12.4 Volume of Prisms and Cylinders

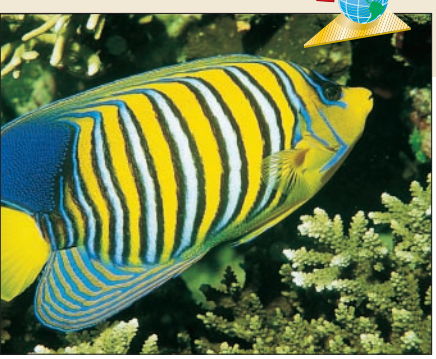
What you should learn

GOAL 1 Use volume postulates.

GOAL 2 Find the volume of prisms and cylinders in **real life**, such as the concrete block in **Example 4**.

Why you should learn it

▼ Learning to find the volumes of prisms and cylinders is important in **real life**, such as in finding the volume of a fish tank in **Exs. 7–9, and 46**.



GOAL 1 EXPLORING VOLUME

The **volume of a solid** is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic meters (m^3).

VOLUME POSTULATES

POSTULATE 27 ***Volume of a Cube***

The volume of a cube is the cube of the length of its side, or $V = s^3$.

POSTULATE 28 ***Volume Congruence Postulate***

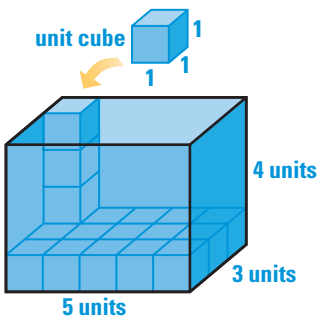
If two polyhedra are congruent, then they have the same volume.

POSTULATE 29 ***Volume Addition Postulate***

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

EXAMPLE 1 *Finding the Volume of a Rectangular Prism*

The box shown is 5 units long, 3 units wide, and 4 units high. How many unit cubes will fit in the box? What is the volume of the box?



SOLUTION

The base of the box is 5 units by 3 units. This means 5 • 3, or 15 unit cubes, will cover the base.

Three more layers of 15 cubes each can be placed on top of the lower layer to fill the box. Because the box contains 4 layers with 15 cubes in each layer, the box contains a total of 4 • 15, or 60 unit cubes.

▶ Because the box is completely filled by the 60 cubes and each cube has a volume of 1 cubic unit, it follows that the volume of the box is 60 • 1, or 60 cubic units.

. . . . . .

In Example 1, the area of the base, 15 square units, multiplied by the height, 4 units, yields the volume of the box, 60 cubic units. So, the volume of the prism can be found by multiplying the area of the base by the height. This method can also be used to find the volume of a cylinder.

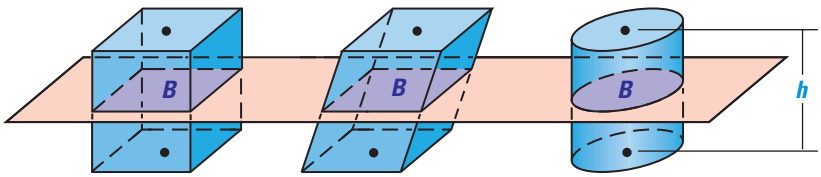
GOAL 2 FINDING VOLUMES OF PRISMS AND CYLINDERS

THEOREM

THEOREM 12.6 ***Cavalieri's Principle***

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

Theorem 12.6 is named after mathematician Bonaventura Cavalieri (1598–1647). To see how it can be applied, consider the solids below. All three have cross sections with equal areas, B, and all three have equal heights, h. By Cavalieri's Principle, it follows that each solid has the same volume.



VOLUME THEOREMS

THEOREM 12.7 ***Volume of a Prism***

The volume V of a prism is $V = Bh$, where B is the area of a base and h is the height.

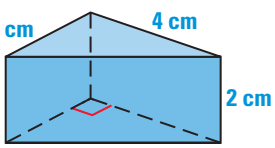
THEOREM 12.8 ***Volume of a Cylinder***

The volume V of a cylinder is $V = Bh = \pi r^2 h$, where B is the area of a base, h is the height, and r is the radius of a base.

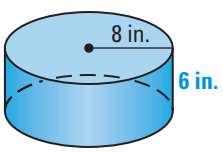
EXAMPLE 2 *Finding Volumes*

Find the volume of the right prism and the right cylinder.

a.



b.



SOLUTION

a. The area B of the base is $\frac{1}{2}(3)(4)$, or 6 cm^2. Use $h = 2$ to find the volume.

$$V = Bh = 6(2) = 12 \text{ cm}^3$$

b. The area B of the base is $\pi \cdot 8^2$, or 64π in.^{2} Use $h = 6$ to find the volume.

