CHAPTER 11

Chapter Summary

WHAT did you learn? / *WHY did you learn it?*

WHAT did you learn?	WHY did you learn it?
Find the measures of the interior and exterior angles of polygons. **(11.1)**	Find the measures of angles in real-world objects, such as a home plate marker. **(p. 664)**
Find the areas of equilateral triangles and other regular polygons. **(11.2)**	Solve problems by finding real-life areas, such as the area of a hexagonal mirror in a telescope. **(p. 674)**
Compare perimeters and areas of similar figures. **(11.3)**	Solve real-life problems, such as estimating a reasonable cost for photographic paper. **(p. 678)**
Find the circumference of a circle and the length of an arc of a circle. **(11.4)**	Find real-life distances, such as the distance around a track. **(p. 685)**
Find the areas of circles and sectors. **(11.5)**	Find areas of real-life regions containing circles or parts of circles, such as the area of the front of a case for a clock. **(p. 693)**
Find a geometric probability. **(11.6)**	Estimate the likelihood that an event will occur, such as the likelihood that divers will find a sunken ship on their first dive. **(p. 703)**

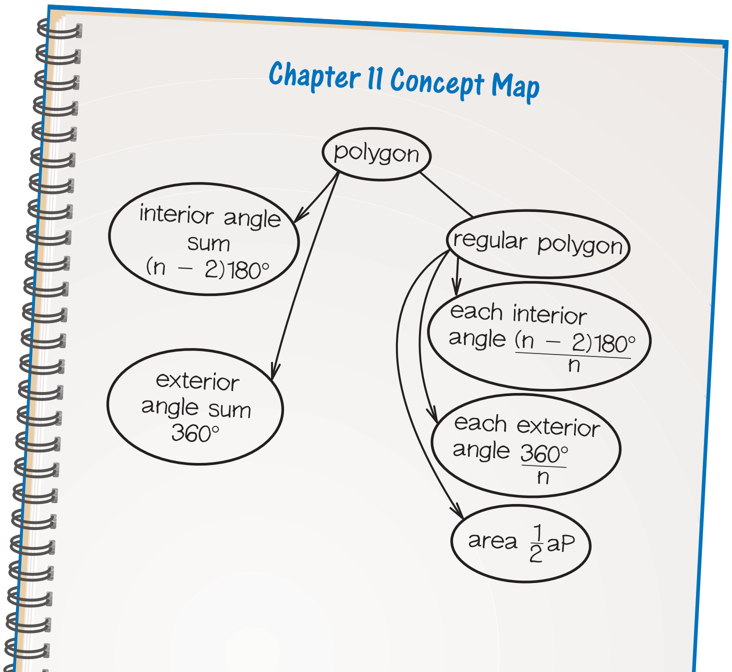
How does Chapter 11 fit into the BIGGER PICTURE of geometry?

The word *geometry* is derived from Greek words meaning "land measurement." The ability to measure angles, arc lengths, perimeters, circumferences, and areas allows you to calculate measurements required to solve problems in the real world. Keep in mind that a region that lies in a plane has two types of measures. The perimeter or circumference of a region is a *one*-dimensional measure that uses units such as centimeters or feet. The area of a region is a *two*-dimensional measure that uses units such as square centimeters or square feet. In the next chapter, you will study a *three*-dimensional measure called *volume*.

STUDY STRATEGY

Did your concept map help you organize your work?

The concept map you made, following the **Study Strategy** on page 660, may include these ideas.



CHAPTER 11

Chapter Review

VOCABULARY

- center of a polygon, p. 670
- radius of a polygon, p. 670
- apothem of a polygon, p. 670
- central angle of a regular polygon, p. 671
- circumference, p. 683
- arc length, p. 683
- sector of a circle, p. 692
- probability, p. 699
- geometric probability, p. 699

11.1 ANGLE MEASURES IN POLYGONS

Examples on pp. 661–664

EXAMPLES If a regular polygon has 15 sides, then the sum of the measures of its interior angles is $(15 - 2) \cdot 180° = 2340°$. The measure of each interior angle is $\frac{1}{15} \cdot 2340° = 156°$. The measure of each exterior angle is $\frac{1}{15} \cdot 360° = 24°$.

In Exercises 1–4, you are given the number of sides of a regular polygon. Find the measure of each interior angle and each exterior angle.

1. 9　**2.** 13　**3.** 16　**4.** 24

In Exercises 5–8, you are given the measure of each interior angle of a regular *n*-gon. Find the value of *n*.

