PROJECT
for Chapters 10 and 11

Geometry on a Sphere

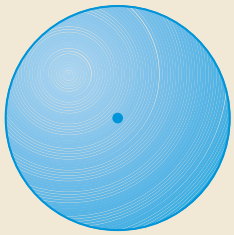
OBJECTIVE **Explore lines and triangles on a sphere, and compare geometry on a sphere to Euclidean geometry.**

Materials: round balloon and markers (or ball and rubber bands), protractor

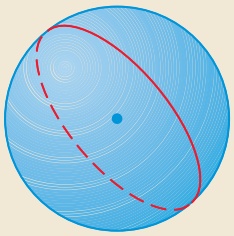
A DIFFERENT UNDERSTANDING OF PLANE AND LINE

In Euclidean geometry, you can think of a plane as a flat surface that extends forever, and you can think of a line that lies in a plane as a set of points that extends forever in two directions.

Geometry on a sphere is different: a *plane* is a spherical surface and a *line* is a special kind of circle on that surface.



a **plane** in the geometry of a sphere



a **line** in the geometry of a sphere

A *line* on a sphere can be defined as a *great circle*, that is, a circle on the sphere whose center is the center of the sphere. Circles on the sphere that do not have the same center as the sphere are not considered lines.

If you imagine Earth as a sphere, then the equator is an example of a great circle. Circles of longitude, which pass through the north and south poles, are also great circles.

INVESTIGATION

Use an inflated balloon as a sphere and use a marker to draw lines, or use a large ball and fit rubber bands around it to represent lines.

1. Draw one line on a sphere. If a point lies on the line, does the point that is opposite it on the sphere also lie on the line?
2. Draw another line on a sphere. Do your two lines intersect?
3. Is it possible to draw two lines on the sphere that do not intersect?
4. In Euclidean geometry, the Parallel Postulate states that given a line and a point not on the line, there is exactly one line through the point parallel to the given line. Does the Parallel Postulate apply to geometry on a sphere? Explain.
5. Is it possible to draw two lines on a sphere that intersect to form right angles?

TRIANGLES ON A SPHERE

If you draw three lines on a sphere, you can divide the sphere into eight 3-sided regions, which can be considered *triangles* in the geometry of a sphere.

INVESTIGATION

6. Draw three lines on a sphere. Find a triangular region enclosed by three lines. Measure the angles of the triangle with a protractor.

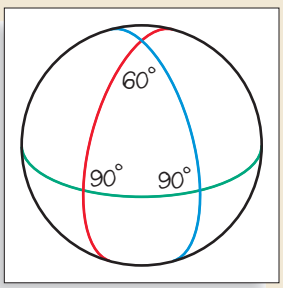
7. What is the sum of the angles of the triangle you drew on the sphere?

8. Draw three lines on a sphere so that the triangular region formed is equiangular. What are the measures of the angles?

9. Is it possible to draw a 60°-60°-60° triangle on a sphere? a 90°-90°-90° triangle? a 120°-120°-120° triangle?

10. What is the range of values for the sum of the angles of a triangle on a sphere?

PRESENT YOUR RESULTS



Make a bulletin board display of your results.

- Include a description of how *plane* and *line* are viewed differently in Euclidean geometry and the geometry of a sphere.
- Include drawings or balloons with markings of lines that intersect on a sphere.
- Summarize your results about the angles of triangles on a sphere.

EXTENSION

These mathematicians investigated non-Euclidean geometries, such as elliptic geometry and hyperbolic geometry. Research and write a report about one of them. How did the mathematician's work challenge the Parallel Postulate?

- Carl Friedrich Gauss (1777–1855)
- Nikolay Lobachevsky (1792–1856)
- János Bolyai (1802–1860)
- G. F. Bernhard Riemann (1826–1866)
- Felix Klein (1849–1925)
- David Hilbert (1862–1943)

Pilots save time and fuel by using great circle routes to travel between cities on the globe.