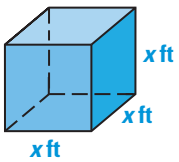
$$V = Bh = 64\pi(6) = 384\pi \approx 1206.37 \text{ in.}^3$$

Using Algebra

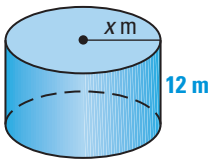
EXAMPLE 3 Using Volumes

Use the measurements given to solve for x.

a. Cube, $V = 100$ ft^3



b. Right cylinder, $V = 4561$ m^3



STUDENT HELP

INTERNET KEYSTROKE HELP

If your calculator does not have a cube root key, you can raise a number to the $\frac{1}{3}$ to find its cube root. For example, the cube root of 8 can be found as follows:

8 ^ (1 ÷ 3)

SOLUTION

a. A side length of the cube is x feet.

$V = s^3$ Formula for volume of cube

$100 = x^3$ Substitute.

$4.64 \approx x$ Take the cube root.

▶ So, the height, width, and length of the cube are about 4.64 feet.

b. The area of the base is πx^2 square meters.

$V = Bh$ Formula for volume of cylinder

$4561 = \pi x^2(12)$ Substitute.

$4561 = 12\pi x^2$ Rewrite.

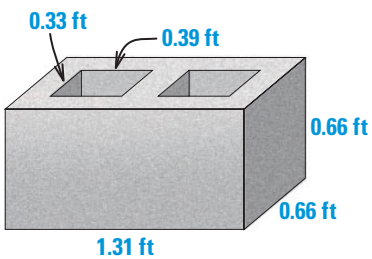
$\frac{4561}{12\pi} = x^2$ Divide each side by 12π.

$11 \approx x$ Find the positive square root.

▶ So, the radius of the cylinder is about 11 meters.

EXAMPLE 4 Using Volumes in Real Life

CONSTRUCTION Concrete weighs 145 pounds per cubic foot. To find the weight of the concrete block shown, you need to find its volume. The area of the base can be found as follows:



$$B = \boxed{\text{Area of large rectangle}} - 2 \cdot \boxed{\text{Area of small rectangle}}$$

$$= (1.31)(0.66) - 2(0.33)(0.39)$$

$$\approx 0.61 \text{ ft}^2$$

Using the formula for the volume of a prism, the volume is

$$V = Bh \approx 0.61(0.66) \approx 0.40 \text{ ft}^3.$$

▶ To find the weight of the block, multiply the pounds per cubic foot, 145 lb/ft^3, by the number of cubic feet, 0.40 ft^3.

$$\text{Weight} = \frac{145 \text{ lb}}{1 \text{ ft}^3} \cdot 0.4 \text{ ft}^3 \approx 58 \text{ lb}$$

FOCUS ON APPLICATIONS

REAL LIFE **CONSTRUCTION** The Ennis-Brown House, shown above, was designed by Frank Lloyd Wright. It was built using concrete blocks.

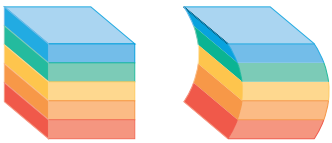
GUIDED PRACTICE

Vocabulary Check ✓

1. Surface area is measured in __?__ and volume is measured in __?__.

Concept Check ✓

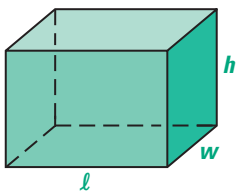
2. Each stack of memo papers shown contains 500 sheets of paper. Explain why the stacks have the same volume. Then calculate the volume, given that each sheet of paper is 3 inches by 3 inches by 0.01 inches.



Skill Check ✓

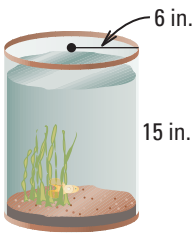
Use the diagram to complete the table.

	ℓ	w	h	Volume
3.	17	3	5	?
4.	?	8	10	160
5.	4.8	6.1	?	161.04
6.	$6t$	?	$3t$	$54t^3$

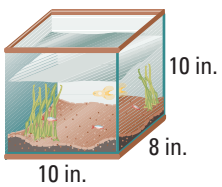


FISH TANKS **Find the volume of the tank.**

7.



8.



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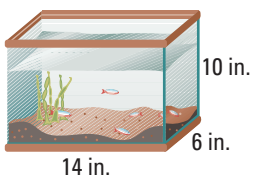


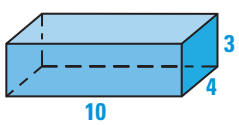
PRACTICE AND APPLICATIONS

STUDENT HELP

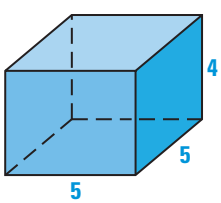
Extra Practice to help you master skills is on p. 826.

