11.6 Geometric Probability

What *you should learn*

GOAL 1 Find a geometric probability.

GOAL 2 Use geometric probability to solve **real-life** problems, as applied in **Example 2**.

Why *you should learn it*

▼ Geometric probability is one model for calculating **real-life** probabilities, such as the probability that a bus will be waiting outside a hotel in **Ex. 28**.

REAL LIFE

GOAL 1 Finding a Geometric Probability

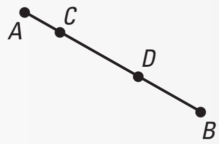
A **probability** is a number from 0 to 1 that represents the chance that an event will occur. Assuming that all outcomes are equally likely, an event with a probability of 0 *cannot* occur. An event with a probability of 1 is *certain* to occur, and an event with a probability of 0.5 is just as likely to occur as not.

In an earlier course, you may have evaluated probabilities by counting the number of favorable outcomes and dividing that number by the total number of possible outcomes. In this lesson, you will use a related process in which the division involves geometric measures such as length or area. This process is called **geometric probability**.

GEOMETRIC PROBABILITY

PROBABILITY AND LENGTH

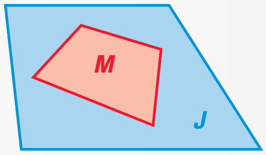
Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is as follows:



$$P(\text{Point } K \text{ is on } \overline{CD}) = \frac{\text{Length of } \overline{CD}}{\text{Length of } \overline{AB}}$$

PROBABILITY AND AREA

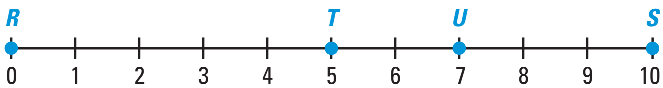
Let J be a region that contains region M. If a point K in J is chosen at random, then the probability that it is in region M is as follows:



$$P(\text{Point } K \text{ is in region } M) = \frac{\text{Area of } M}{\text{Area of } J}$$

EXAMPLE 1 *Finding a Geometric Probability*

Find the probability that a point chosen at random on $\overline{RS}$ is on $\overline{TU}$.



SOLUTION

$$P(\text{Point is on } \overline{TU}) = \frac{\text{Length of } \overline{TU}}{\text{Length of } \overline{RS}} = \frac{2}{10} = \frac{1}{5}$$

▶ The probability can be written as $\frac{1}{5}$, 0.2, or 20%.

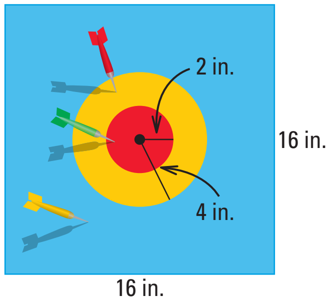
STUDENT HELP

Study Tip
When applying a formula for geometric probability, it is important that every point on the segment or in the region is equally likely to be chosen.

GOAL 2 USING GEOMETRIC PROBABILITY IN REAL LIFE

EXAMPLE 2 *Using Areas to Find a Geometric Probability*

DART BOARD A dart is tossed and hits the dart board shown. The dart is equally likely to land on any point on the dart board. Find the probability that the dart lands in the red region.



SOLUTION

Find the ratio of the area of the red region to the area of the dart board.

$$P(\text{Dart lands in red region}) = \frac{\text{Area of red region}}{\text{Area of dart board}} = \frac{\pi(2^2)}{16^2} = \frac{4\pi}{256} \approx 0.05$$

▶ The probability that the dart lands in the red region is about 0.05, or 5%.

EXAMPLE 3 *Using a Segment to Find a Geometric Probability*

TRANSPORTATION You are visiting San Francisco and are taking a trolley ride to a store on Market Street. You are supposed to meet a friend at the store at 3:00 P.M. The trolleys run every 10 minutes and the trip to the store is 8 minutes. You arrive at the trolley stop at 2:48 P.M. What is the probability that you will arrive at the store by 3:00 P.M.?

