12.3 Surface Area of Pyramids and Cones

What *you should learn*

GOAL 1 Find the surface area of a pyramid.

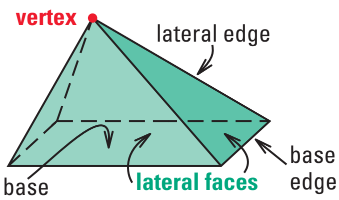
GOAL 2 Find the surface area of a cone.

Why *you should learn it*

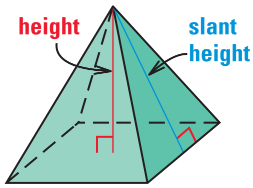
▼ To find the surface area of solids in **real life**, such as the Pyramid Arena in Memphis, Tennessee, shown below and in **Example 1**.

GOAL 1 FINDING THE SURFACE AREA OF A PYRAMID

A **pyramid** is a polyhedron in which the *base* is a polygon and the *lateral faces* are triangles with a common *vertex*. The intersection of two lateral faces is a *lateral edge*. The intersection of the base and a lateral face is a *base edge*. The *altitude*, or *height*, of the pyramid is the perpendicular distance between the base and the vertex.



Pyramid

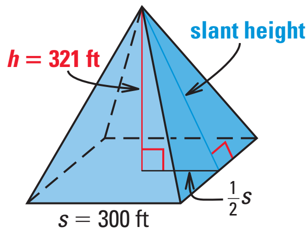


Regular pyramid

A **regular pyramid** has a regular polygon for a base and its height meets the base at its center. The *slant height* of a regular pyramid is the altitude of any lateral face. A nonregular pyramid does not have a slant height.

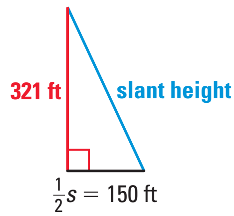
EXAMPLE 1 *Finding the Area of a Lateral Face*

ARCHITECTURE The lateral faces of the Pyramid Arena in Memphis, Tennessee, are covered with steel panels. Use the diagram of the arena at the right to find the area of each lateral face of this regular pyramid.



SOLUTION

To find the slant height of the pyramid, use the Pythagorean Theorem.



$$(\text{Slant height})^2 = h^2 + \left(\tfrac{1}{2}s\right)^2$$ **Write formula.**

$$(\text{Slant height})^2 = 321^2 + 150^2$$ **Substitute.**

$$(\text{Slant height})^2 = 125{,}541$$ **Simplify.**

$$\text{Slant height} = \sqrt{125{,}541}$$ **Take the positive square root.**

$$\text{Slant height} \approx 354.32$$ **Use a calculator.**

▶ So, the area of each lateral face is $\frac{1}{2}$(base of lateral face)(slant height), or about $\frac{1}{2}(300)(354.32)$, which is about 53,148 square feet.

STUDENT HELP

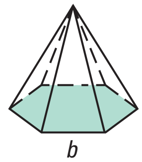
Study Tip

A *regular pyramid* is considered a regular polyhedron only if *all* its faces, including the base, are congruent. So, the only pyramid that is a regular polyhedron is the regular triangular pyramid, or *tetrahedron*. See page 721.

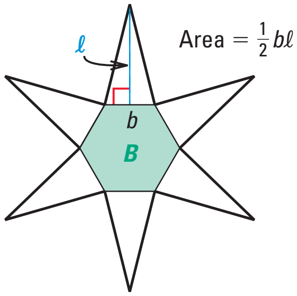
STUDENT HELP

Study Tip
When sketching the net of a pyramid, first sketch the base. Then sketch the lateral faces.

A regular hexagonal pyramid and its net are shown at the right. Let b represent the length of a base edge, and let ℓ represent the slant height of the pyramid.



The area of each lateral face is $\frac{1}{2}b\ell$ and the perimeter of the base is $P = 6b$. So, the surface area is as follows:



$S =$ (Area of base) + 6(Area of lateral face)

$S = B + 6\left(\frac{1}{2}b\ell\right)$ **Substitute.**

$S = B + \frac{1}{2}(6b)\ell$ **Rewrite $6\left(\frac{1}{2}b\ell\right)$ as $\frac{1}{2}(6b)\ell$.**

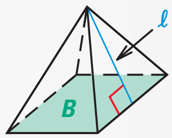
$S = B + \frac{1}{2}P\ell$ **Substitute P for $6b$.**

THEOREM

THEOREM 12.4 ***Surface Area of a Regular Pyramid***

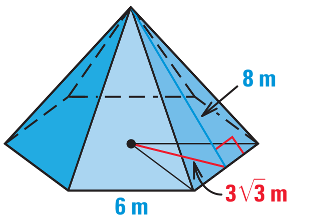
The surface area S of a regular pyramid is

$S = B + \frac{1}{2}P\ell$, where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.