USING UNIT CUBES **Find the number of unit cubes that will fit in the box. Explain your reasoning.**

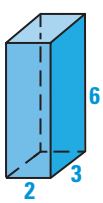
10.



11.



12.



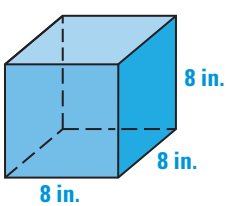
STUDENT HELP

HOMEWORK HELP

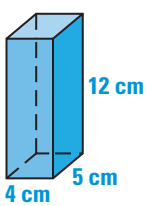
Example 1: Exs. 10–12
Example 2: Exs. 13–27
Example 3: Exs. 28–33
Example 4: Exs. 34–37

VOLUME OF A PRISM **Find the volume of the right prism.**

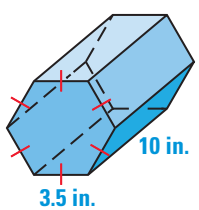
13.



14.

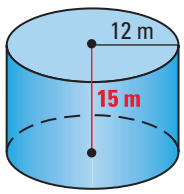


15.

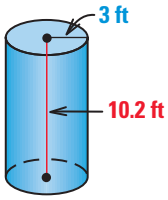


VOLUME OF A CYLINDER **Find the volume of the right cylinder. Round the result to two decimal places.**

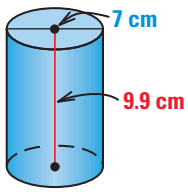
16.



17.



18.



VISUAL THINKING **Make a sketch of the solid and find its volume. Round the result to two decimal places.**

19. A prism has a square base with 4 meter sides and a height of 15 meters.

20. A pentagonal prism has a base area of 24 square feet and a height of 3 feet.

21. A prism has a height of 11.2 centimeters and an equilateral triangle for a base, where each base edge measures 8 centimeters.

22. A cylinder has a radius of 4 meters and a height of 8 meters.

23. A cylinder has a radius of 21.4 feet and a height of 33.7 feet.

24. A cylinder has a diameter of 15 inches and a height of 26 inches.

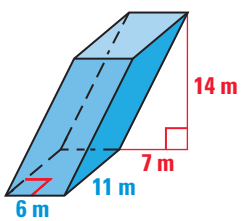
STUDENT HELP

INTERNET HOMEWORK HELP

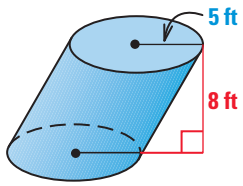
Visit our Web site www.mcdougallittell.com for help with Exs. 25–27.

VOLUMES OF OBLIQUE SOLIDS **Use Cavalieri's Principle to find the volume of the oblique prism or cylinder.**

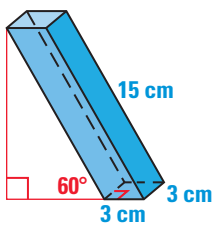
25.



26.

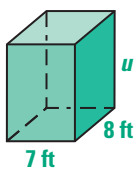


27.

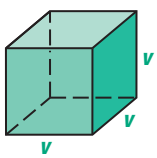


USING ALGEBRA **Solve for the variable using the given measurements. The prisms and the cylinders are right.**

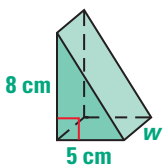
28. Volume = 560 ft^3



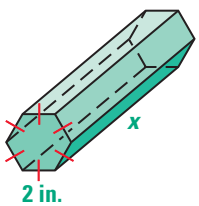
29. Volume = 2700 yd^3



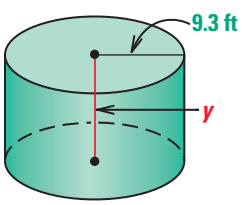
30. Volume = 80 cm^3



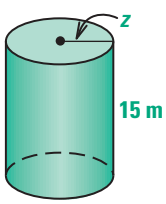
31. Volume = 72.66 $in.^3$



32. Volume = 3000 ft^3



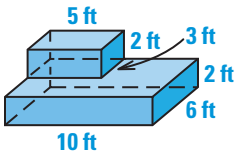
33. Volume = 1696.5 m^3



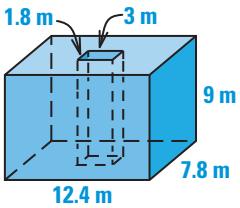
34. CONCRETE BLOCK In Example 4 on page 745, find the volume of the entire block and subtract the volume of the two rectangular prisms. How does your answer compare with the volume found in Example 4?

FINDING VOLUME Find the volume of the entire solid. The prisms and cylinders are right.

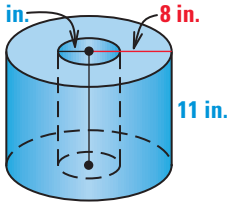
35.



36.



37.