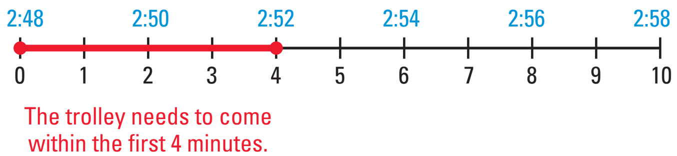
Logical Reasoning

SOLUTION

To begin, find the greatest amount of time you can afford to wait for the trolley and still get to the store by 3:00 P.M.

Because the ride takes 8 minutes, you need to catch the trolley no later than 8 minutes before 3:00 P.M., or in other words by 2:52 P.M.

So, you can afford to wait 4 minutes (2:52 − 2:48 = 4 min). You can use a line segment to model the probability that the trolley will come within 4 minutes.



STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

$$P(\text{Get to store by 3:00}) = \frac{\text{Favorable waiting time}}{\text{Maximum waiting time}} = \frac{4}{10} = \frac{2}{5}$$

▶ The probability is $\frac{2}{5}$, or 40%.

FOCUS ON CAREERS

EMPLOYMENT COUNSELORS help people make decisions about career choices. A counselor evaluates a client's interests and skills and works with the client to locate and apply for appropriate jobs.

CAREER LINK
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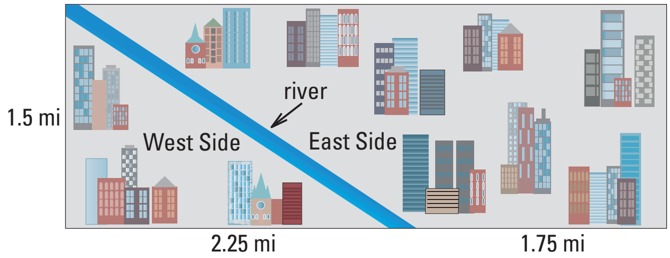
EXAMPLE 4 *Finding a Geometric Probability*

JOB LOCATION You work for a temporary employment agency. You live on the west side of town and prefer to work there. The work assignments are spread evenly throughout the rectangular region shown. Find the probability that an assignment chosen at random for you is on the west side of town.



SOLUTION

The west side of town is approximately triangular. Its area is $\frac{1}{2} \cdot 2.25 \cdot 1.5$, or about 1.69 square miles. The area of the rectangular region is $1.5 \cdot 4$, or 6 square miles. So, the probability that the assignment is on the west side of town is

$$P(\text{Assignment is on west side}) = \frac{\text{Area of west side}}{\text{Area of rectangular region}} \approx \frac{1.69}{6} \approx 0.28.$$

▶ So, the probability that the work assignment is on the west side is about 28%.

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain how a *geometric probability* is different from a *probability* found by dividing the number of favorable outcomes by the total number of possible outcomes.

Concept Check ✓

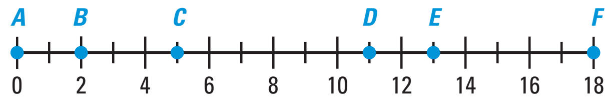
Determine whether you would use the *length method* or *area method* to find the geometric probability. Explain your reasoning.

2. The probability that an outcome lies in a triangular region

3. The probability that an outcome occurs within a certain time period

Skill Check ✓

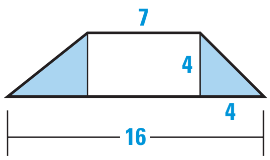
In Exercises 4–7, K is chosen at random on $\overline{AF}$. Find the probability that K is on the indicated segment.



4. $\overline{AB}$ **5.** $\overline{BD}$ **6.** $\overline{BF}$

7. Explain the relationship between your answers to Exercises 4 and 6.

8. Find the probability that a point chosen at random in the trapezoid shown lies in either of the shaded regions.

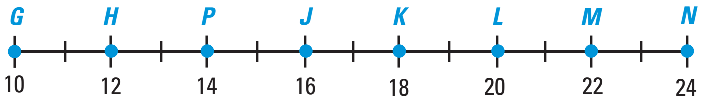


PRACTICE AND APPLICATIONS

STUDENT HELP

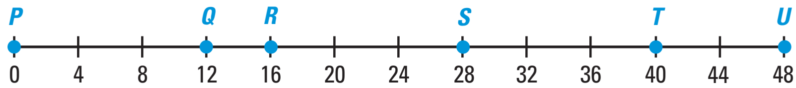
Extra Practice to help you master skills is on p. 824.

