 milliliters per second, how long will it be before the funnel overflows? $(1\ mL = 1\ cm^3)$

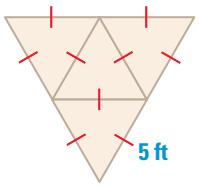
FOCUS ON CAREERS

VOLCANOLOGY Volcanologists collect and interpret data about volcanoes to help them predict when a volcano will erupt.

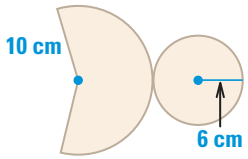
CAREER LINK
www.mcdougallittell.com

USING NETS **In Exercises 30–32, use the net to sketch the solid. Then find the volume of the solid. Round the result to two decimal places.**

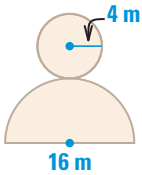
30.



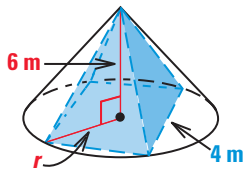
31.



32.



33. FINDING VOLUME In the diagram at the right, a regular square pyramid with a base edge of 4 meters is inscribed in a cone with a height of 6 meters. Use the dimensions of the pyramid to find the volume of the cone.



34. VOLCANOES Before 1980, Mount St. Helens was cone shaped with a height of about 1.83 miles and a base radius of about 3 miles. In 1980, Mount St. Helens erupted. The tip of the cone was destroyed, as shown, reducing the volume by 0.043 cubic mile. The cone-shaped tip that was destroyed had a radius of about 0.4 mile. How tall is the volcano today? (*Hint:* Find the height of the destroyed cone-shaped tip.)

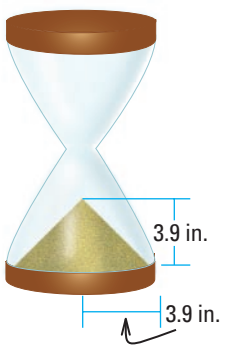
Test Preparation

MULTI-STEP PROBLEM **Use the diagram of the hourglass below.**

35. Find the volume of the cone-shaped pile of sand.

36. The sand falls through the opening at a rate of one cubic inch per minute. Is the hourglass a true "hour"-glass? Explain. (1 hr = 60 min)

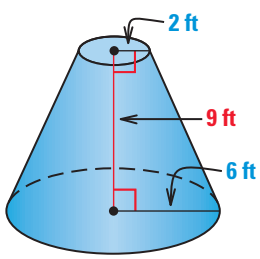
37. *Writing* The sand in the hourglass falls into a conical shape with a one-to-one ratio between the radius and the height. Without doing the calculations, explain how to find the radius and height of the pile of sand that has accumulated after 30 minutes.



Challenge

FRUSTUMS **A *frustum* of a cone is the part of the cone that lies between the base and a plane parallel to the base, as shown. Use the information below to complete Exercises 38 and 39.**

One method for calculating the volume of a frustum is to add the areas of the two bases to their geometric mean, then multiply the result by $\frac{1}{3}$ the height.



38. Use the measurements in the diagram to calculate the volume of the frustum.

39. Write a formula for the volume of a frustum that has bases with radii r_1 and r_2 and a height h.

STUDENT HELP

Look Back
For help with finding geometric means, see p. 466.

MIXED REVIEW

FINDING ANGLE MEASURES **Find the measure of each interior and exterior angle of a regular polygon with the given number of sides.** (Review 11.1)

40. 9 **41.** 10 **42.** 19

43. 22 **44.** 25 **45.** 30

FINDING THE AREA OF A CIRCLE **Find the area of the described circle.** (Review 11.5 for 12.6)

46. The diameter of the circle is 25 inches.

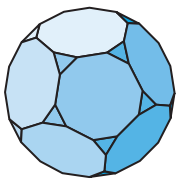
47. The radius of the circle is 16.3 centimeters.

48. The circumference of the circle is 48π feet.

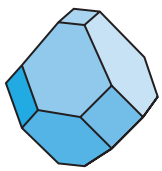
49. The length of a 36° arc of the circle is 2π meters.

USING EULER'S THEOREM **Calculate the number of vertices of the solid using the given information.** (Review 12.1)

50. 32 faces; 12 octagons and 20 triangles



51. 14 faces; 6 squares and 8 hexagons

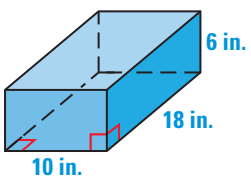


QUIZ 2

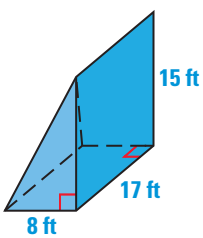
Self-Test for Lessons 12.4 and 12.5

In Exercises 1–6, find the volume of the solid. (Lessons 12.4 and 12.5)

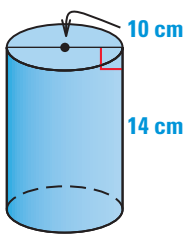
1.



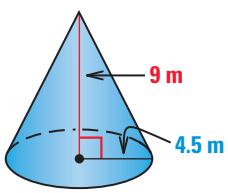
2.



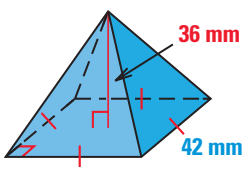
3.



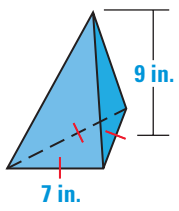
4.



5.



6.



7. **STORAGE BUILDING** A road-salt storage building is composed of a regular octagonal pyramid and a regular octagonal prism as shown. Find the volume of salt that the building can hold. (Lesson 12.5)