EXAMPLE 2 *Finding the Surface Area of a Pyramid*

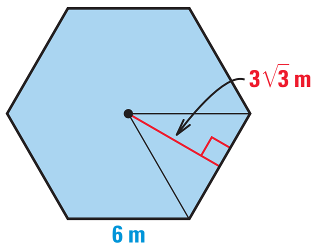
To find the surface area of the regular pyramid shown, start by finding the area of the base.



STUDENT HELP

Look Back
For help with finding the area of regular polygons see pp. 669–671.

Use the formula for the area of a regular polygon, $\frac{1}{2}$(apothem)(perimeter). A diagram of the base is shown at the right. After substituting, the area of the base is $\frac{1}{2}(3\sqrt{3})(6 \cdot 6)$, or $54\sqrt{3}$ square meters.



Now you can find the surface area, using $54\sqrt{3}$ for the area of the base, B.

$S = B + \frac{1}{2}P\ell$ **Write formula.**

$= 54\sqrt{3} + \frac{1}{2}(36)(8)$ **Substitute.**

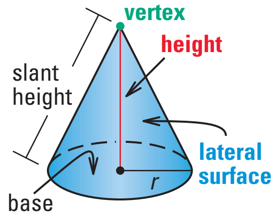
$= 54\sqrt{3} + 144$ **Simplify.**

≈ 237.5 **Use a calculator.**

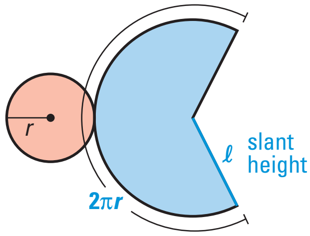
▶ So, the surface area is about 237.5 square meters.

GOAL 2 FINDING THE SURFACE AREA OF A CONE

A **circular cone**, or **cone**, has a circular *base* and a *vertex* that is not in the same plane as the base. The *altitude*, or *height*, is the perpendicular distance between the vertex and the base. In a **right cone**, the height meets the base at its center and the *slant height* is the distance between the vertex and a point on the base edge.



The **lateral surface** of a cone consists of all segments that connect the vertex with points on the base edge. When you cut along the slant height and lie the cone flat, you get the net shown at the right. In the net, the circular base has an area of πr^2 and the lateral surface is the sector of a circle. You can find the area of this sector by using a proportion, as shown below.



$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference of circle}} \qquad \textbf{Set up proportion.}$$

$$\frac{\text{Area of sector}}{\pi \ell^2} = \frac{2\pi r}{2\pi \ell} \qquad \textbf{Substitute.}$$

$$\text{Area of sector} = \pi \ell^2 \cdot \frac{2\pi r}{2\pi \ell} \qquad \textbf{Multiply each side by } \pi\ell^2.$$

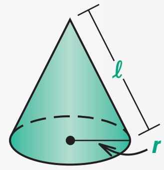
$$\text{Area of sector} = \pi r \ell \qquad \textbf{Simplify.}$$

The surface area of a cone is the sum of the base area and the lateral area, $\pi r \ell$.

THEOREM

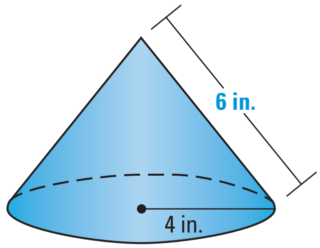
THEOREM 12.5 ***Surface Area of a Right Cone***

The surface area S of a right cone is $S = \pi r^2 + \pi r \ell$, where r is the radius of the base and ℓ is the slant height.



EXAMPLE 3 *Finding the Surface Area of a Right Cone*

To find the surface area of the right cone shown, use the formula for the surface area.



$S = \pi r^2 + \pi r \ell$ **Write formula.**

$= \pi 4^2 + \pi(4)(6)$ **Substitute.**

$= 16\pi + 24\pi$ **Simplify.**

$= 40\pi$ **Simplify.**

▶ The surface area is 40π square inches, or about 125.7 square inches.