PROBABILITY ON A SEGMENT **In Exercises 9–12, find the probability that a point *A*, selected randomly on $\overline{GN}$, is on the given segment.**



9. $\overline{GH}$ **10.** $\overline{JL}$ **11.** $\overline{JN}$ **12.** $\overline{GJ}$

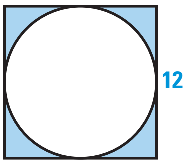
PROBABILITY ON A SEGMENT **In Exercises 13–16, find the probability that a point *K*, selected randomly on $\overline{PU}$, is on the given segment.**



13. $\overline{PQ}$ **14.** $\overline{PS}$ **15.** $\overline{SU}$ **16.** $\overline{PU}$

FINDING A GEOMETRIC PROBABILITY **Find the probability that a randomly chosen point in the figure lies in the shaded region.**

17.



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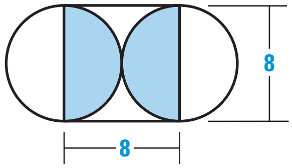

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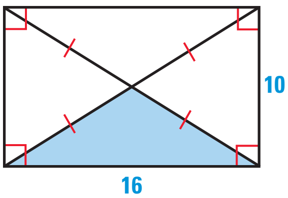

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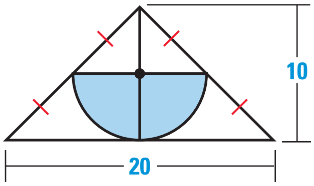
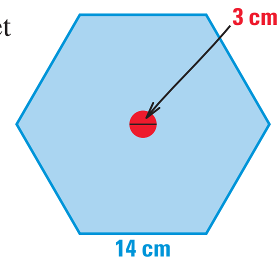


TARGETS **A regular hexagonal shaped target with sides of length 14 centimeters has a circular bull's eye with a diameter of 3 centimeters. In Exercises 21–23, darts are thrown and hit the target at random.**

21. What is the probability that a dart that hits the target will hit the bull's eye?

22. Estimate how many times a dart will hit the bull's eye if 100 darts hit the target.

23. Find the probability that a dart will hit the bull's eye if the bull's eye's radius is doubled.



24. **LOGICAL REASONING** The midpoint of $\overline{JK}$ is M. What is the probability that a randomly selected point on $\overline{JK}$ is closer to M than to J or to K?

25. **LOGICAL REASONING** A circle with radius $\sqrt{2}$ units is circumscribed about a square with side length 2 units. Find the probability that a randomly chosen point will be inside the circle but outside the square.

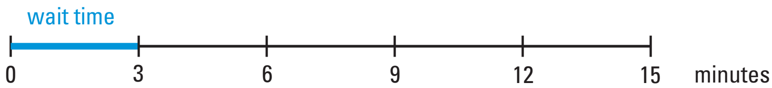
STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 9–16
Example 2: Exs. 17–23, 29–34
Example 3: Exs. 26–28
Example 4: Exs. 40–42

26. **FIRE ALARM** Suppose that your school day begins at 7:30 A.M. and ends at 3:00 P.M. You eat lunch at 11:00 A.M. If there is a fire drill at a random time during the day, what is the probability that it begins before lunch?

27. **PHONE CALL** You are expecting a call from a friend anytime between 6:00 P.M. and 7:00 P.M. Unexpectedly, you have to run an errand for a relative and are gone from 5:45 P.M. until 6:10 P.M. What is the probability that you missed your friend's call?

28. **TRANSPORTATION** Buses arrive at a resort hotel every 15 minutes. They wait for three minutes while passengers get on and get off, and then the buses depart. What is the probability that there is a bus waiting when a hotel guest walks out of the door at a randomly chosen time?



