NAME ______________________________ DATE __________

# Reteaching with Practice

**For use with pages 669–675**

**GOAL** **Find the area of an equilateral triangle and a regular polygon**

### VOCABULARY

The **center of a regular polygon** is the center of its circumscribed circle.

The **radius of a regular polygon** is the radius of its circumscribed circle.

The distance from the center to any side of a regular polygon is called the **apothem of the polygon**.

A **central angle of a regular polygon** is an angle whose vertex is the center and whose sides contain two consecutive vertices of the polygon.

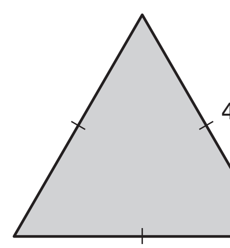
**Theorem 11.3 Area of an Equilateral Triangle** The area of an equilateral triangle is one fourth the square of the length of the side times $\sqrt{3}$.

$A = \frac{1}{4}\sqrt{3}s^2$

**Theorem 11.4 Area of a Regular Polygon** The area of a regular $n$-gon with side length $s$ is half the product of the apothem $a$ and the perimeter $P$, so $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$.

## EXAMPLE 1 *Finding the Area of an Equilateral Triangle*

Find the area of an equilateral triangle with 4 foot sides.




### SOLUTION

Use $s = 4$ in the formula of Theorem 11.3.

$$A = \frac{1}{4}\sqrt{3}s^2 = \frac{1}{4}\sqrt{3}(4^2)$$

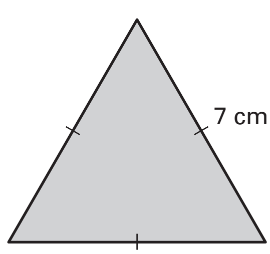
$$= \frac{1}{4}\sqrt{3}(16) = \frac{1}{4}(16)\sqrt{3} = 4\sqrt{3} \text{ square feet}$$

Using a calculator, the area is about 6.9 square feet.

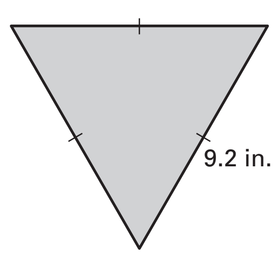
### *Exercises for Example 1*

**Find the area of the triangle.**

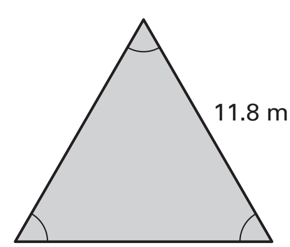
**1.**




**2.**




**3.**



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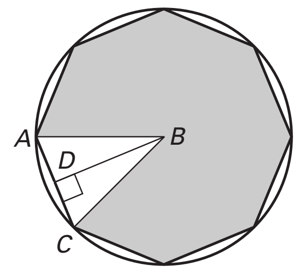
## EXAMPLE 2 *Finding the Area of a Regular Polygon*




A regular octagon is inscribed in a circle with radius 2 units. Find the area of the octagon.

### Solution

To apply the formula for the area of a regular octagon, you must find its apothem and perimeter.

The measure of central $\angle ABC$ is $\frac{1}{8} \cdot 360° = 45°$.

In isosceles triangle $ABC$, the altitude to base $\overline{AC}$ also bisects $\angle ABC$ and side $\overline{AC}$. The measure of $\angle DBC$ is 22.5°. In $\triangle BDC$, you can use trigonometric ratios to find the lengths of the legs.

$$\cos 22.5° = \frac{BD}{BC} = \frac{BD}{2} \text{ and } \sin 22.5° = \frac{DC}{BC} = \frac{DC}{2}$$

So, the octagon has an apothem of $a = BD = 2 \cdot \cos 22.5°$ and perimeter of $P = 8(AC) = 8(2 \cdot DC) = 8(2 \cdot 2 \cdot \sin 22.5°) = 32 \cdot \sin 22.5°$. The area of the octagon is

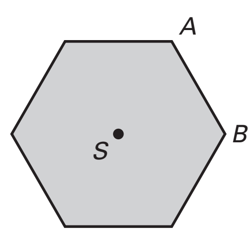
$$A = \frac{1}{2}aP = \frac{1}{2}(2 \cdot \cos 22.5°)(32 \cdot \sin 22.5°) \approx 11.3 \text{ square units.}$$

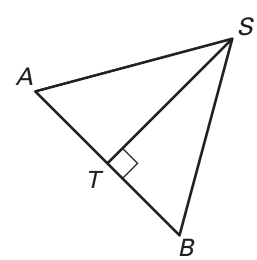
### *Exercise for Example 2*

**4.** Find the area of a regular pentagon inscribed in a circle with radius 3 units.

## EXAMPLE 3 *Finding the Perimeter and Area of a Regular Polygon*

Find the perimeter and area of a regular hexagon with side length of 4 cm and radius 4 cm.




### Solution

A hexagon has 6 sides. So, the perimeter is $P = 6(4) = 24$ cm.

To determine the apothem, consider the triangle $SBT$.

$BT = \frac{1}{2}(BA) = \frac{1}{2}(4) = 2$ cm.

Use the Pythagorean Theorem to find the apothem $ST$.

$a = \sqrt{4^2 - 2^2} = 2\sqrt{3}$ cm.

So, the area of the hexagon is $A = \frac{1}{2}aP = \frac{1}{2}(2\sqrt{3})(24) = 24\sqrt{3}$ cm$^2$.

### *Exercise for Example 3*

**Find the perimeter and area of the regular polygon described.**

**5.** Regular octagon with side length 9.18 feet and radius 12 feet.