Guided Practice

Vocabulary Check ✓

1. Describe the differences between pyramids and cones. Describe their similarities.

Concept Check ✓

2. Can a pyramid have rectangles for lateral faces? Explain.

Skill Check ✓

Match the expression with the correct measurement.

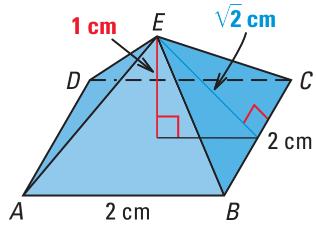
3. Area of base — **A.** $4\sqrt{2}$ cm^2

4. Height — **B.** $\sqrt{2}$ cm

5. Slant height — **C.** 4 cm^2

6. Lateral area — **D.** $(4 + 4\sqrt{2})$ cm^2

7. Surface area — **E.** 1 cm



In Exercises 8–11, sketch a right cone with $r = 3$ ft and $h = 7$ ft.

8. Find the area of the base.

9. Find the slant height.

10. Find the lateral area.

11. Find the surface area.

Find the surface area of the regular pyramid described.

12. The base area is 9 square meters, the perimeter of the base is 12 meters, and the slant height is 2.5 meters.

13. The base area is $25\sqrt{3}$ square inches, the perimeter of the base is 30 inches, and the slant height is 12 inches.

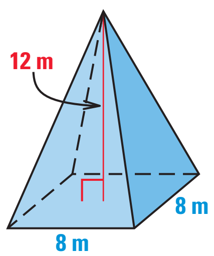
Practice and Applications

Student Help

Extra Practice to help you master skills is on p. 825.

Area of a Lateral Face Find the area of a lateral face of the regular pyramid. Round the result to one decimal place.

14.



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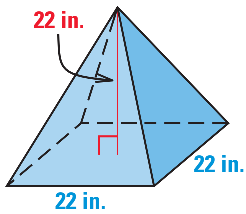

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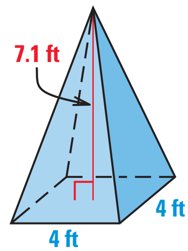
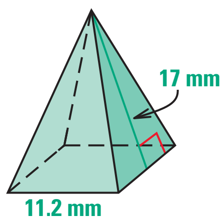


Surface Area of a Pyramid Find the surface area of the regular pyramid.

Student Help

Homework Help
Example 1: Exs. 14–16
Example 2: Exs. 17–19
Example 3: Exs. 20–25

17.



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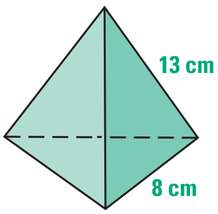

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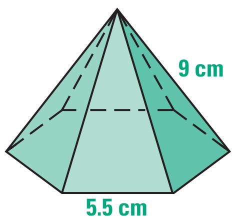
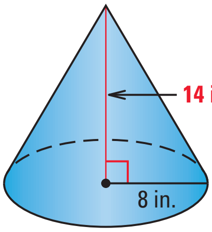
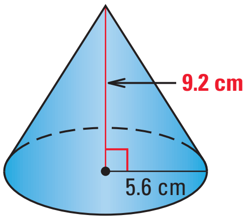


FINDING SLANT HEIGHT Find the slant height of the right cone.

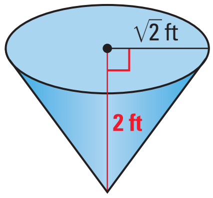
20.



21.

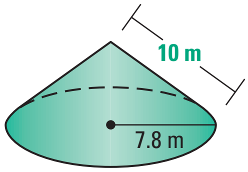


22.

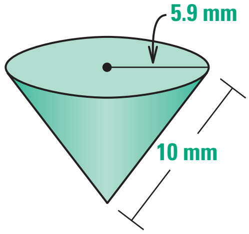


SURFACE AREA OF A CONE Find the surface area of the right cone. Leave your answers in terms of π.

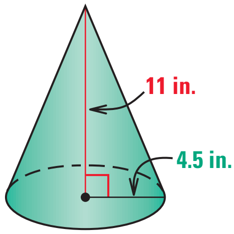
23.



24.

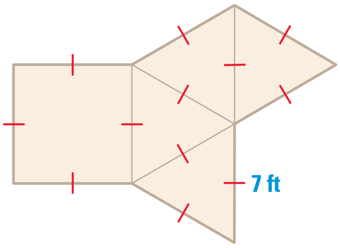


25.