5. 172°　**6.** 135°　**7.** 150°　**8.** 170°

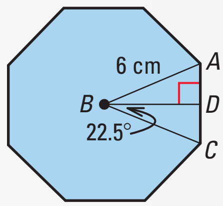
11.2 AREAS OF REGULAR POLYGONS

Examples on pp. 669–671

EXAMPLES The area of an equilateral triangle with sides of length 14 inches is

$A = \frac{1}{4}\sqrt{3}\,(14^2) = \frac{1}{4}\sqrt{3}\,(196) = 49\sqrt{3} \approx 84.9 \text{ in.}^2$

In the regular octagon at the right, $m\angle ABC = \frac{1}{8} \cdot 360° = 45°$ and $m\angle DBC = 22.5°$. The apothem BD is $6 \cdot \cos 22.5°$. The perimeter of the octagon is $8 \cdot 2 \cdot DC$, or $16(6 \cdot \sin 22.5°)$. The area of the octagon is



$A = \frac{1}{2}aP = \frac{1}{2}(6 \cos 22.5°) \cdot 16(6 \sin 22.5°) \approx 101.8 \text{ cm}^2.$

9. An equilateral triangle has 12 centimeter sides. Find the area of the triangle.

10. An equilateral triangle has a height of 6 inches. Find the area of the triangle.

11. A regular hexagon has 5 meter sides. Find the area of the hexagon.

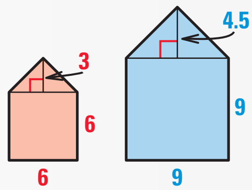
12. A regular decagon has 1.5 foot sides. Find the area of the decagon.

11.3 PERIMETERS AND AREAS OF SIMILAR FIGURES

Examples on pp. 677–678

EXAMPLE The two pentagons at the right are similar. Their corresponding sides are in the ratio $2:3$, so the ratio of their areas is $2^2:3^2 = 4:9$.

$$\frac{\text{Area (smaller)}}{\text{Area (larger)}} = \frac{6 \cdot 6 + \frac{1}{2}(3 \cdot 6)}{9 \cdot 9 + \frac{1}{2}(4.5 \cdot 9)} = \frac{45}{101.25} = \frac{4}{9}$$



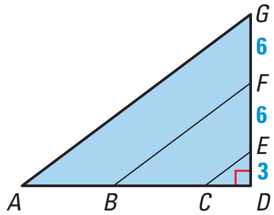
Complete the statement using *always*, *sometimes*, or *never*.

13. If the ratio of the perimeters of two rectangles is $3:5$, then the ratio of their areas is __?__ $9:25$.

14. Two parallelograms are __?__ similar.

15. Two regular dodecagons with perimeters in the ratio 4 to 7 __?__ have areas in the ratio 16 to 49.

In the diagram at the right, $\triangle ADG$, $\triangle BDF$, and $\triangle CDE$ are similar.



16. Find the ratio of the perimeters and of the areas of $\triangle CDE$ and $\triangle BDF$.

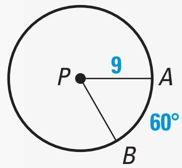
17. Find the ratio of the perimeters and of the areas of $\triangle ADG$ and $\triangle BDF$.

11.4 CIRCUMFERENCE AND ARC LENGTH

Examples on pp. 683–685

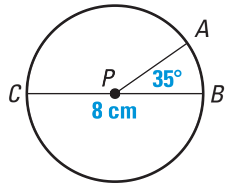
EXAMPLES The circumference of the circle at the right is $C = 2\pi(9) = 18\pi$.

The length of $\widehat{AB} = \dfrac{m\widehat{AB}}{360^\circ} \cdot 2\pi r = \dfrac{60^\circ}{360^\circ} \cdot 18\pi = 3\pi$.



In Exercises 18–20, find the circumference of $\odot P$ and the length of $\widehat{AB}$.

18.



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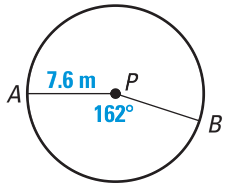

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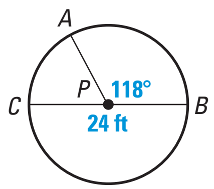


21. Find the radius of a circle with circumference 12 inches.

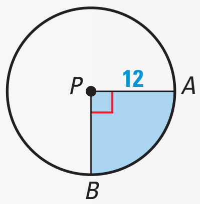
22. Find the diameter of a circle with circumference 15π meters.

11.5 Areas of Circles and Sectors

Examples on pp. 691–694

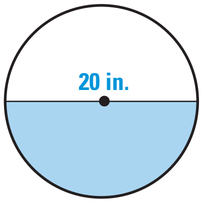
EXAMPLES The area of $\odot P$ at the right is $A = \pi(12^2) = 144\pi$. To find the area A of the shaded sector of $\odot P$, use

$$\frac{A}{\pi r^2} = \frac{m\overparen{AB}}{360^\circ}, \text{ or } A = \frac{m\overparen{AB}}{360^\circ} \cdot \pi r^2 = \frac{90^\circ}{360^\circ} \cdot 144\pi = 36\pi.$$



In Exercises 23–26, find the area of the shaded region.

23.