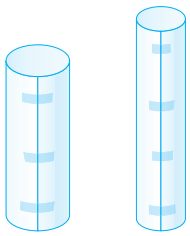
CONCRETE In Exercises 38–40, determine the number of yards of concrete you need for the given project. To builders, a "yard" of concrete means a cubic yard. (A cubic yard is equal to $(36 \text{ in.})^3$, or $46{,}656 \text{ in.}^3$.)

38. A driveway that is 30 feet long, 18 feet wide, and 4 inches thick

39. A tennis court that is 100 feet long, 50 feet wide, and 6 inches thick

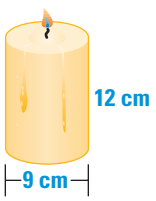
40. A circular patio that has a radius of 24 feet and is 8 inches thick

41. LOGICAL REASONING Take two sheets of paper that measure $8\frac{1}{2}$ inches by 11 inches and form two cylinders; one with the height as $8\frac{1}{2}$ inches and one with the height as 11 inches. Do the cylinders have the same volume? Explain.

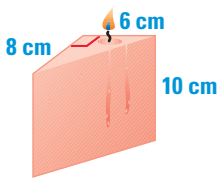


CANDLES In Exercises 42–44, you are melting a block of wax to make candles. How many candles of the given shape can be made using a block that measures 10 cm by 9 cm by 20 cm? The prisms and cylinder are right.

42.



43.



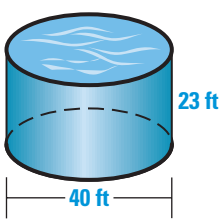
44.

45. CANNED GOODS Find the volume and surface area of a prism with a height of 4 inches and a 3 inch by 3 inch square base. Compare the results with the volume and surface area of a cylinder with a height of 5.1 inches and a diameter of 3 inches. Use your results to explain why canned goods are usually packed in cylindrical containers.

AQUARIUM TANK The Caribbean Coral Reef Tank at the New England Aquarium is a cylindrical tank that is 23 feet deep and 40 feet in diameter, as shown.

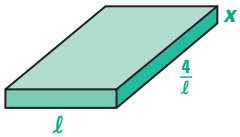
46. How many gallons of water are needed to fill the tank? (One gallon of water equals 0.1337 cubic foot.)

47. Determine the weight of the water in the tank. (One gallon of salt water weighs about 8.56 pounds.)



Test Preparation

48. MULTIPLE CHOICE If the volume of the rectangular prism at the right is 1, what does x equal?



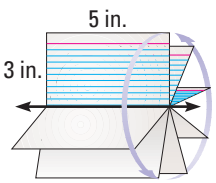
Ⓐ $\frac{1}{4}$ Ⓑ $\frac{\ell}{4}$ Ⓒ ℓ Ⓓ 4 Ⓔ 4ℓ

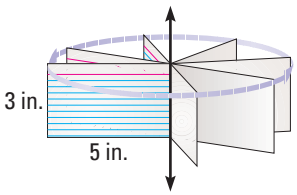
49. MULTIPLE CHOICE What is the volume of a cylinder with a radius of 6 and a height of 10?

Ⓐ 60π Ⓑ 90π Ⓒ 120π Ⓓ 180π Ⓔ 360π

Challenge

50. Suppose that a 3 inch by 5 inch index card is rotated around a horizontal line and a vertical line to produce two different solids, as shown. Which solid has a greater volume? Explain your reasoning.





EXTRA CHALLENGE
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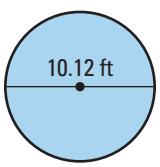
MIXED REVIEW

USING RATIOS **Find the measures of the angles in the triangle whose angles are in the given extended ratio.** (Review 8.1)

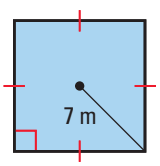
51. 2:5:5 **52.** 1:2:3 **53.** 3:4:5

FINDING AREA **In Exercises 54–56, find the area of the figure. Round your result to two decimal places.** (Review 11.2, 11.5 for 12.5)

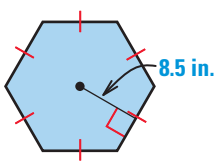
54.



55.



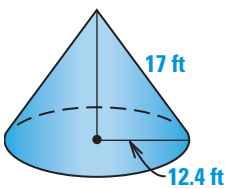
56.



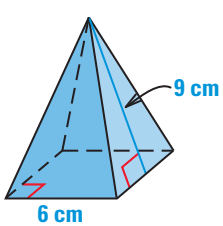
57. SURFACE AREA OF A PRISM A right rectangular prism has a height of 13 inches, a length of 1 foot, and a width of 3 inches. Sketch the prism and find its surface area. (Review 12.2)

SURFACE AREA **Find the surface area of the solid. The cone is right and the pyramids are regular.** (Review 12.3)

58.



59.



60.