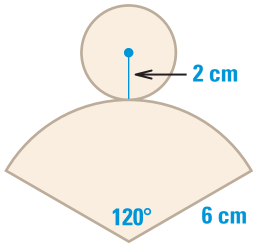


USING NETS Name the figure that is represented by the net. Then find its surface area. Round the result to one decimal place.

26.



27.



VISUAL THINKING Sketch the described solid and find its surface area. Round the result to one decimal place.

28. A regular pyramid has a triangular base with a base edge of 8 centimeters, a height of 12 centimeters, and a slant height of 12.2 centimeters.

29. A regular pyramid has a hexagonal base with a base edge of 3 meters, a height of 5.8 meters, and a slant height of 6.2 meters.

30. A right cone has a diameter of 11 feet and a slant height of 7.2 feet.

31. A right cone has a radius of 9 inches and a height of 12 inches.

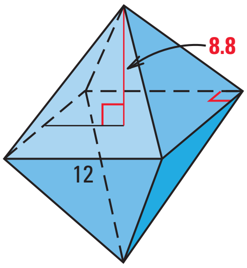
STUDENT HELP

INTERNET

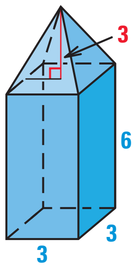
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 32–34.

COMPOSITE SOLIDS Find the surface area of the solid. The pyramids are regular and the prisms, cylinders, and cones are right. Round the result to one decimal place.

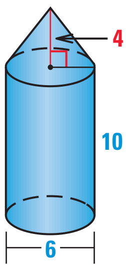
32.



33.

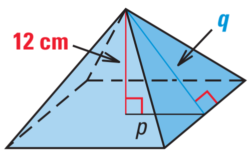


34.

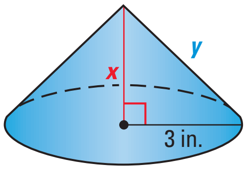


USING ALGEBRA In Exercises 35–37, find the missing measurements of the solid. The pyramids are regular and the cones are right.

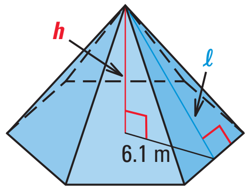
35. $P = 72$ cm



36. $S = 75.4$ in.^{2}



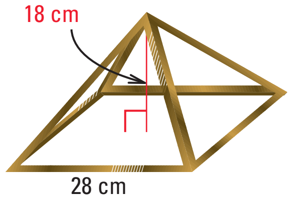
37. $S = 333$ m^2, $P = 42$ m



FOCUS ON APPLICATIONS

REAL LIFE **LAMPSHADES** Many stained-glass lampshades are shaped like cones or pyramids. These shapes help direct the light down.

38. LAMPSHADES Some stained-glass lampshades are made out of decorative pieces of glass. Estimate the amount of glass needed to make the lampshade shown at the right by calculating the lateral area of the pyramid formed by the framing. The pyramid has a square base.



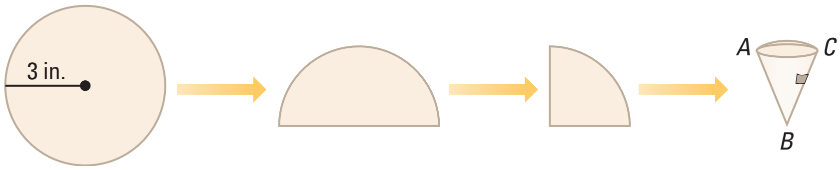
39. PYRAMIDS The three pyramids of Giza, Egypt, were built as regular square pyramids. The pyramid in the middle of the photo is Chephren's Pyramid and when it was built its base edge was $707\frac{3}{4}$ feet, and it had a height of 471 feet. Find the surface area of Chephren's Pyramid, including its base, when it was built.

40. DATA COLLECTION Find the dimensions of Chephren's Pyramid today and calculate its surface area. Compare this surface area with the surface area you found in Exercise 39.

41. SQUIRREL BARRIER Some bird feeders have a metal cone that prevents squirrels from reaching the bird seed, as shown. You are planning to manufacture this metal cone. The slant height of the cone is 12 inches and the radius is 8 inches. Approximate the amount of sheet metal you need.

42. CRITICAL THINKING A regular hexagonal pyramid with a base edge of 9 feet and a height of 12 feet is inscribed in a right cone. Find the lateral area of the cone.

43. PAPER CUP To make a paper drinking cup, start with a circular piece of paper that has a 3 inch radius, then follow the steps below. How does the surface area of the cup compare to the original paper circle? Find $m\angle ABC$.