FOCUS ON APPLICATIONS

SHIP SALVAGE Searchers for sunken items such as ships, planes, or even a space capsule, use charts, sonar, and video cameras in their search and recovery expeditions.

APPLICATION LINK
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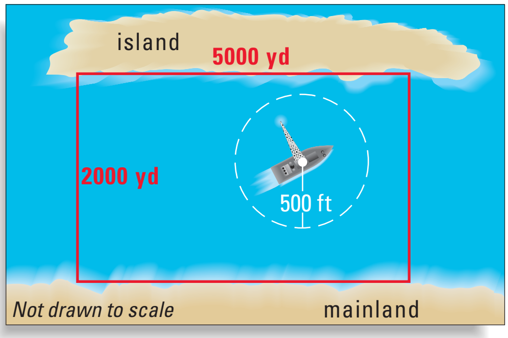
SHIP SALVAGE **In Exercises 29 and 30, use the following information.**
A ship is known to have sunk off the coast, between an island and the mainland as shown. A salvage vessel anchors at a random spot in this rectangular region for divers to search for the ship.

29. Find the approximate area of the rectangular region where the ship sank.

30. The divers search 500 feet in all directions from a point on the ocean floor directly below the salvage vessel. Estimate the probability that the divers will find the sunken ship on the first try.



ARCHERY **In Exercises 31–35, use the following information.**
Imagine that an arrow hitting the target shown is equally likely to hit any point on the target. The 10-point circle has a 4.8 inch diameter and each of the other rings is 2.4 inches wide. Find the probability that the arrow hits the region described.

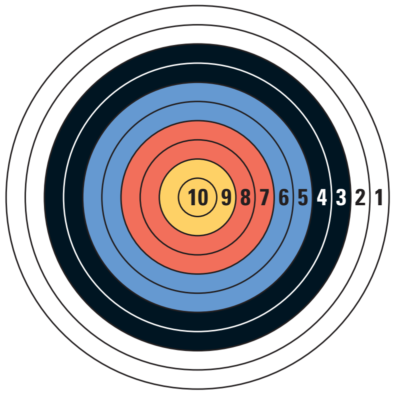
31. The 10-point region

32. The yellow region

33. The white region

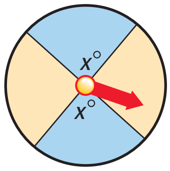
34. The 5-point region

35. **CRITICAL THINKING** Does the geometric probability model hold true when an expert archer shoots an arrow? Explain your reasoning.



36. **USING ALGEBRA** If $0 < y < 1$ and $0 < x < 1$, find the probability that $y < x$. Begin by sketching the graph, and then use the *area method* to find the probability.

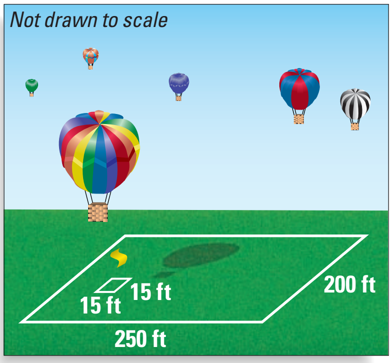
USING ALGEBRA Find the value of x so that the probability of the spinner landing on a blue sector is the value given.



37. $\frac{1}{3}$ **38.** $\frac{1}{4}$ **39.** $\frac{1}{6}$

BALLOON RACE In Exercises 40–42, use the following information.
In a "Hare and Hounds" balloon race, one balloon (the hare) leaves the ground first. About ten minutes later, the other balloons (the hounds) leave. The hare then lands and marks a square region as the target. The hounds each try to drop a marker in the target zone.

40. Suppose that a hound's marker dropped onto a rectangular field that is 200 feet by 250 feet is equally likely to land anywhere in the field. The target region is a 15 foot square that lies in the field. What is the probability that the marker lands in the target region?



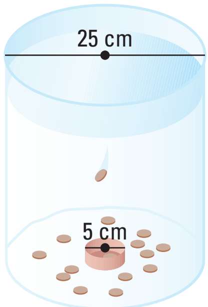
41. If the area of the target region is doubled, how does the probability change?