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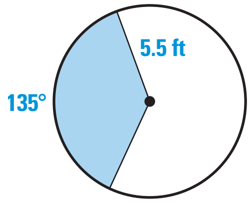

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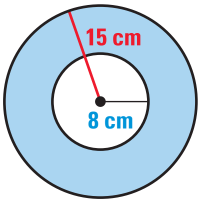

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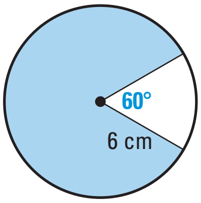


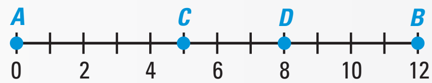
27. What is the area of a circle with diameter 28 feet?

28. What is the radius of a circle with area 40 square inches?

11.6 Geometric Probability

Examples on pp. 699–701

EXAMPLES The probability that a randomly chosen point on $\overline{AB}$ is on $\overline{CD}$ is

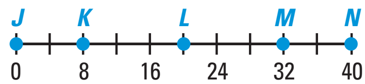


$$P(\text{Point is on } \overline{CD}) = \frac{\text{Length of } \overline{CD}}{\text{Length of } \overline{AB}} = \frac{3}{12} = \frac{1}{4}.$$

Suppose a circular target has radius 12 inches and its bull's eye has radius 2 inches. If a dart that hits the target hits it at a random point, then

$$P(\text{Dart hits bull's eye}) = \frac{\text{Area of bull's eye}}{\text{Area of target}} = \frac{4\pi}{144\pi} = \frac{1}{36}.$$

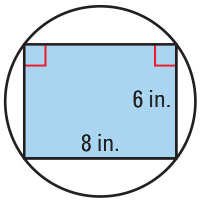
Find the probability that a point *A*, selected randomly on $\overline{JN}$, is on the given segment.



29. $\overline{LM}$ **30.** $\overline{JL}$ **31.** $\overline{KM}$

Find the probability that a randomly chosen point in the figure lies in the shaded region.

32.



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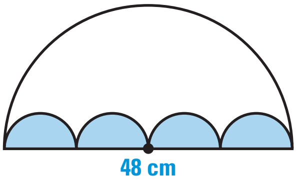

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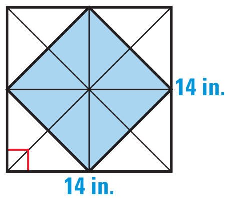
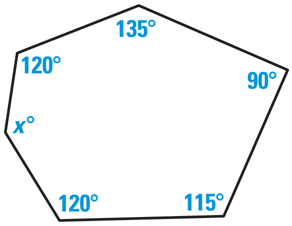


CHAPTER 11

Chapter Test

In Exercises 1 and 2, use the figure at the right.



1. What is the value of x?

2. Find the sum of the measures of the exterior angles, one at each vertex.

3. What is the measure of each interior angle of a regular 30-gon?

4. What is the measure of each exterior angle of a regular 27-gon?

In Exercises 5–8, find the area of the regular polygon to two decimal places.

5. An equilateral triangle with perimeter 30 feet

6. A regular pentagon with apothem 8 inches

7. A regular hexagon with 9 centimeter sides

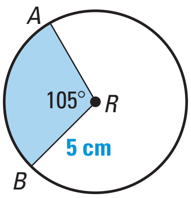
8. A regular nonagon (9-gon) with radius 1 meter

Rhombus *ABCD* has sides of length 8 centimeters. *EFGH* is a similar rhombus with sides of length 6 centimeters.

9. Find the ratio of the perimeters of *ABCD* to *EFGH*. Then find the ratio of their areas.

10. The area of *ABCD* is 56 square centimeters. Find the area of *EFGH*.

Use the diagram of $\odot R$.



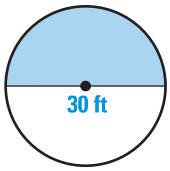
11. Find the circumference and the area of $\odot R$.

12. Find the length of $\widehat{AB}$.

13. Find the area of the sector *ARB*.

Find the area of the shaded region.

14.



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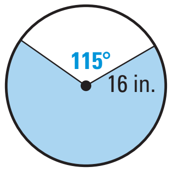

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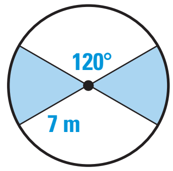
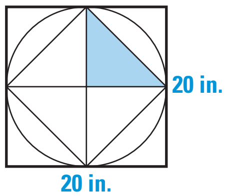


In Exercises 17 and 18, a point is chosen randomly in the 20 inch by 20 inch square at the right.



17. Find the probability that the point is inside the circle.

18. Find the probability that the point is in the shaded area.

19. **WATER-SKIER** A boat that is pulling a water-skier drives in a circle that has a radius of 80 feet. The skier is moving outside the path of the boat in a circle that has a radius of 110 feet. Find the distance traveled by the boat when it has completed a full circle. How much farther has the skier traveled?

20. **WAITING TIME** You are expecting friends to come by your house any time between 6:00 P.M. and 8:00 P.M. Meanwhile, a problem at work has delayed you. If you get home at 6:20 P.M., what is the probability that your friends are already there?