Test Preparation

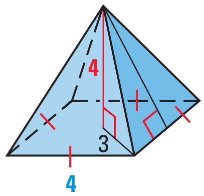
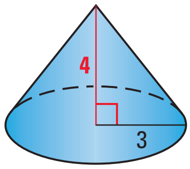
QUANTITATIVE COMPARISON **Choose the statement that is true about the given quantities.**

Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

Ⓒ The two quantities are equal.

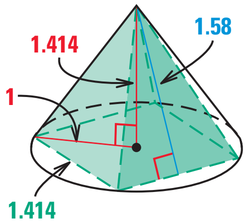
Ⓓ The relationship cannot be determined from the given information.

	Column A	Column B
		
44.	Area of base	Area of base
45.	Lateral edge length	Slant height
46.	Lateral area	Lateral area

★ Challenge

INSCRIBED PYRAMIDS **Each of three regular pyramids are inscribed in a right cone whose radius is 1 unit and height is $\sqrt{2}$ units. The dimensions of each pyramid are listed in the table and the square pyramid is shown.**

Base	Base edge	Slant height
Square	1.414	1.58
Hexagon	1	1.65
Octagon	0.765	1.68



47. Find the surface area of the cone.

48. Find the surface area of each of the three pyramids.

49. What happens to the surface area as the number of sides of the base increases? If the number of sides continues to increase, what number will the surface area approach?

EXTRA CHALLENGE
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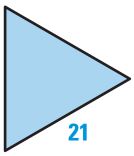
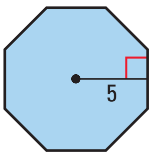
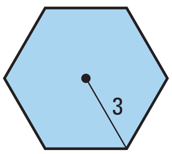
MIXED REVIEW

FINDING AREA **In Exercises 50–52, find the area of the regular polygon. Round your result to two decimal places.** (Review 11.2 for 12.4)

50. 

51. 

52. 

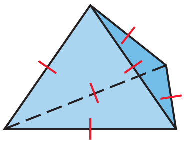
53. **AREA OF A SEMICIRCLE** A semicircle has an area of 190 square inches. Find the approximate length of the radius. (Review 11.5 for 12.4)

QUIZ 1

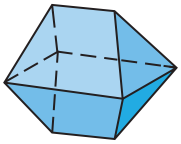
Self-Test for Lessons 12.1–12.3

State whether the polyhedron is regular and/or convex. Then calculate the number of vertices of the solid using the given information. (Lesson 12.1)

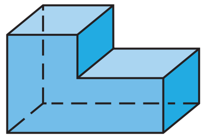
1. 4 faces; all triangles



2. 8 faces; 4 triangles and 4 trapezoids

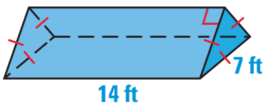


3. 8 faces; 2 hexagons and 6 rectangles

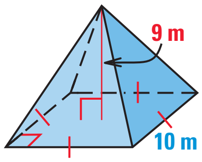


Find the surface area of the solid. Round your result to two decimal places. (Lesson 12.2 and 12.3)

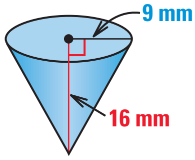
4.



5.



6.



MATH & History

History of Containers

APPLICATION LINK
www.mcdougallittell.com

THEN

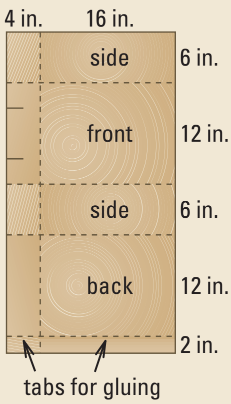
THROUGHOUT HISTORY, people have created containers for items that were important to store, such as liquids and grains. In ancient civilizations, large jars called *amphorae* were used to store water and other liquids.

NOW

TODAY, containers are no longer used just for the bare necessities. People use containers of many shapes and sizes to store a variety of objects.

1. How much paper is required to construct a paper grocery bag using the pattern at the right?

2. The sections on the left side of the pattern are folded to become the rectangular base of the bag. Find the dimensions of the base. Then find the surface area of the completed bag.



c. 525 B.C.
Amphorae are used in Ancient Greece to store water and oils.

1810
Tin containers are first used to package food.

1870
Margaret Knight patents machine to make paper bags.

1990s
Water bottles come in all shapes and sizes.