42. If each side of the target region is doubled, how does the probability change?

Test Preparation

43. MULTI-STEP PROBLEM Use the following information.
You organize a fund-raiser at your school. You fill a large glass jar that has a 25 centimeter diameter with water. You place a dish that has a 5 centimeter diameter at the bottom of the jar. A person donates a coin by dropping it in the jar. If the coin lands in the dish, the person wins a small prize.

a. Calculate the probability that a coin dropped, with an equally likely chance of landing anywhere at the bottom of the jar, lands in the dish.



b. Use the probability in part (a) to estimate the average number of coins needed to win a prize.

c. From past experience, you expect about 250 people to donate 5 coins each. How many prizes should you buy?

d. *Writing* Suppose that instead of the dish, a circle with a diameter of 5 centimeters is painted on the bottom of the jar, and any coin touching the circle wins a prize. Will the probability change? Explain.

★ **Challenge**

44. USING ALGEBRA Graph the lines $y = x$ and $y = 3$ in a coordinate plane. A point is chosen randomly from within the boundaries $0 < y < 4$ and $0 < x < 4$. Find the probability that the coordinates of the point are a solution of this system of inequalities:

$$y < 3$$
$$y > x$$

EXTRA CHALLENGE
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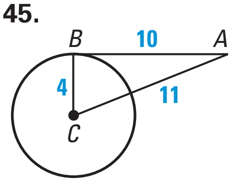
MIXED REVIEW

DETERMINING TANGENCY **Tell whether $\overleftrightarrow{AB}$ is tangent to $\odot C$. Explain your reasoning.** **(Review 10.1)**

45.



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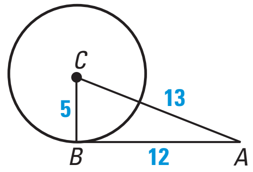

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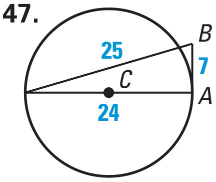


DESCRIBING LINES **In Exercises 48–51, graph the line with the circle $(x - 2)^2 + (y + 4)^2 = 16$. Is the line a *tangent* or a *secant*?** **(Review 10.6)**

48. $x = -y$

49. $y = 0$

50. $x = 6$

51. $y = x - 1$

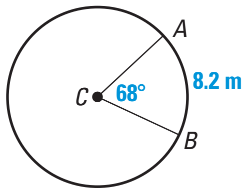
52. **LOCUS** Find the locus of all points in the coordinate plane that are equidistant from points $(3, 2)$ and $(1, 2)$ and within $\sqrt{2}$ units of the point $(1, -1)$. **(Review 10.7)**

QUIZ 2

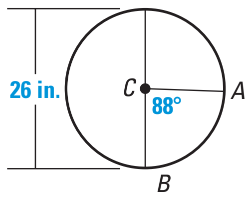
Self-Test for Lessons 11.4–11.6

Find the indicated measure. **(Lesson 11.4)**

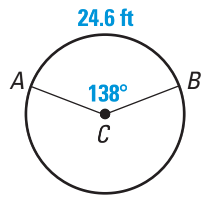
1. Circumference



2. Length of $\overset{\frown}{AB}$

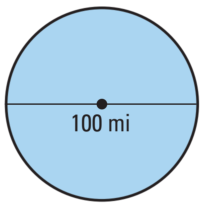


3. Radius



In Exercises 4–6, find the area of the shaded region. **(Lesson 11.5)**

4.



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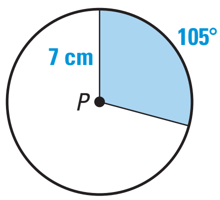
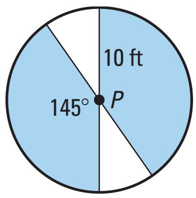


6.



7. **TARGETS** A square target with 20 cm sides includes a triangular region with equal side lengths of 5 cm. A dart is thrown and hits the target at random. Find the probability that the dart hits the triangle. **(Lesson 11.6)**